Quicksort Revisited

Corky Cartwright

Vivek Sarkar

Department of Computer Science

Rice University

Acknowledgments

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www.cis.upenn.edu/~matuszek/cit594-2003/.../34-quicksort.ppt

The Sorting Problem

Given a array of n objects (records) R, construct an array R’ containing the same set of records as R but in ascending (non-descending) order according to a specified comparison function

Example of comparison function: compareTo() method for Java objects that implement the Comparable interface

Quick Sort (Lecture 11)

- Invented by C.A.R. ("Tony") Hoare
- Functional version is derived from the imperative (destructive) algorithm; less efficient but still works very well
- Idea:
  - Base case: list of length 0 or 1
  - Inductive case:
  - partition the list into the singleton list containing first, the list of all items <= first, and the list of all items > first
  - sort the the lists of lesser and greater items
  - return (sorted lesser) || (first) || (sorted greater) where || means list concatenation (append)

Quicksort in Scheme (Lecture 11)

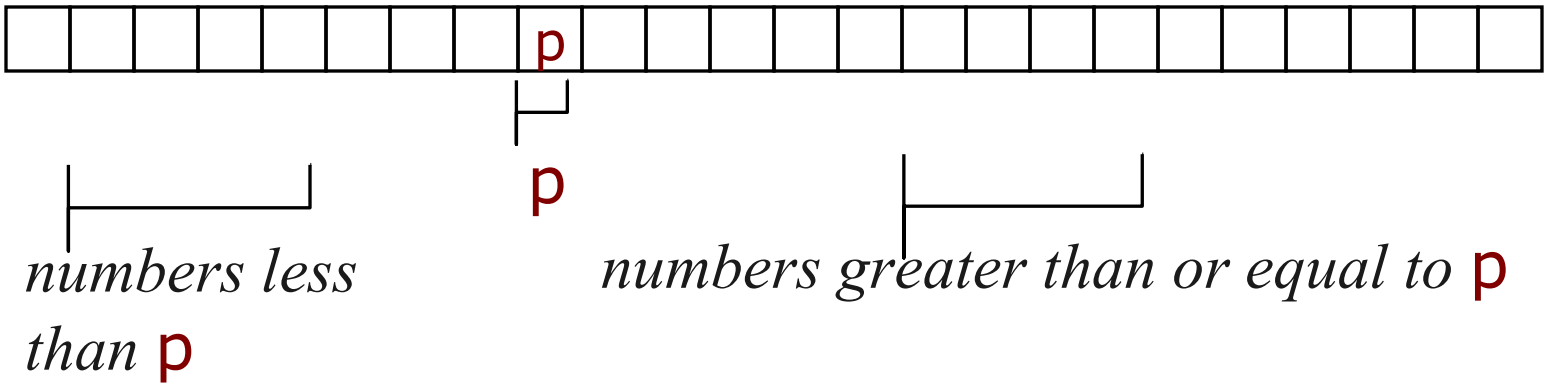
```
(define (qsort alon)
  (cond
    [(empty? alon) empty]
    [else
      (local ((define pivot (first alon))
              (define other (rest alon)))
        (append
          (qsort [filter (lambda (x) (<= x pivot)) other])
          (list pivot)
          (qsort [filter (lambda (x) (> x pivot)) other])))]))
```

Partitioning

A key step in the Quicksort algorithm is partitioning the array

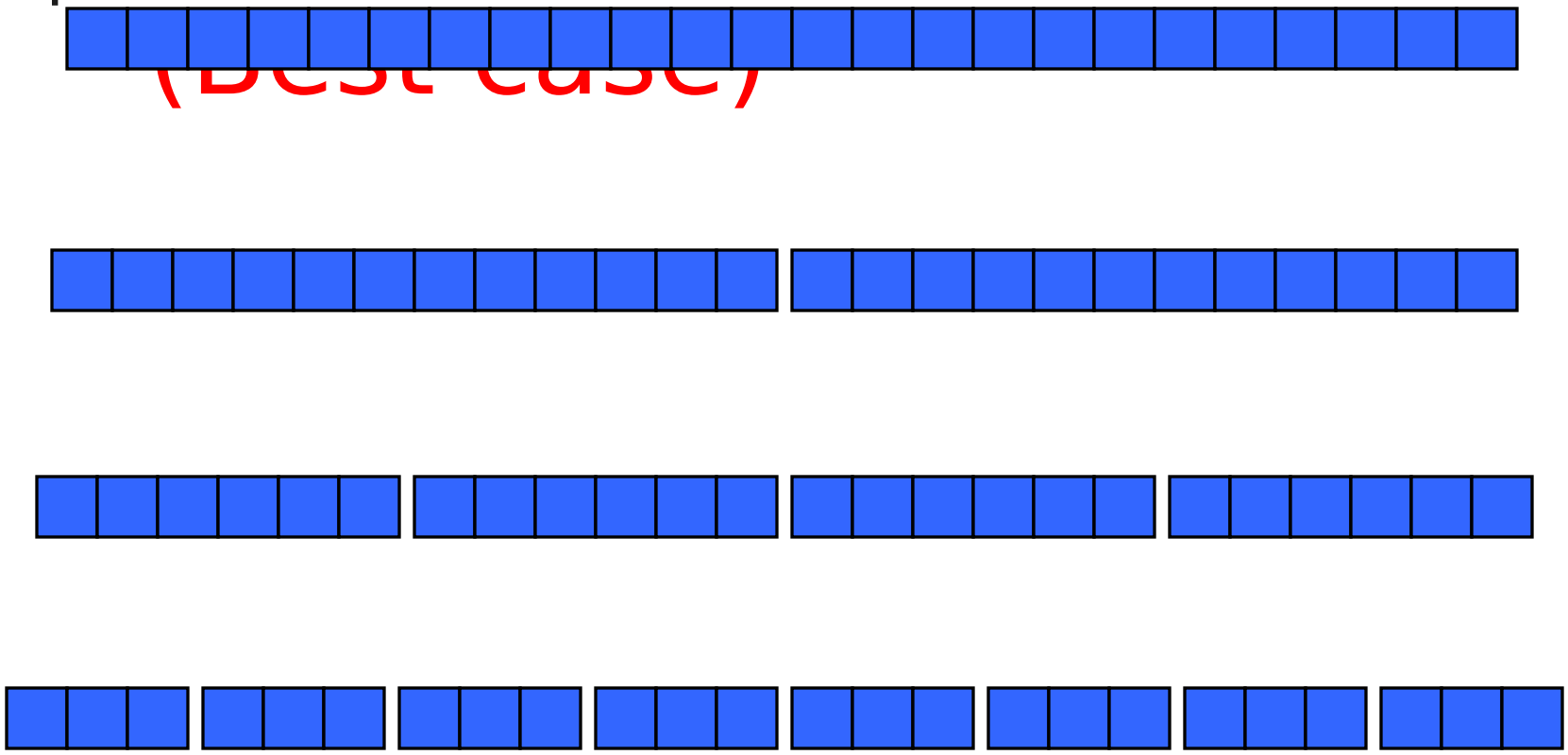
We choose some (any) number p in the array to use as a pivot

We partition the array into three parts:



Partitioning at various levels

(Best case)



Best case Partitioning

We cut the array size in half each time

So the depth of the recursion in log2n

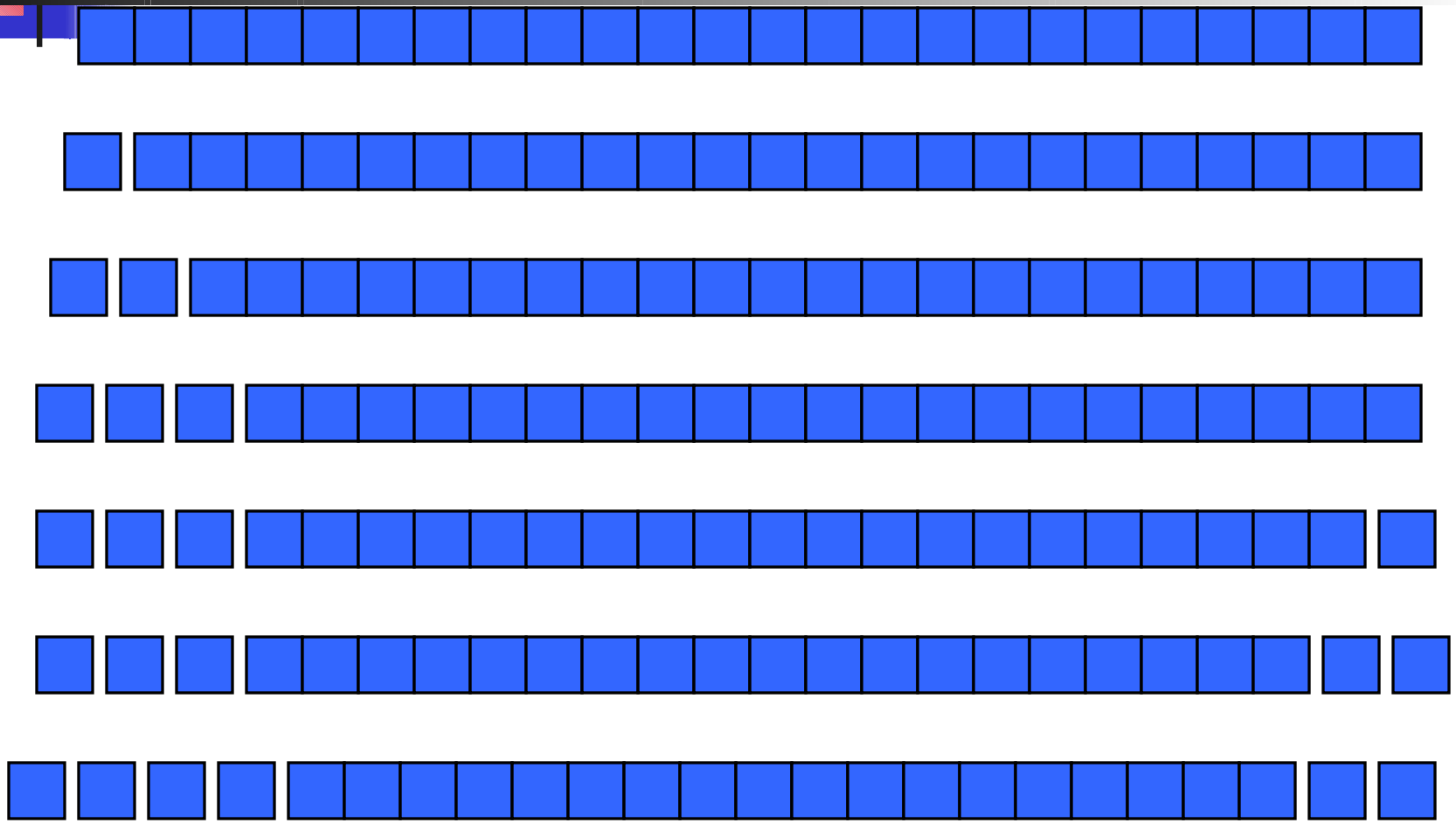
At each level of the recursion, all the partitions at that level do work that is linear in n

O(log2n) * O(n) = O(n log2n)

Hence in the best case, quicksort has time complexity O(n log2n)

What about the worst case?

Worst case partitioning



Worst case for quicksort

In the worst case, recursion may be n levels deep (for an array of size n)

But the partitioning work done at each level is still n

O(n) * O(n) = O(n^2)

So worst case for Quicksort is O(n^2)

When can this happen?

e.g., when the array is sorted to begin with!

Typical case for quicksort

If the array is sorted to begin with, Quicksort is terrible: O(n2)

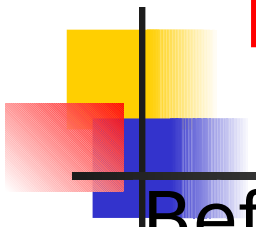
It is possible to construct other bad cases

However, Quicksort is *usually* O(n log2n)

The constants are so good that Quicksort is generally the fastest algorithm known

A lot of real-world sorting is done by Quicksort

Picking a better pivot



- Before, we picked the *first* element of the subarray to use as a pivot
  - If the array is already sorted, this results in O(n2) behavior
  - It's no better if we pick the *last* element
- We could do an *optimal* quicksort (guaranteed O(n log n)) if we always picked a pivot value that exactly cuts the array in half
  - Such a value is called a median: half of the values in the array are larger, half are smaller
  - The easiest way to find the median is to *sort* the array and pick the value in the middle (!)

Median of three

Obviously, it doesn't make sense to sort the array in order to find the median to use as a pivot

There are faster more advanced algorithms to find the median that we'll ignore for today

Instead, compare just *three* elements of our (sub)array—the first, the last, and the middle

Take the *median* (middle value) of these three as pivot

It's possible (but less likely) to construct cases which will make this technique O(n2)

For simplicity, we will continue with first element as pivot in the rest of this lecture.

Functional version of Quicksort in Java

```
public static ArrayList<Integer> quickSort(ArrayList<Integer> a) {
    if (a.isEmpty()) return new ArrayList<Integer>();
    ArrayList<Integer> left = new ArrayList<Integer>();
    ArrayList<Integer> mid = new ArrayList<Integer>();
    ArrayList<Integer> right = new ArrayList<Integer>();
    for (Integer i : a)
        if ( i < a.get(0) ) left.add(i); // Use element 0 as pivot
        else if ( i > a.get(0)) right.add(i);
        else mid.add(i)
    ArrayList<Integer> left_s = quickSort(left);
    ArrayList<Integer> right_s = quickSort(right);
    return left_s.addAll(mid).addAll(right_s);
}
```

Reprise: Task Decomposition

```
public static ArrayList<Integer> quickSort(ArrayList<Integer> a) {
    if (a.isEmpty()) return new ArrayList<Integer>();
    ArrayList<Integer> left = new ArrayList<Integer>();
    ArrayList<Integer> mid = new ArrayList<Integer>();
    ArrayList<Integer> right = new ArrayList<Integer>();
    for (Integer i : a)
        if ( i < a.get(0) ) left.add(i); // Use element 0 as pivot
        else if ( i > a.get(0)) right.add(i);
        else mid.add(i)
    final ArrayList<Integer> left_f = left, right_f = right; // QUESTION: why do we need these?
    Callable<ArrayList<Integer>> left_c = new Callable<ArrayList<Integer>>() {
        public ArrayList<Integer> call() { return quickSort(left_f); } } ;
    Callable<ArrayList<Integer>> right_c = new Callable<ArrayList<Integer>>() {
        public ArrayList<Integer> call() { return quickSort(right_f); } } ;
   // QUESTION: where can we place left_c.call() and right_c.call()?
  . . .
}
```

Original Task Order

```
public static ArrayList<Integer> quickSort(ArrayList<Integer> a) {
    if (a.isEmpty()) return new ArrayList<Integer>();
    ArrayList<Integer> left = new ArrayList<Integer>();
    ArrayList<Integer> mid = new ArrayList<Integer>();
    ArrayList<Integer> right = new ArrayList<Integer>();
    for (Integer i : a)
       if ( i < a.get(0) ) left.add(i); // Use element 0 as pivot
       else if ( i > a.get(0)) right.add(i);
       else mid.add(i)
    final ArrayList<Integer> left_f = left, right_f = right;
   Callable<ArrayList<Integer>> left_c = new Callable<ArrayList<Integer>>() {
        public ArrayList<Integer> call() { return quickSort(left_f); } } ;
    Callable<ArrayList<Integer>> right_c = new Callable<ArrayList<Integer>>() {
        public ArrayList<Integer> call() { return quickSort(right_f); } } ;
   ArrayList<Integer> left_s = left_s.call(); ArrayList<Integer> right_s = right_c.call();
    return left_s.addAll(mid).addAll(right_s);
}
```

Alternate Task Order

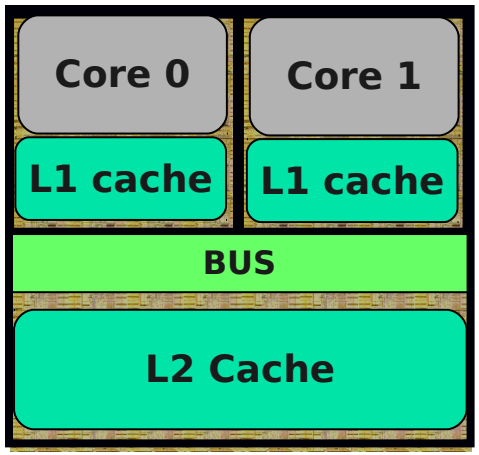
```
public static ArrayList<Integer> quickSort(ArrayList<Integer> a) {
    if (a.isEmpty()) return new ArrayList<Integer>();
    ArrayList<Integer> left = new ArrayList<Integer>();
    ArrayList<Integer> mid = new ArrayList<Integer>();
    ArrayList<Integer> right = new ArrayList<Integer>();
    for (Integer i : a)
       if ( i < a.get(0) ) left.add(i); // Use element 0 as pivot
       else if ( i > a.get(0)) right.add(i);
       else mid.add(i)
    final ArrayList<Integer> left_f = left, right_f = right;
    Callable<ArrayList<Integer>> left_c = new Callable<ArrayList<Integer>>() {
        public ArrayList<Integer> call() { return quickSort(left_f); } } ;
    Callable<ArrayList<Integer>> right_c = new Callable<ArrayList<Integer>>() {
        public ArrayList<Integer> call() { return quickSort(right_f); } } ;
    ArrayList<Integer> right_s = right_c.call(); ArrayList<Integer> left_s = left_s.call();
    return left_s.addAll(mid).addAll(right_s);
}
```

From Sequential to Parallel Task Decomposition

Key Observation:

If two *functional* tasks can be executed in any order, they can also be executed *in parallel*

Task A Task B



Schematic of a Dual-core Processor

How can we express Task Parallelism in Java?

Answer: there are many ways, but they all ultimately involve execution on Java *threads*

The Java main program starts as a single thread

The code executed by the main thread can create other threads

Either explicitly (as in the following slides); or

Implicitly via library use:

- AWT/Swing, Applets, RMI, image loading, Servlets, web services, `Executor usage (thread pools), …`

Executing a Callable task in a parallel Java Thread

```
// 1. Create a callable closure (lambda)
Callable<ArrayList<Integer>> left_c = ...
// 2. Package the closure as a task
final FutureTask<ArrayList<Integer>> task_A =
    new FutureTask<ArrayList<Integer>>(left_c);
// 3. Start executing the task in a parallel thread
new Thread(task_A).start();
// 4. Wait for task to complete, and get its result
left_s = task_A.get();
```

Quicksort with Parallel Tasks

```
public static ArrayList<Integer> quickSort(ArrayList<Integer> a) {
     if (a.isEmpty()) return new ArrayList<Integer>();
    ArrayList<Integer> left = new ArrayList<Integer>();
    ArrayList<Integer> mid = new ArrayList<Integer>();
    ArrayList<Integer> right = new ArrayList<Integer>();
    for (Integer i : a)
       if ( i < a.get(0) ) left.add(i); // Use element 0 as pivot
       else if ( i > a.get(0)) right.add(i);
       else mid.add(i)
    final ArrayList<Integer> left_f = left, right_f = right;
    FutureTask<ArrayList<Integer>> left_t = new FutureTask<ArrayList<Integer>>(
       new Callable<ArrayList<Integer>>() {
          public ArrayList<Integer> call() { return quickSort(left_f); } } );
    FutureTask<ArrayList<Integer>> right_t = new FutureTask<ArrayList<Integer>>(
       new Callable<ArrayList<Integer>>() {
           public ArrayList<Integer> call() { return quickSort(right_f); } } );
   new Thread(left_t).start(); new Thread(right_t).start();
   ArrayList<Integer> left_s = left_t.get(); ArrayList<Integer> right_s = right_t.call();
    return left_s.addAll(mid).addAll(right_s);
}
```

Discussion

Why must the tasks be functional? What would happen if two parallel tasks attempted to mutate the same object?

It is strongly recommended that each FutureTask declaration be final. Why? Can you create a cyclic wait structure with blocking get() operations?

Sometimes, a parallel program may run slower than a sequential program. Why? Note that it can take a large number (> 104) of machine instructions just to create a thread.