Design Patterns for Self-Balancing Trees

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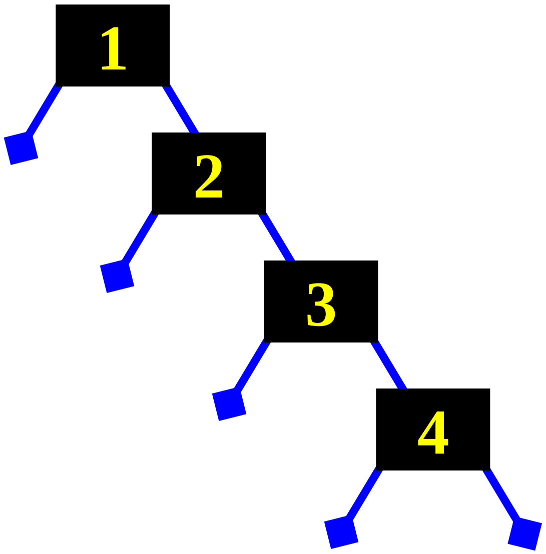
Motivations

- A binary search tree of n elements can be skewed resulting in O(n) search.



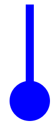
Need a way to maintain the tree's balance in order to guarantee O(logn) search.

Balanced Trees

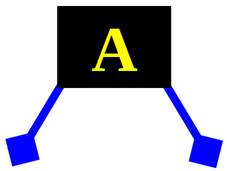
- Classic balanced tree structures
  - 2-3-4 tree (see next slide)
  - red-black tree (binary tree equivalent of 2-3-4 tree)
  - B-tree (generalized 2-3-4 tree)
  - Difficult and complex. Where's the code?
- What's the proper abstraction?
  - Need to decouple algorithms from data structures.

A 2-3-4 Tree is...

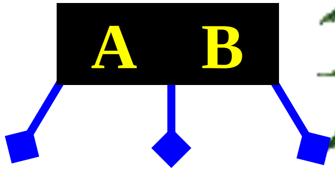
- **Empty**
  - **0-State: no data element + no sub-trees.**



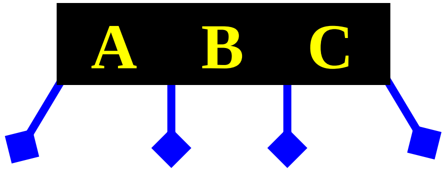
- **Non-Empty, in 3 possible states:**
  - **1-State: 1 data element + 2 sub-trees.**



  - **2-State: 2 data elements + 3 sub-trees.**



  - **3-State: 3 data elements + 4 sub-trees.**

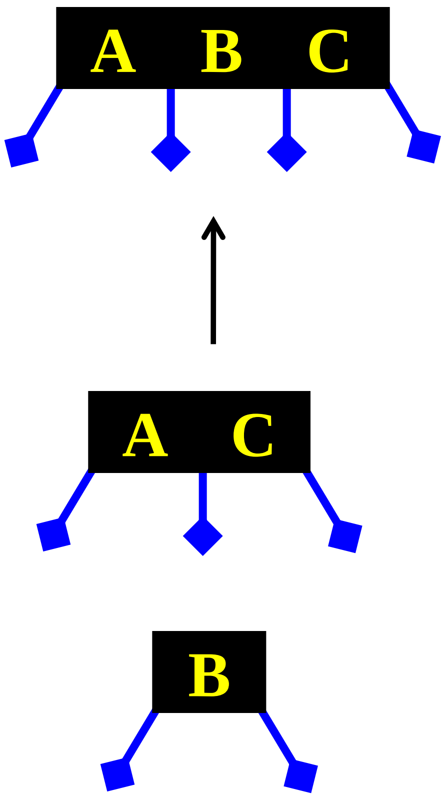


Variant *vs.* Invariant Operations

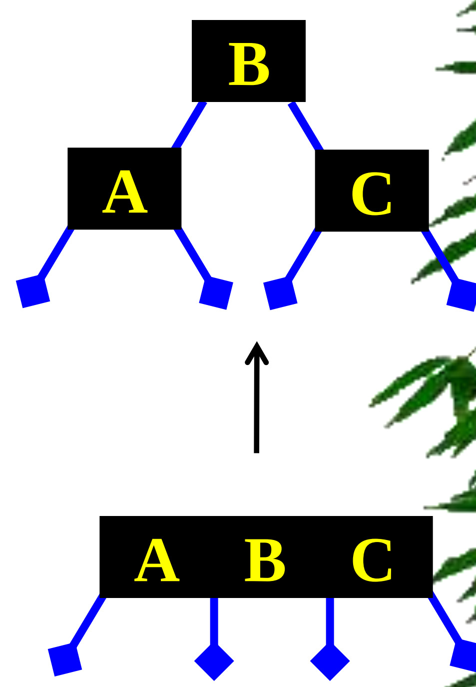
- Self-balancing insertion is <u>not</u> an intrinsic (invariant) operation of a tree.
- **What are the invariant operations?**
  - Gettors & Constructors.
  - Constructive and Destructive operations:
    - ***Constructive*: Splice a tree into another.**
    - ***Destructive*: Split a tree into a 2-state.**

Splittin' and Splicin'

Splice:

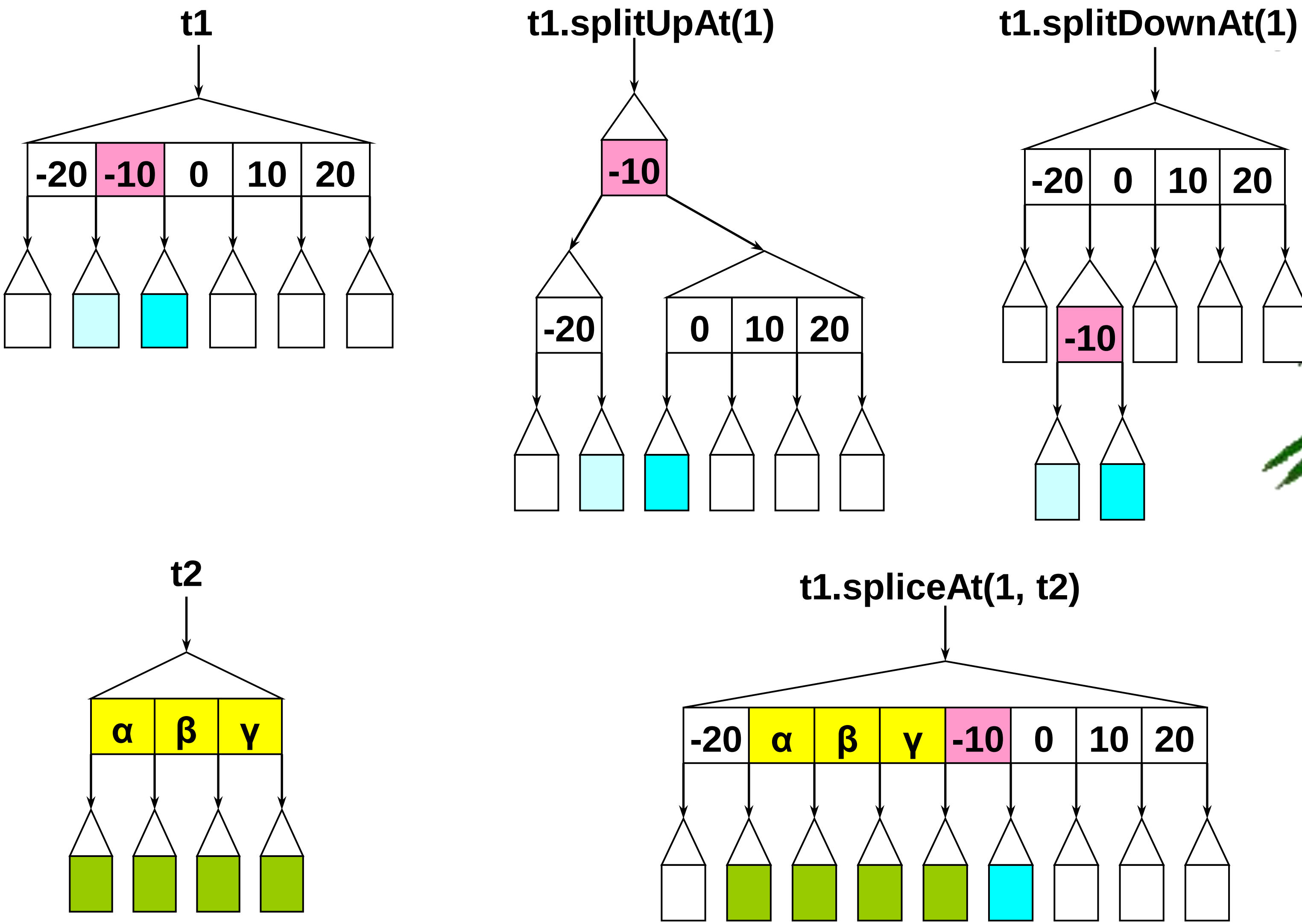


Split Up:

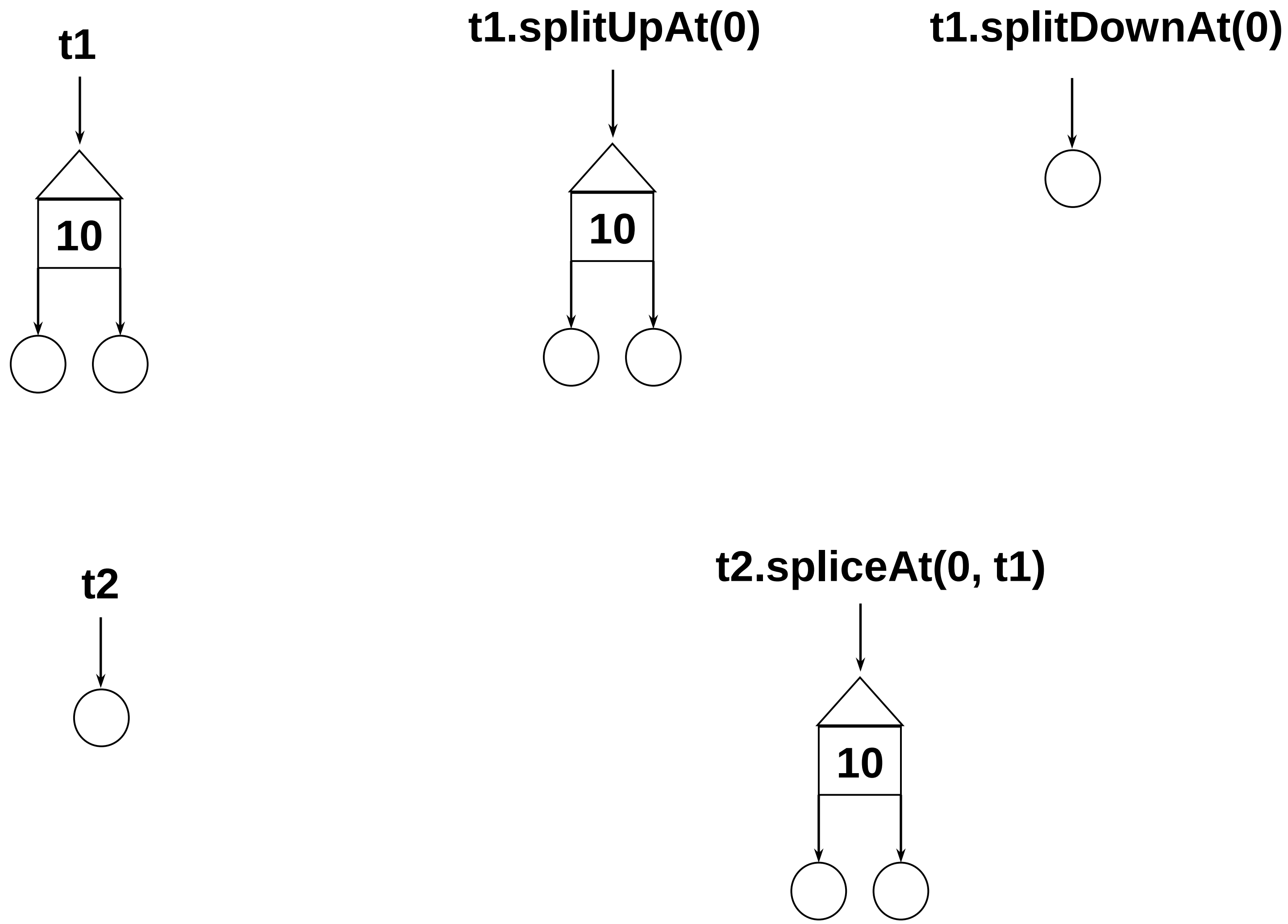


Intrinsic operations on the tree STRUCTURE, not the data!

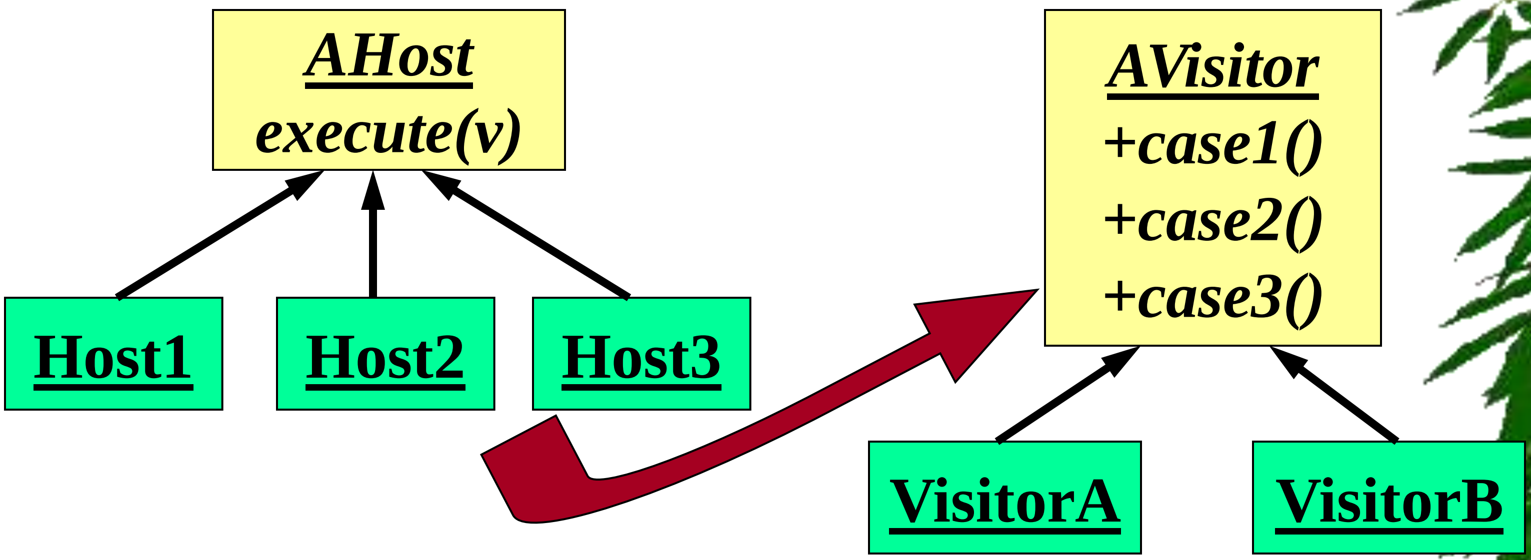
Structural Operations



Con/De-struction



Visitor Design Pattern

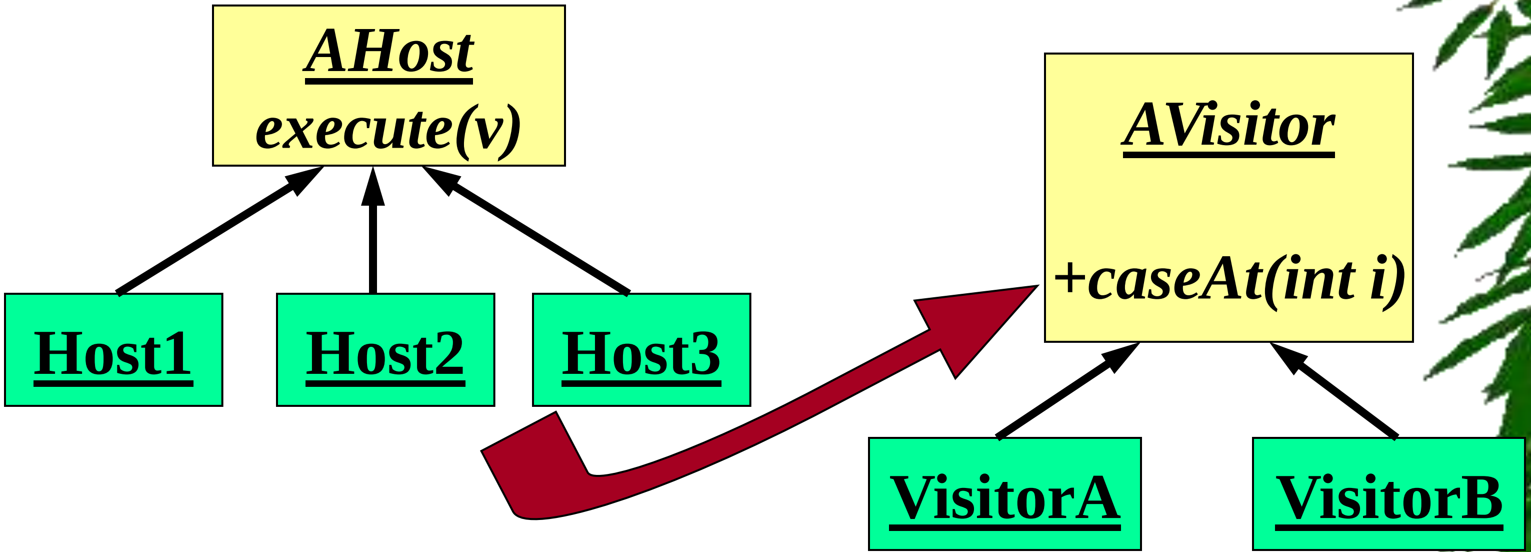


<u>Invariant</u>: Host$_i$ calls case$_i$ of the visitor.

Fixed # of methods → fixed # of hosts

Non-Extensible!

Generalized Visitors

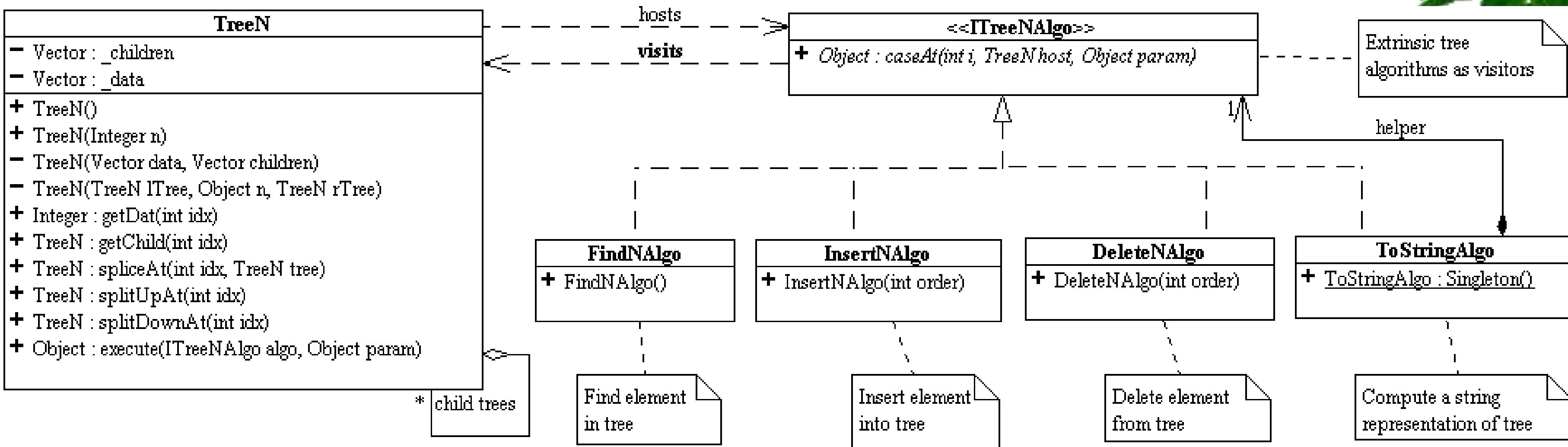


Invariant: Host$_i$ calls caseAt(i) of the visitor.

Unbounded # of hosts!

TreeN and Algorithms

Visitor Design Pattern



Composite Design Pattern: A non-empty TreeN has sub-trees that are TreeN.

toString() Algorithm

```
public class ToStringAlgo implements ITreeNAlgo {
   // Constructors omitted

   public Object caseAt(int idx, TreeN host
      switch(idx) {
         case 0: {  return "[ ]";  }
         default: {
            String sData= "", sTrees="";
            for(int i = 0;i<idx;i++) {
              sData += host.getDat(i)+" ";
              sTrees += host.getChild(i).execute(toStringHelp,"|  ")+"\n";
             }
             sTrees += host.getChild(idx).execute(toStringHelp,"   ").toString();
             return sData +"\n"+sTrees;
           }
        }
   }
   ITreeNAlgo toStringHelp =  …see next slide….
}
```

Empty Tree

Non-Empty Tree

“Prefix” data

ToString() Helper

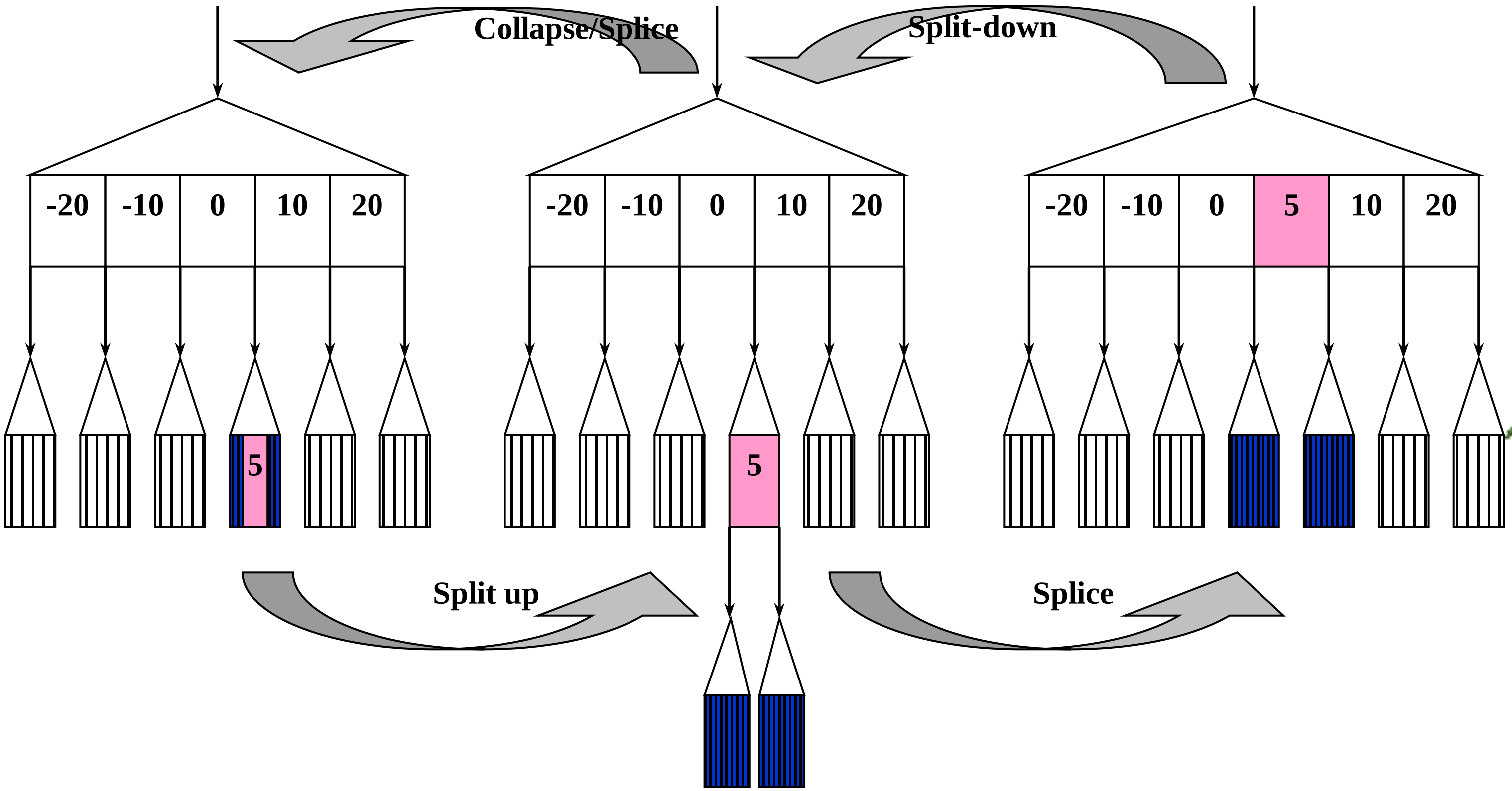
```
private final static ITreeNAlgo toStringHelp = new ITreeNAlgo() {
   public Object caseAt(int idx, TreeN host,
      switch(idx) {
         case 0: {  return "|_[ ]"; }
         default: {
            String sData= "", sTrees="";
            for(int i = 0;i<idx;i++) {
               sData += host.getDat(i)+" ";
               sTrees += prefix
                             + (String) host.getChild(i).execute(this, prefix+"|  ")+"\n";
            }
            sTrees += prefix
                          + host.getChild(idx).execute(this, prefix+"   " ).toString();
            return "|_ "+sData +"\n"+sTrees;
         }
      }
   }
};
```

Empty Tree

Non-Empty Tree

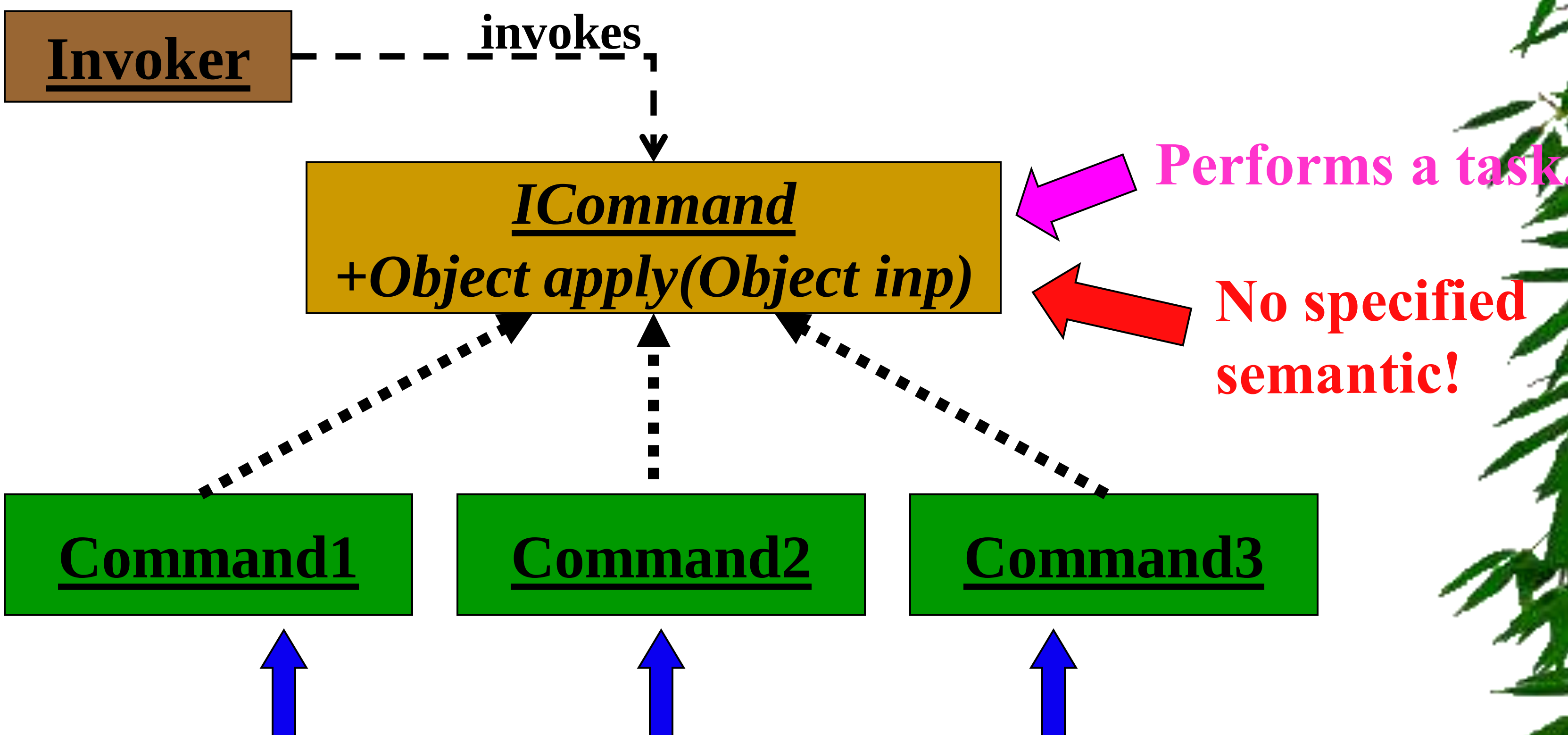
“Prefix” data

Vertical Data Transport



No net height change except at root and leaves!

Command Design Pattern



Well-defined, but unrelated semantics.

Commands = Lambda Functions

Anonymous inner classes provide closures for lambdas!

Insertion Heuristics

- Insertion must take place at the leaf.
- Tree must grow only at the root.

Must transport data
from the leaves to the root
without affecting the height balance.

Problem: If a child node is too wide, it needs to split up and splice into its parent, but…

- The child node does not know *where* to splice into its parent
- The child does not even have a reference to its parent.

Solution: Pass a command (lambda) forward from the parent to the child during the recursive call.

Split-up and Splice (Apply)

Max width of node

```
class SplitUpAndApply implements ITreeNAlgo {
    int _order;
    public SplitUpAndApply(int order) {_order = order;}

    public Object caseAt(int i, TreeN host, Object param) {
        if(i <= _order) return host;
        else {
            host.splitUpAt(i / 2);
            return ((ILambda)param).apply(host); }
    }
}
```

Not too wide? → no-op

Too wide? → Split up then apply lambda

The lambda splices this child into its parent.

Lambda/commands enable decoupled communication!

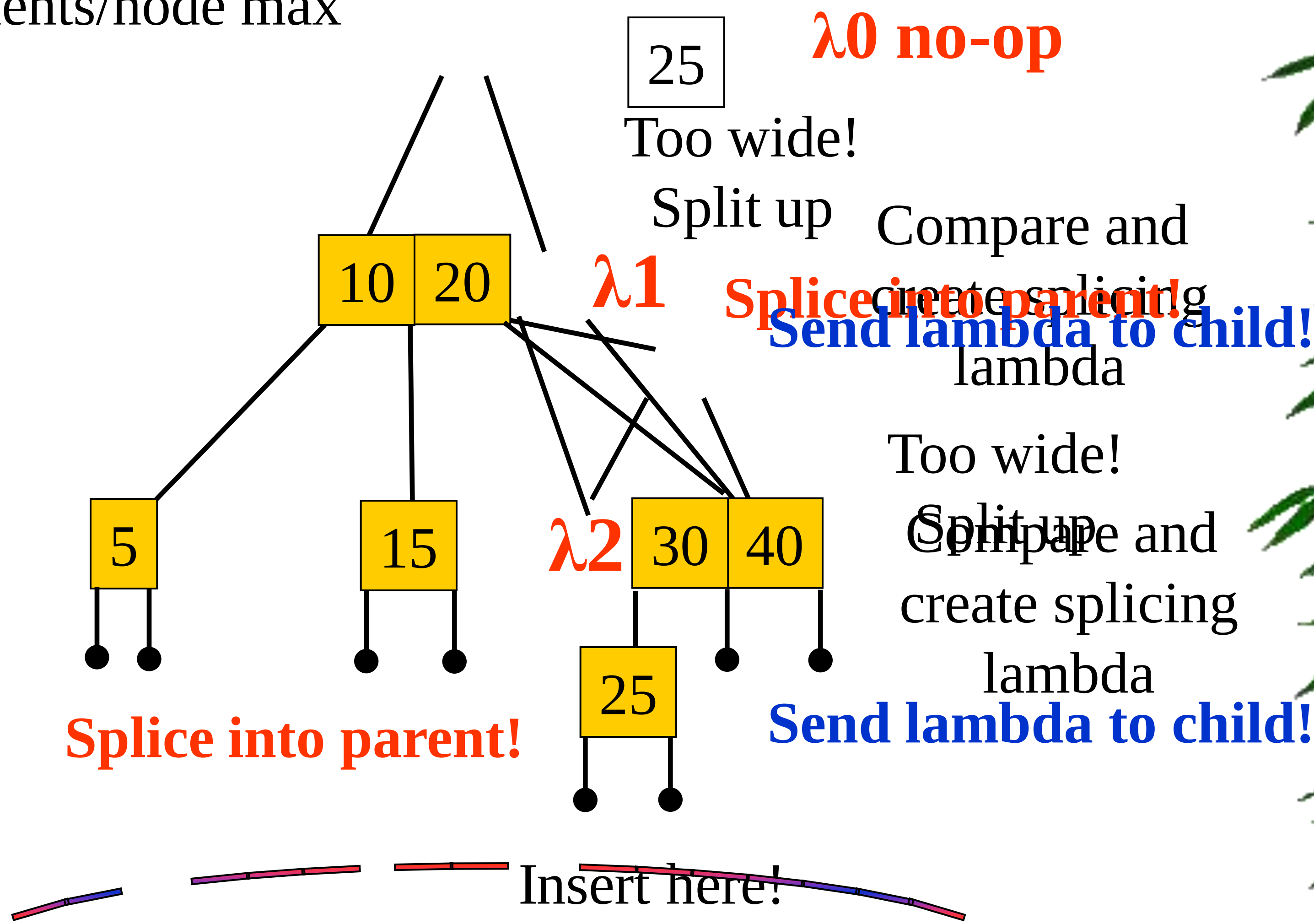
Insertion Algorithm

- Find insertion point at the leaf and splice new data in.
- Use Split-and-Apply visitor to transport excess data upwards.
  - Visitor passed as parameter to recursive call.
  - Non-root: split-and-splice
  - Root node: split-and-no-op will cause entire tree to grow in height.
  - Abstract the splice/no-op as a command passed to the visitor!

Lambdas simplify code!

Insertion Dynamics

2 elements/node max



```
public Object caseAt(int s, final TreeN host, final Object key) {
  switch(s) {
    case 0: {  return host.spliceAt(0, new TreeN((Integer) key)); }
    default: {
      host.execute(new ITreeNAlgo() {
        public Object caseAt(int s_help, final TreeN h, final Object cmd) {
          switch(s_help) {
            case 0: { return ((ILambda)cmd).apply(new TreeN((Integer)key)) ;}
            default: {
              final int[] x={0};  // hack to get around final
              for(; x[0] < s_help; x[0]++) {
                int d = h.getDat(x[0]);
                if (d >= (Integer)key) {
                  if (d == (Integer)key)) return h; // no duplicate keys
                  else break;  } }
              h.getChild(x[0]).execute(this, new ILambda() {
                public Object apply(Object child) {
                  return h.spliceAt(x[0], (TreeN) child); } } );
              return h.execute(splitUpAndSplice, cmd); } } } },
      new ILambda() {
        public Object apply(Object child){ return host; }} );
    return host;  } } }
```

Find the insertion/splice location

x[0] has the splice location

Recurse into the child, passing on the splicing lambda

The beauty of closure!

O(log n) insertion!

Deletion Heuristics

- Deletion only well-defined at leaf.
- Data might exist anywhere in the tree.
- Tree can only shorten at root.

→ **Push "candidate" data down from the root to the leaves.**

→ **Bubble "excess" data back to the root.**

Must transport data
from the root to the leaves and
from the leaves to the root
without affecting the height balance.

Deletion Algorithm

- Identify candidate data
  - split down at candidate and collapse with children.
  - If root is a 2-node, then tree will shorten.
- Data to delete will appear as 2-node below leaves.
- Use Split-and-Apply to transport excess data upwards.

No Rotations!

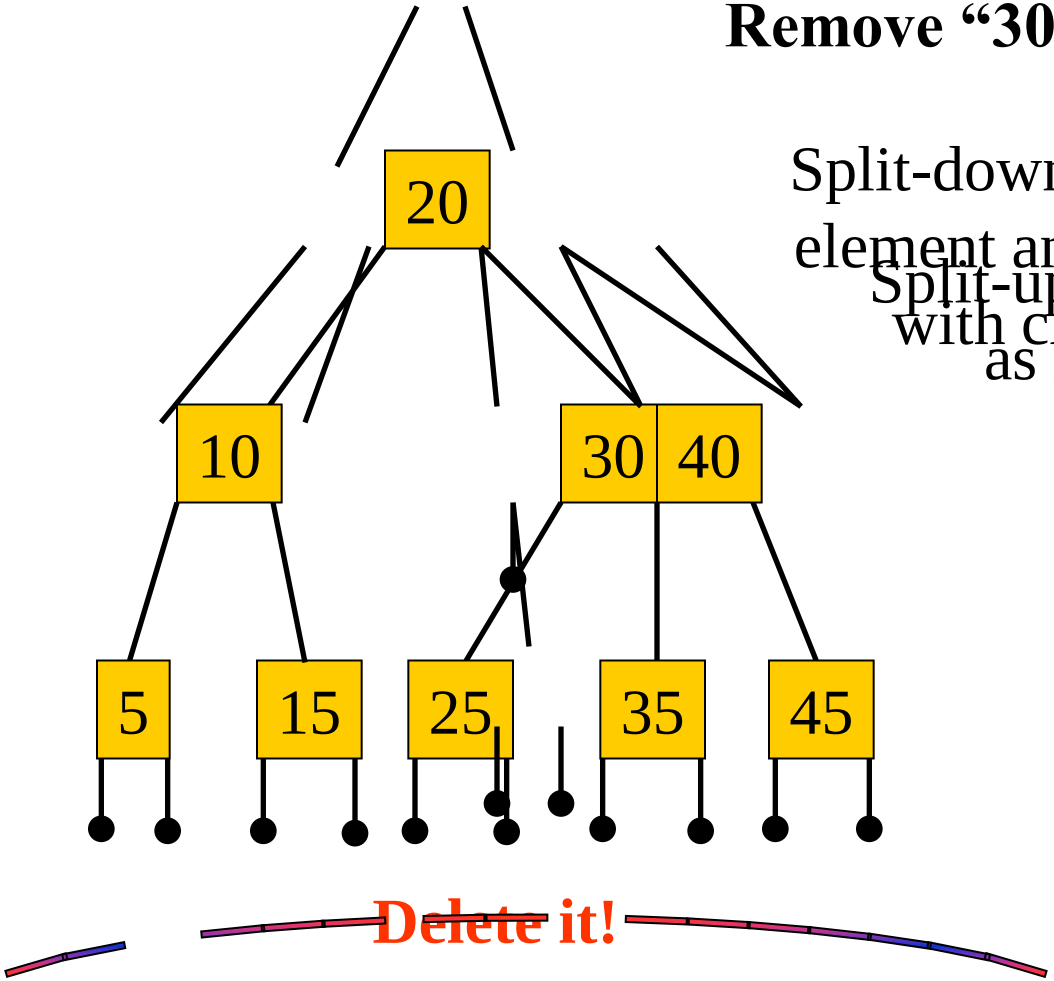
Deletion Dynamics

2 elements/node max

Remove "30"

Split-down candidate element and collapse with children

Split-up and splice as needed



Conclusions

- Proper abstraction leads to
  - Decoupling
  - Simplicity
  - Flexibility & extensibility
- Generalized Visitors open up new possibilities.
- Self-balancing trees illustrate
  - Abstract decomposition
  - Design patterns
  - Component-frameworks
  - Lambda calculus
  - Proof-of-correctness & complexity analysis