Comp 311
Functional Programming

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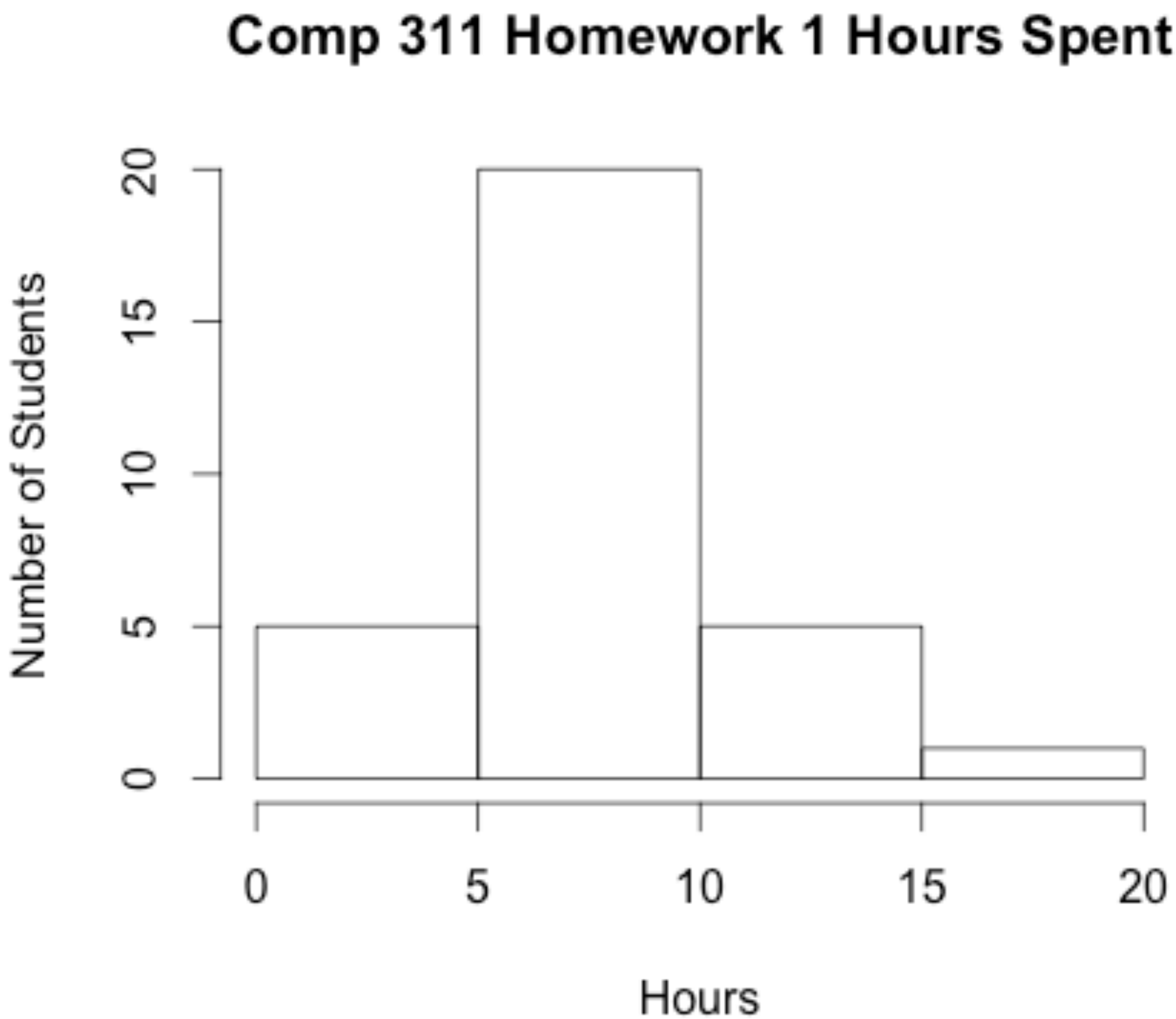
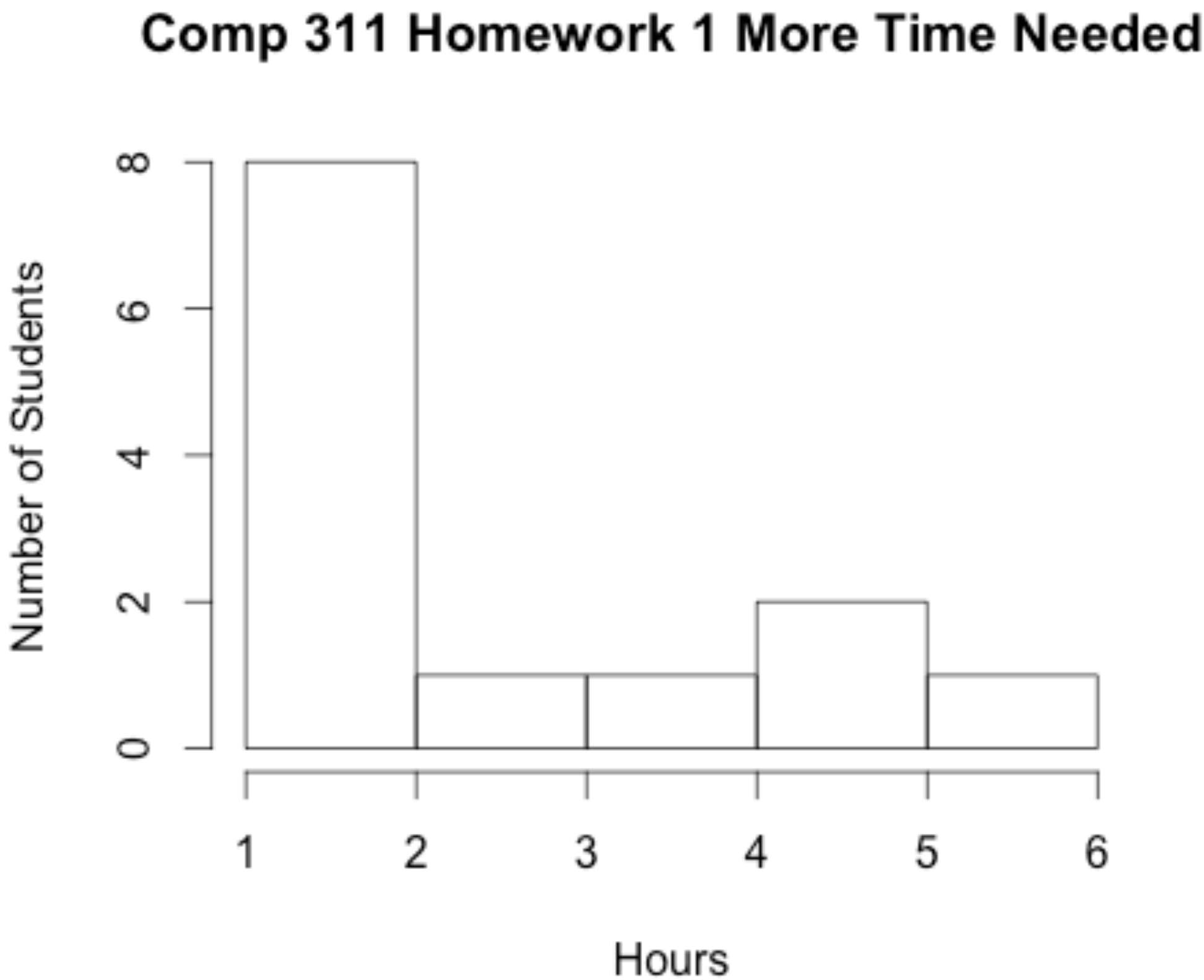
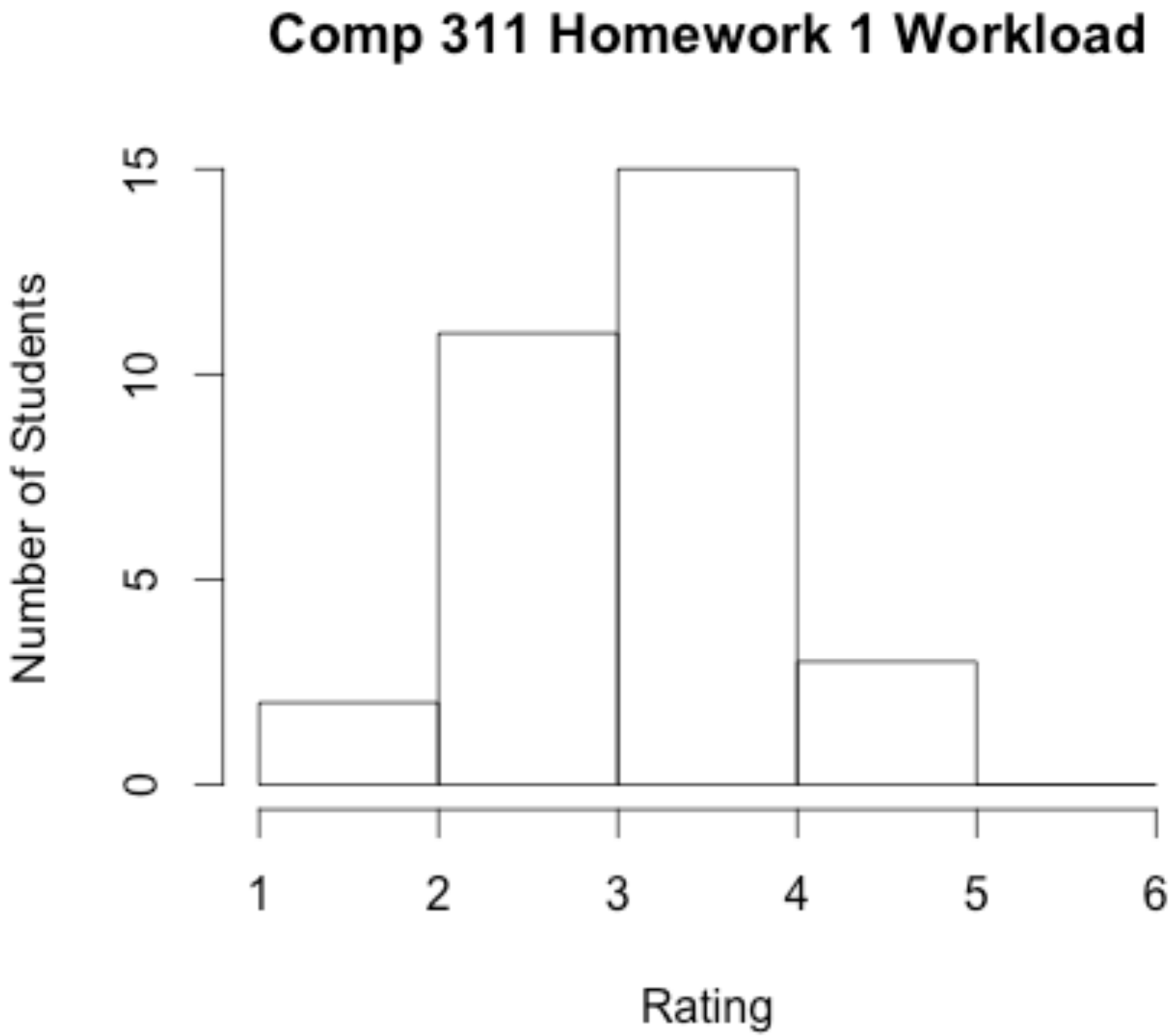
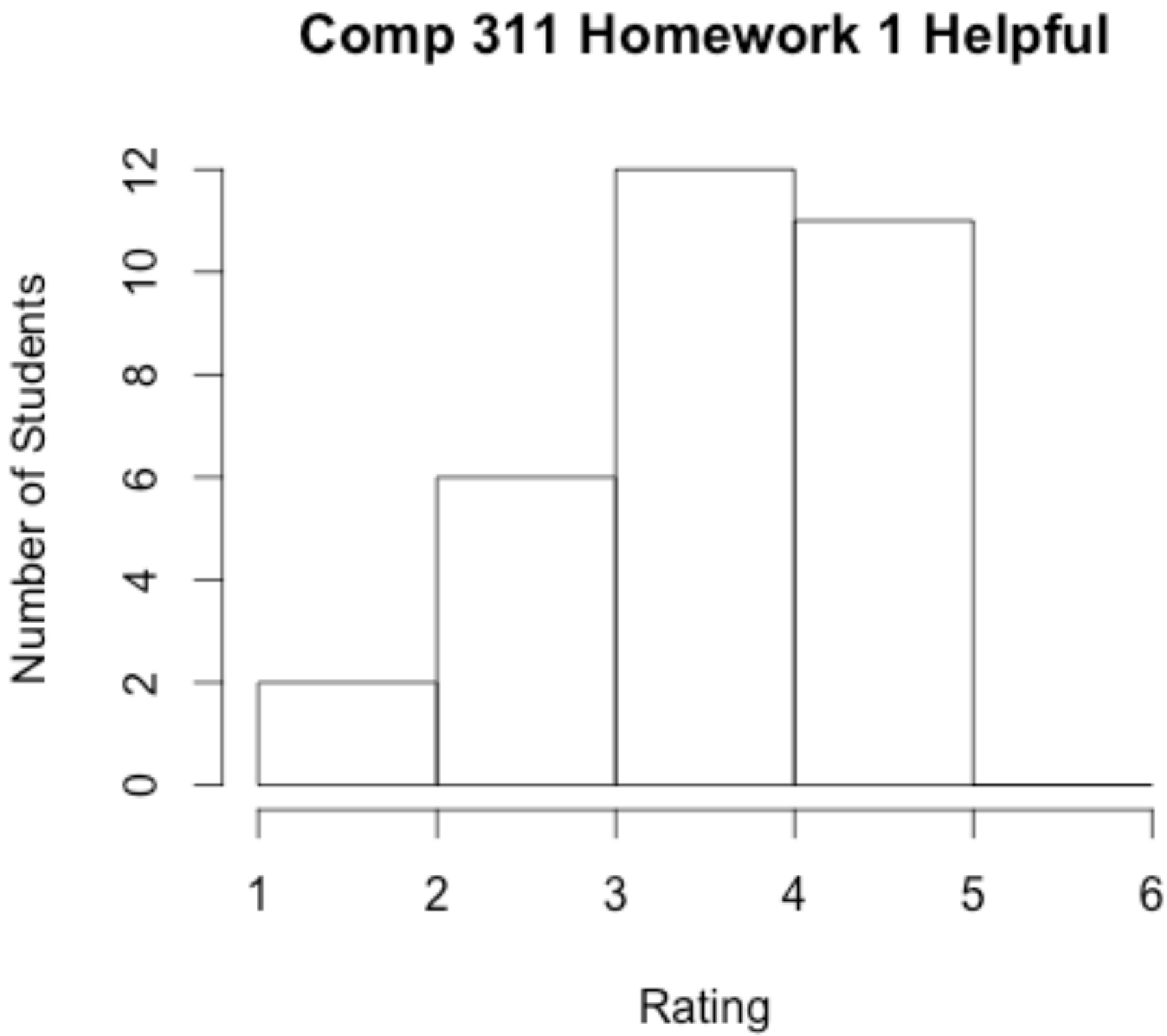
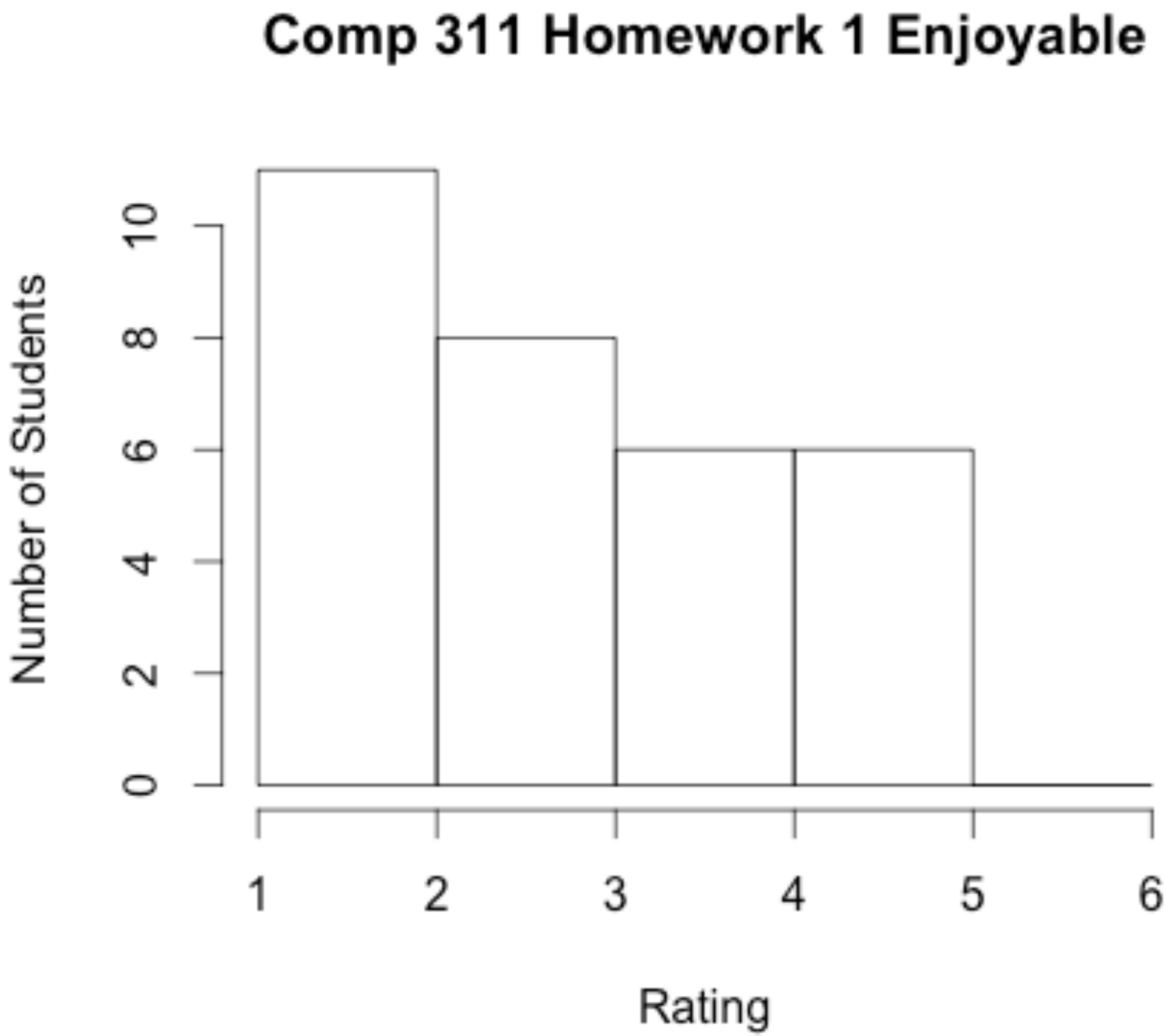
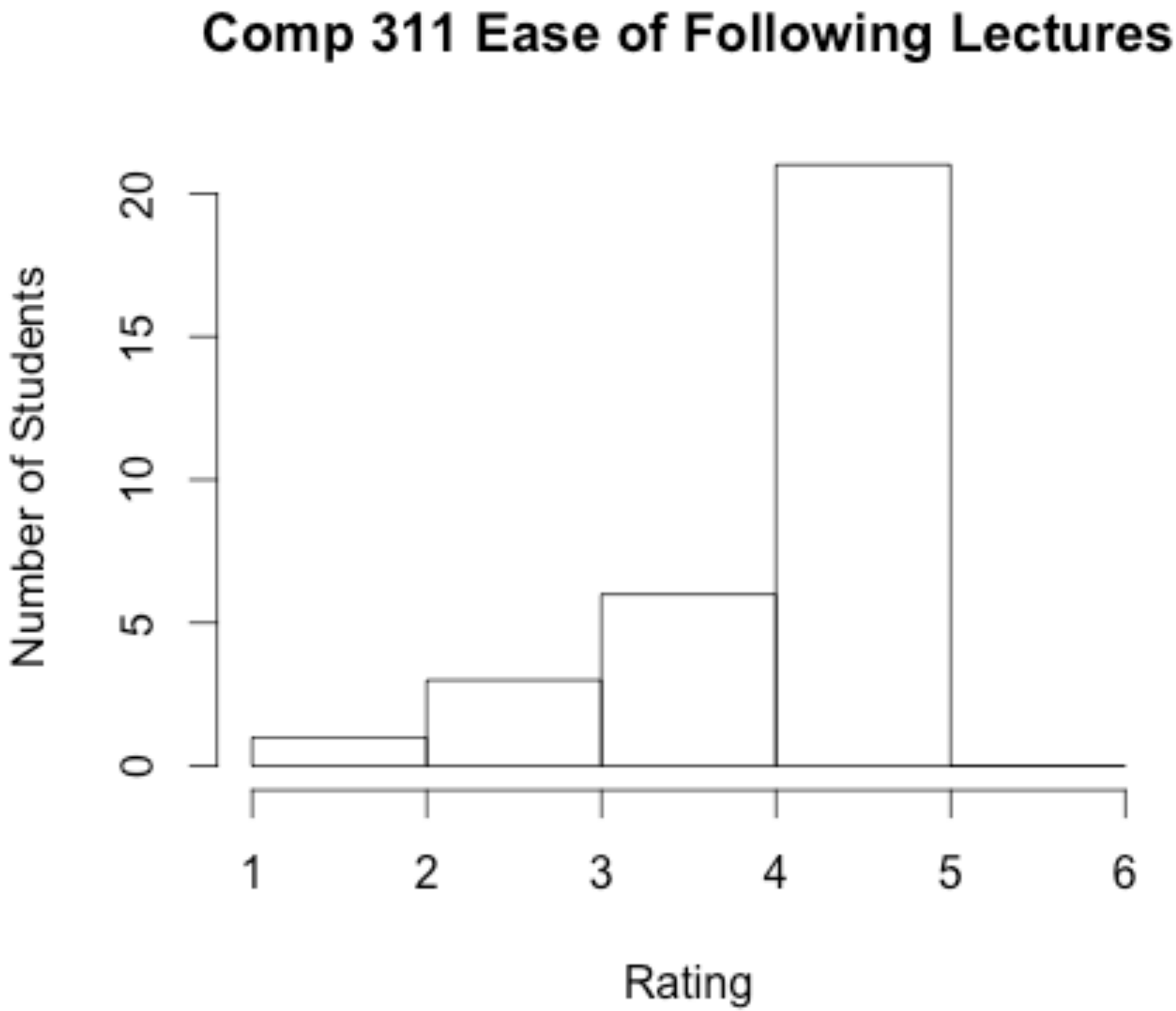

Comp 311 Homework 1 Hours Spent
Number of Students
0
5
10
15
20
0
5
10
15
20
Hours

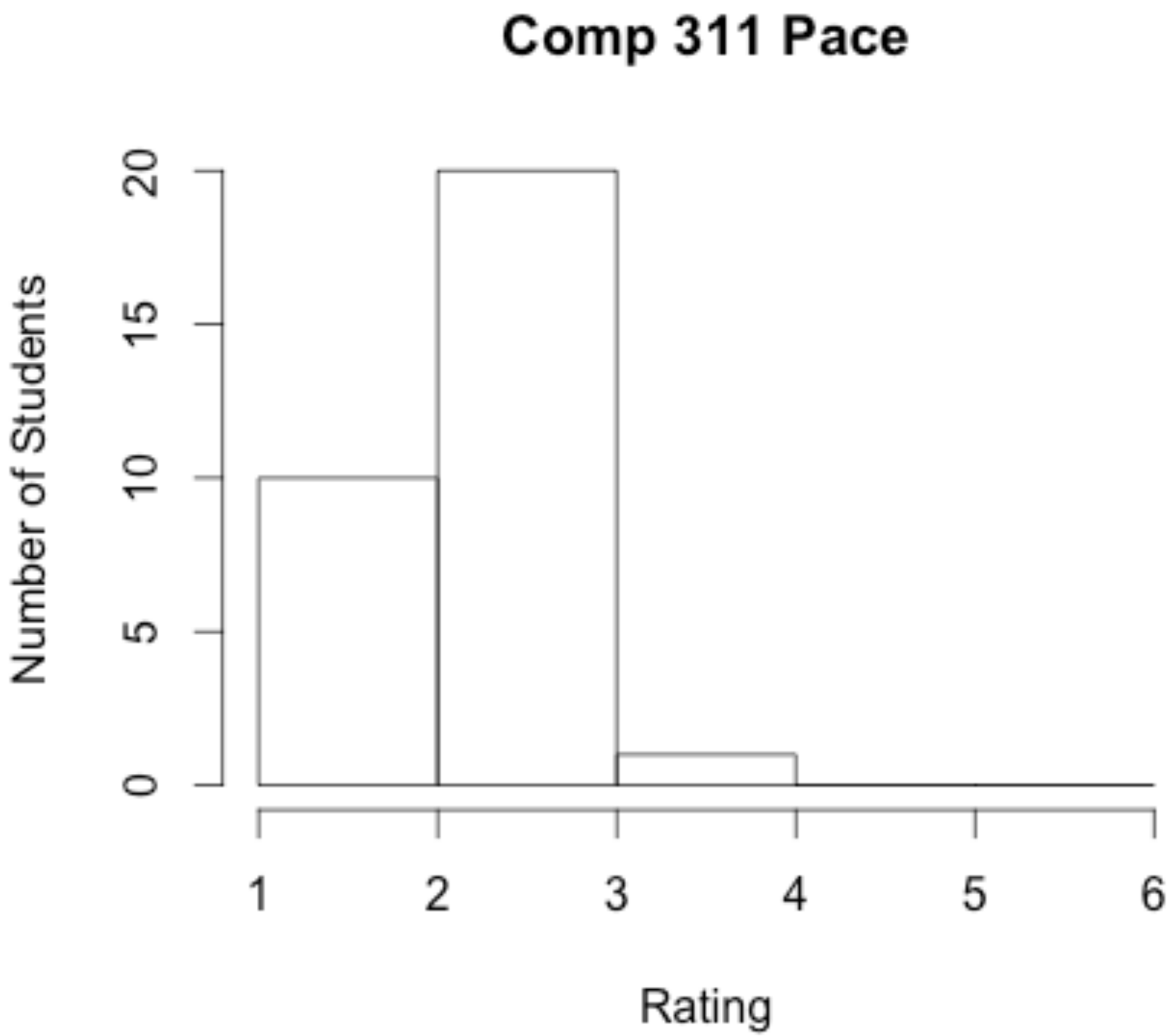

Comp 311 Homework 1 More Time Needed
Number of Students
0
2
4
6
8
1
2
3
4
5
6
Hours

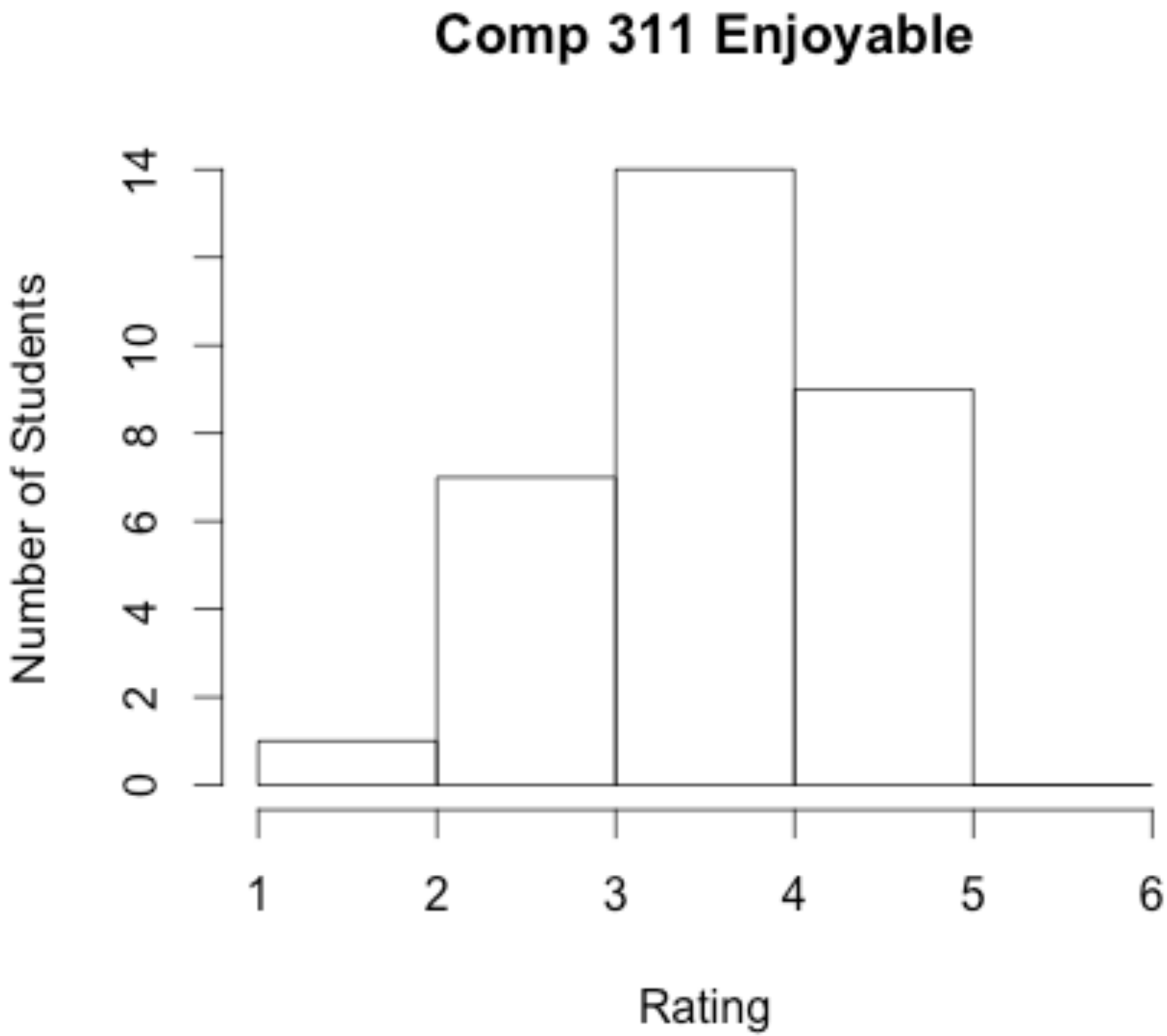

Comp 311 Homework 1 Workload
Number of Students
0
5
10
15
1
2
3
4
5
6
Rating


Comp 311 Homework 1 Helpful
Number of Students
0
2
4
6
8
10
12
1
2
3
4
5
6
Rating


Comp 311 Homework 1 Enjoyable
Number of Students
0
2
4
6
8
10
1
2
3
4
5
6
Rating


Comp 311 Ease of Following Lectures
Number of Students
0
5
10
15
20
1
2
3
4
5
6
Rating


Comp 311 Pace
Number of Students
0
5
10
15
20
1
2
3
4
5
6
Rating


Comp 311 Enjoyable
Number of Students
0
2
4
6
8
10
14
1
2
3
4
5
6
Rating

Actions

- Switch to two week assignments
- Double the weighting on subsequent assignments
- Keep Thursday at 2:30pm deadline

Importing a Member of a Package

```
import scala.collection.immutable.List
```

Importing Multiple Members of a Package

```
import scala.collection.immutable.{List, Vector}
```

Importing and Renaming Members of a Package

```
import scala.collection.immutable.{List=>SList, Vector}
```

Importing All Members of a Package

```
import scala.collection.immutable._
```

Note that * is a valid identifier in Scala!

Combining Notations

```
import scala.collection.immutable.{_}
```

same meaning as:

```
import scala.collection.immutable._
```

Combining Notations

```
import scala.collection.immutable.{List=>SList,_}
```

Imports all members of the package but renames `List` to `SList`

Combining Notations

```
import scala.collection.immutable.{List=>_,_}
```

Imports all members of the package except for `List`

Importing a Package

```
import scala.collection.immutable
```

Now sub-packages can be denoted by shorter names:

```
immutable.List
```

Importing and Renaming Packages

```
import scala.collection.{immutable => I}
```

Allows members to be written like this:

```
I.List
```

Importing Members of An Object

```
import Arithmetic._
```

Allows members such as `Arithmetic.gcd` to be write like this:

```
gcd
```

Implicit Imports

The following imports are implicitly included in your program:

```
import java.lang._
import scala._
import Predef._
```

Package java.lang

- Contains all the standard Java classes
- This import allows you to write things like:

```
Thread
```

instead of:

```
java.lang.Thread
```

Package scala

- Provides access to the standard Scala classes:

```
BigInt, BigDecimal, List, etc.
```

Object Predef

- Definitions of many commonly used types and methods, such as:

```
require, ensuring, assert
```

Visibility Modifier Private

For a method `Arithmetic.reduce` in package `Rationals`

Modifier	Explanation
no modifier	public access
private	private to class Arithmetic

Local Definitions

- As with constant definitions, we can make function definitions local to the body of a function
- The functions can be referred to only in the body of the enclosing function

Local Definitions

```
def reduce() = {
  val isPositive =
    ((numerator < 0) & (denominator < 0)) |
      ((numerator > 0) & (denominator > 0))

  def reduceFromInts(num: Int, denom: Int) = {
    require ((num >= 0) & (denom > 0))
    val gcd = Arithmetic.gcd(num, denom)
    val newNum = num/gcd
    val newDenom = denom/gcd

    if (isPositive) Rational(newNum, newDenom)
    else Rational(-newNum, newDenom)
  }
  reduceFromInts(Arithmetic.abs(numerator), Arithmetic.abs(denominator))

} ensuring (_ match {
  case Rational(n,d) => Arithmetic.gcd(n,d) == 1 & (d > 0)
})
```

Design Templates for Abstract Datatypes (Part 2)

Case Two

We Expect Many New Functions But Few New Variants

Case 2: We Expect Many New Functions But Few New Variants

- This is a case that traditional functional programming handles well
- Classic example domains: Compilers, theorem provers, numeric algorithms, machine learning
- Declare a top-level function with cases for each data variant

a.k.a., The Visitor Pattern

Again We Turn to Pattern Matching

```
val pi = 3.14

def area(shape: Shape) = {
  shape match {
    case Circle(r) => pi * r * r
    case Square(x) => x * x
    case Rectangle(x,y) => x * y
  }
}
```

We Can Define Arbitrary Functions Without Modifying Data Definitions

```
def makeLikeFirst(shape0: Shape, shape1: Shape) = {
  (shape0, shape1) match {
    case (Circle(r), Square(s)) => Circle(s)
    case (Circle(r), Rectangle(l,w)) => Circle((l+w)/2)

    case (Square(s), Circle(r)) => Square(r)
    case (Square(s), Rectangle(l,w)) => Square((l+w)/2)

    case (Rectangle(l,w), Circle(r)) => Rectangle(r,r)
    case (Rectangle(l,w), Square(s)) => Rectangle(s,s)

    case _ => shape1
  }
}
```

But A New Data Variant Requires Us To Modify All Functions Over the Datatype

```
val pi = 3.14

def area(shape: Shape) = {
  shape match {
    case Circle(r) => pi * r * r
    case Square(x) => x * x
    case Rectangle(x,y) => x * y
    case Triangle(b,h) => b*h/2
  }
}
```

But A New Data Variant Requires Us To Modify All Functions Over the Datatype

```
def makeLikeFirst(shape0: Shape, shape1: Shape) = {
  (shape0, shape1) match {
    case (Circle(r), Square(s)) => Circle(s)
    case (Circle(r), Rectangle(l,w)) => Circle((l+w)/2)
    case (Circle(r), Triangle(b,h)) => Circle(b)

    case (Square(s), Circle(r)) => Square(r)
    case (Square(s), Rectangle(l,w)) => Square((l+w)/2)
    case (Square(s), Triangle(b,h)) => Square(b+h/2)

    case (Rectangle(l,w), Circle(r)) => Rectangle(r,r)
    case (Rectangle(l,w), Square(s)) => Rectangle(s,s)
    case (Rectangle(l,w), Triangle(b,h)) => Rectangle(b,h)

    // plus all the cases for Triangle on the left (omitted)
    case _ => shape1
  }
}
```

Recursively Defined Datatypes

Recursively Defined Datatypes

- Case classes allow us to combine multiple pieces of a data into a single object
- But sometimes we don't know how many things we wish to combine
- We can use recursion to define datatypes of unbounded size
- This case corresponds to the Composite Design Pattern

Backus-Naur Form For Lists of Ints

```
List ::= Empty
       | Cons(Int,List)
```

Examples of Lists

```
Empty
Cons(3, Empty)
Cons(3, Cons(1, Empty))
Cons(3, Cons(1, Cons(4, Empty)))
```

Defining Lists With Scala Case Classes

```scala
abstract class List
case object Empty extends List
case class Cons(head: Int, tail: List) extends List
```

Where Do We Put Functions Over Lists?

- We do not expect to define new subtypes of lists
- We do expect to define many new functions over lists
- Similar to our Case Two Design Template for Abstract Datatypes
- Thus, we will start with our pattern matching template

An Example Function for Lists

```
def containsZero(xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => {
      if (n == 0) true
      else containsZero(ys)
    }
  }
}
```

An Example Function for Lists

```
def containsZero(xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => (n == 0) || containsZero(ys)
  }
}
```

Generalizing to Our First Template Function for Lists

```
def ourFunction(xs: List): Boolean = {
  xs match {
    case Empty => …
    case Cons(n, ys) => … n … ourFunction(ys) …
  }
}
```

Generalizing to Our First Template Function for Lists

```
def ourFunction(xs: List): Boolean = {
  xs match {
    case Empty => …
    case Cons(n, ys) => … n … ourFunction(ys) …
  }
}
```

We need to determine our *base case*

Generalizing to Our First Template Function for Lists

```
def ourFunction(xs: List): Boolean = {
  xs match {
    case Empty => …
    case Cons(n, ys) => … n … ourFunction(ys) …
  }
}
```

We must determine how to combine these values

Generalizing to Our First Template Function for Lists

```
def ourFunction(xs: List): Boolean = {
  xs match {
    case Empty => …
    case Cons(n, ys) => … n … ourFunction(ys) …
  }
}
```

This template is an example of *natural recursion* or *structural recursion*: We recursively decompose and then recombine a computation according to the natural structure of the data.

Filling in the Template

```
def containsZero(xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => (n == 0) || containsZero(ys)
  }
}
```

Here the base case is easy:
An empty list does not contain zero
(or anything else)

Filling in the Template

```
def containsZero(xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => (n == 0) || containsZero(ys)
  }
}
```

We break into cases based on the pieces from match: Either our first element *n* is zero or the answer lies with the rest of the list

Another Example: How Many Elements?

```
def length(xs: List): Int = {
  xs match {
    case Empty => 0
    case Cons(n, ys) => 1 + length(ys)
  }
}
```

Another Example: The Sum of the Elements

```
def sum(xs: List): Int = {
  xs match {
    case Empty => 0
    case Cons(n, ys) => n + sum(ys)
  }
}
```

Another Example: The Product of the Elements

```
def product(xs: List): Int = {
  xs match {
    case Empty => 1
    case Cons(n, ys) => n * product(ys)
  }
}
```

Converting Hours to Seconds

Problem Statement: Given a list of times measured in hours, we want to construct a list of corresponding times measured in seconds

Converting Hours to Seconds

```
def hoursToSeconds(xs: List): List = {
  xs match {
    case Empty => Empty
    case Cons(n, ys) => Cons(seconds(n), hoursToSeconds(ys))
  }
}

def seconds(hours: Int) = 3600 * hours
```

Generalizing to a Template

```
def ourFunction(xs: List): List = {
  xs match {
    case Empty => …
    case Cons(n, ys) => Cons(…n…,
                             ourFunction(ys))
  }
}
```

Really, this is the same template as before, but now Cons is our combining operation

The Natural Numbers

```
Nat ::= 0
      | Next(Nat)
```

The Natural Numbers

```
Nat ::= 0
      | Next(Nat)
```

Here we are between Cases One and Two for Abstract Datatypes:

- No new variants expected
- Many new functions expected
- But some basic functions are intrinsic to the type

Defining The Natural Numbers in Scala

```
abstract class Nat
case object Zero extends Nat
case class Next(n: Nat) extends Nat
```

Defining The Natural Numbers in Scala

```
abstract class Nat {
  def +(n: Nat): Nat
  def *(n: Nat): Nat
}
```

Defining The Natural Numbers in Scala

```scala
case object Zero extends Nat {
  def +(n: Nat) = n
  def *(n: Nat) = Zero
}

case class Next(n: Nat) extends Nat {
  def +(m: Nat) = Next(n + m)
  def *(m: Nat) = m + (n * m)
}
```

Defining The Natural Numbers in Scala

```scala
case object Zero extends Nat {
  def +(n: Nat) = n
  def *(n: Nat) = Zero
}

case class Next(n: Nat) extends Nat {
  def +(m: Nat) = Next(n + m)
  def *(m: Nat) = m + (n * m)
}
```

Again we have natural recursion: base case, recursion, combination

Example Reduction
(3 + 2)

```
Next(Next(Next(Zero)) + Next(Next(Zero))  ↦
Next(Next(Next(Zero)) + Next(Next(Zero))) ↦
Next(Next(Next(Zero) + Next(Next(Zero)))) ↦
Next(Next(Next(Zero + Next(Next(Zero))))) ↦
    Next(Next(Next(Next(Next(Zero)))))
```

Factorial

```
def factorial(n: Nat): Nat = {
  n match {
    case Zero => Next(Zero)
    case Next(m) => n * factorial(m)
  }
}
```

Transferring The Pattern To Ints

```
def factorial(n: Int): Int = {
  require (n >= 0)

  if (n == 0) 1
  else n * factorial(n - 1)

} ensuring (n > 0)
```

Combining Via Auxiliary Functions

Combining Via Auxiliary Functions

- As our examples with natural numbers shows, it is often necessary to define the combining operation of a natural recursion as an auxiliary function
- We can apply this insight to lists and use our template to cover yet more cases

Sorting Lists

```
def sort(xs: List): List = {
  xs match {
    case Empty => Empty
    case Cons(n, ys) => insert(n, sort(ys))
  }
}
```

We need to explain how to insert into a sorted list

Insertion

```
def insert(n: Int, xs: List): List = {
  xs match {
    case Empty => Cons(n, Empty)
    case Cons(m, ys) => {
      if (n <= m) Cons(n, xs)
      else Cons(m, insert(n, ys))
    }
  }
}
```

Insertion

```
def insert(n: Int, xs: List): List = {
  xs match {
    case Empty => Cons(n, Empty)
    case Cons(m, ys) => {
      if (n <= m) Cons(n, xs)
      else Cons(m, insert(n, ys))
    }
  }
}
```

This parameter is not traversed,
but is used for combination and comparison
Other functions follow this pattern.

Appending Two Lists

```
abstract class List {
  /**
   * Returns a new list with the elements of
   * this list appended to the given list.
   */
  def ++(ys: List): List
}
```

Appending Two Lists

```
case object Empty extends List {
  def ++(ys: List) = ys
}
```

Appending Two Lists

```
case class Cons(first: Int, rest: List) extends List {
  def ++(ys: List) = Cons(first, rest ++ ys)
}
```

Family Trees

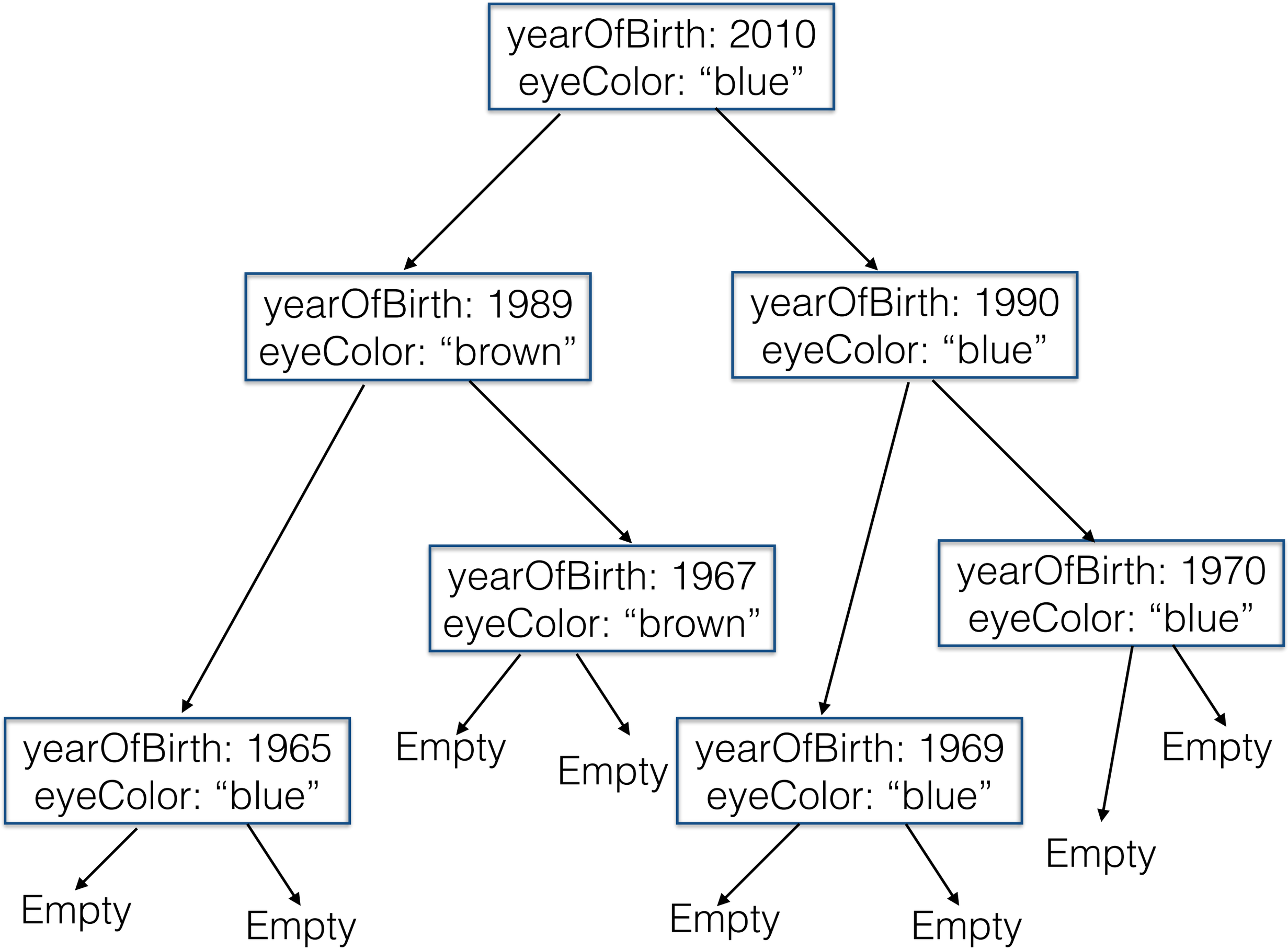
```
TreeNode ::= Empty
           | Child(TreeNode,
                   TreeNode,
                   Int,
                   String)
```

Family Trees

```
abstract class TreeNode

case object EmptyNode extends TreeNode

case class Child(mother: TreeNode,
                 father: TreeNode,
                 yearOfBirth: Int,
                 eyeColor: String)
extends TreeNode
```


yearOfBirth: 2010
eyeColor: “blue”
yearOfBirth: 1989
eyeColor: “brown”
yearOfBirth: 1990
eyeColor: “blue”
yearOfBirth: 1967
eyeColor: “brown”
yearOfBirth: 1970
eyeColor: “blue”
yearOfBirth: 1965
eyeColor: “blue”
Empty
Empty
yearOfBirth: 1969
eyeColor: “blue”
Empty
Empty
Empty
Empty
Empty
Empty

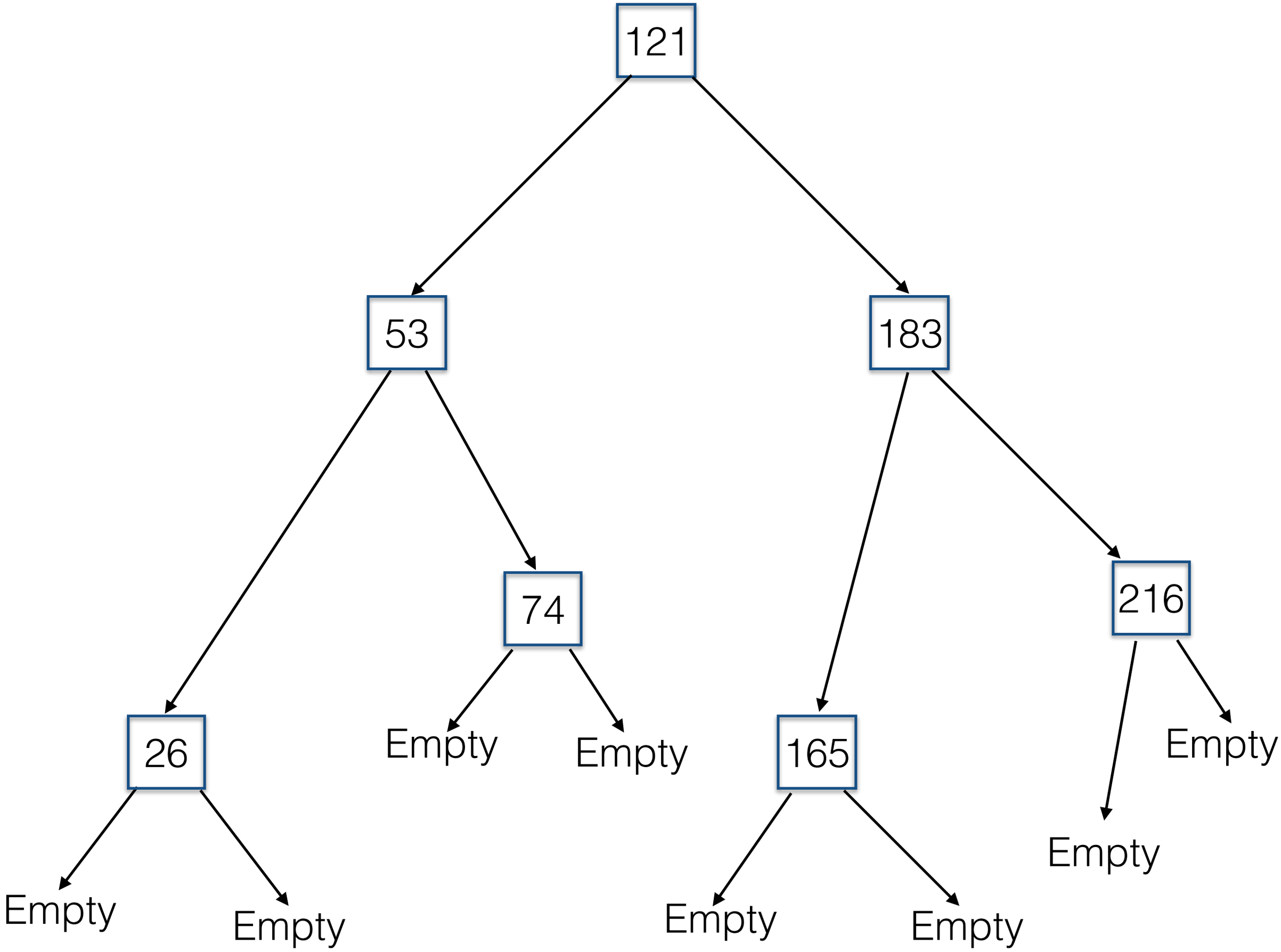
Family Trees

```
def hasBlueEyedAncestor(t: TreeNode): Boolean = {
  t match {
    case EmptyNode => false
    case Child(m,f,b,e) => ((e == "Blue") ||
                            hasBlueEyedAncestor(m) ||
                            hasBlueEyedAncestor(f))
  }
}
```

Binary Search Trees

Binary Search Trees

- We will define trees containing only Ints
- To help us find elements quickly, we will abide by the following invariant:
  - At a given node containing value *n*:
    - All values in the left subtree are less than *n*
    - All values in the right subtree are greater than *n*


121
53
183
26
74
165
216
Empty
Empty
Empty
Empty
Empty
Empty
Empty
Empty

Binary Search Trees

```
abstract class BinarySearchTree {
  def contains(n: Int): Boolean
  def insert(n: Int): BinarySearchTree
}
```

Binary Search Trees

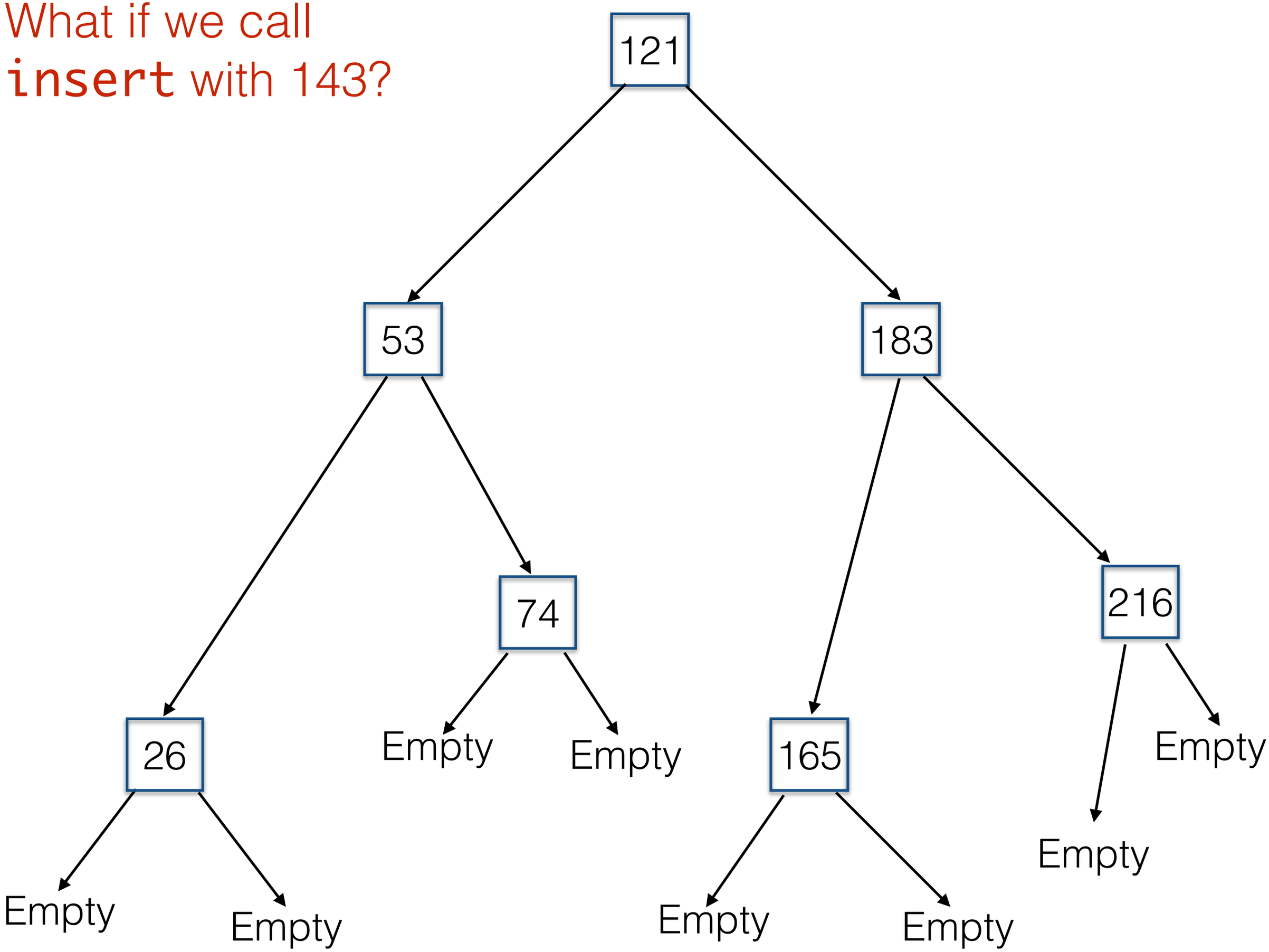
```
case object EmptyTree extends BinarySearchTree {
  def contains(n: Int) = false
  def insert(n: Int) = ConsTree(n, EmptyTree, EmptyTree)
}
```

Binary Search Trees

```
case class ConsTree(m: Int,
                    left: BinarySearchTree,
                    right: BinarySearchTree)
extends BinarySearchTree {

  def contains(n: Int): Boolean = {
    if (n < m) left.contains(n)
    else if (n > m) right.contains(n)
    else true // n == m
  }
  def insert(n: Int) = {
    if (n < m) ConsTree(m, left.insert(n), right)
    else if (n > m) ConsTree(m, left, right.insert(n))
    else this // n == m
  }
}
```

What if we call `insert` with 143?



What if we call `insert` with 143?