

Comp 311

Functional Programming

Eric Allen, Two Sigma Investments
Robert “Corky” Cartwright, Rice University
Sagnak Tasirlar, Two Sigma Investments

Design Templates for Abstract Datatypes (Part 2)

Case Two

We Expect Many New
Functions But Few New Variants

Case 2: We Expect Many New Functions But Few New Variants

- This is a case that traditional functional programming handles well
- Classic example domains: Compilers, theorem provers, numeric algorithms, machine learning
- Declare a top-level function with cases for each data variant

a.k.a., The Visitor Pattern

Again We Turn to Pattern Matching

```
val pi = 3.14
```

```
def area(shape: Shape) = {  
  shape match {  
    case Circle(r) => pi * r * r  
    case Square(x) => x * x  
    case Rectangle(x,y) => x * y  
  }  
}
```

We Can Define Arbitrary Functions Without Modifying Data Definitions

```
def makeLikeFirst(shape0: Shape, shape1: Shape) = {  
  (shape0, shape1) match {  
    case (Circle(r), Square(s)) => Circle(s)  
    case (Circle(r), Rectangle(l,w)) => Circle((l+w)/2)  
  
    case (Square(s), Circle(r)) => Square(r)  
    case (Square(s), Rectangle(l,w)) => Square((l+w)/2)  
  
    case (Rectangle(l,w), Circle(r)) => Rectangle(r,r)  
    case (Rectangle(l,w), Square(s)) => Rectangle(s,s)  
  
    case _ => shape1  
  }  
}
```

But A New Data Variant Requires Us To Modify All Functions Over the Datatype

```
val pi = 3.14
```

```
def area(shape: Shape) = {  
  shape match {  
    case Circle(r) => pi * r * r  
    case Square(x) => x * x  
    case Rectangle(x,y) => x * y  
    case Triangle(b,h) => b*h/2  
  }  
}
```

But A New Data Variant Requires Us To Modify All Functions Over the Datatype

```
def makeLikeFirst(shape0: Shape, shape1: Shape) = {  
  (shape0, shape1) match {  
    case (Circle(r), Square(s)) => Circle(s)  
    case (Circle(r), Rectangle(l,w)) => Circle((l+w)/2)  
    case (Circle(r), Triangle(b,h)) => Circle(b)  
  
    case (Square(s), Circle(r)) => Square(r)  
    case (Square(s), Rectangle(l,w)) => Square((l+w)/2)  
    case (Square(s), Triangle(b,h)) => Square(b+h/2)  
  
    case (Rectangle(l,w), Circle(r)) => Rectangle(r,r)  
    case (Rectangle(l,w), Square(s)) => Rectangle(s,s)  
    case (Rectangle(l,w), Triangle(b,h)) => Rectangle(b,h)  
  
    // plus all the cases for Triangle on the left (omitted)  
    case _ => shape1  
  }  
}
```


Recursively Defined Datatypes

Recursively Defined Datatypes

- Case classes allow us to combine multiple pieces of a data into a single object
- But sometimes we don't know how many things we wish to combine
- We can use recursion to define datatypes of unbounded size
- This case corresponds to the Composite Design Pattern

Backus-Naur Form For Lists of Ints

```
List ::= Empty  
      | Cons(Int, List)
```

Examples of Lists

Empty

Cons(3, Empty)

Cons(3, Cons(1, Empty))

Cons(3, Cons(1, Cons(4, Empty)))

Defining Lists With Scala Case Classes

```
abstract class List  
case object Empty extends List  
case class Cons(head: Int, tail: List) extends List
```

Where Do We Put Functions Over Lists?

- We do not expect to define new subtypes of lists
- We do expect to define many new functions over lists
- Similar to our Case Two Design Template for Abstract Datatypes
- Thus, we will start with our pattern matching template

An Example Function for Lists

```
def containsZero(xs: List): Boolean = {  
  xs match {  
    case Empty => false  
    case Cons(n, ys) => {  
      if (n == 0) true  
      else containsZero(ys)  
    }  
  }  
}
```

An Example Function for Lists

```
def containsZero(xs: List): Boolean = {  
  xs match {  
    case Empty => false  
    case Cons(n, ys) => (n == 0) || containsZero(ys)  
  }  
}
```


Generalizing to Our First Template Function for Lists

```
def ourFunction(xs: List): Boolean = {  
  xs match {  
    case Empty => ...  
    case Cons(n, ys) => ... n ... ourFunction(ys) ...  
  }  
}
```

Generalizing to Our First Template Function for Lists


```
def ourFunction(xs: List): Boolean = {  
  xs match {  
    case Empty => ...  
    case Cons(n, ys) => ... n ... ourFunction(ys) ...  
  }  
}
```

We need to determine our *base case*



Generalizing to Our First Template Function for Lists

```
def ourFunction(xs: List): Boolean = {  
  xs match {  
    case Empty => ...  
    case Cons(n, ys) => ... n ... ourFunction(ys) ...  
  }  
}
```



We must determine how to combine these values


Generalizing to Our First Template Function for Lists

```
def ourFunction(xs: List): Boolean = {  
  xs match {  
    case Empty => ...  
    case Cons(n, ys) => ... n ... ourFunction(ys) ...  
  }  
}
```

This template is an example of *natural recursion* or *structural recursion*: We recursively decompose and then recombine a computation according to the natural structure of the data.

Filling in the Template


```
def containsZero(xs: List): Boolean = {  
  xs match {  
    case Empty => false  
    case Cons(n, ys) => (n == 0) || containsZero(ys)  
  }  
}
```



Here the base case is easy:
An empty list does not contain zero
(or anything else)

Filling in the Template

```
def containsZero(xs: List): Boolean = {  
  xs match {  
    case Empty => false  
    case Cons(n, ys) => (n == 0) || containsZero(ys)  
  }  
}
```



We break into cases based on the pieces from match: Either our first element n is zero or the answer lies with the rest of the list

Another Example: How Many Elements?

```
def length(xs: List): Int = {  
  xs match {  
    case Empty => 0  
    case Cons(n, ys) => 1 + length(ys)  
  }  
}
```

Another Example: The Sum of the Elements

```
def sum(xs: List): Int = {  
  xs match {  
    case Empty => 0  
    case Cons(n, ys) => n + sum(ys)  
  }  
}
```


Another Example: The Product of the Elements

```
def product(xs: List): Int = {  
  xs match {  
    case Empty => 1  
    case Cons(n, ys) => n * product(ys)  
  }  
}
```

Converting Hours to Seconds

Problem Statement: Given a list of times measured in hours, we want to construct a list of corresponding times measured in seconds

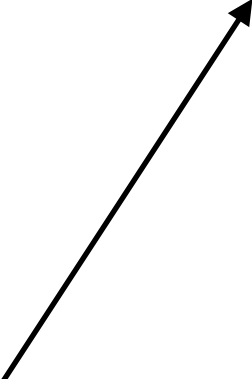
Converting Hours to Seconds

```
def hoursToSeconds(xs: List): List = {  
  xs match {  
    case Empty => Empty  
    case Cons(n, ys) => Cons(seconds(n), hoursToSeconds(ys))  
  }  
}
```

```
def seconds(hours: Int) = 3600 * hours
```

Generalizing to a Template

```
def ourFunction(xs: List): List = {  
  xs match {  
    case Empty => ...  
    case Cons(n, ys) => Cons(...n...,  
                             ourFunction(ys))  
  }  
}
```



Really, this is the same template as before, but now Cons is our combining operation

The Natural Numbers

$\text{Nat} ::= 0$
 $| \text{Next}(\text{Nat})$

The Natural Numbers

$$\text{Nat} ::= 0$$
$$| \text{Next}(\text{Nat})$$

Here we are between Cases One and Two for Abstract Datatypes:

- No new variants expected
- Many new functions expected
- But some basic functions are intrinsic to the type

Defining The Natural Numbers in Scala

```
abstract class Nat  
case object Zero extends Nat  
case class Next(n: Nat) extends Nat
```

Defining The Natural Numbers in Scala

```
abstract class Nat {  
  def +(n: Nat): Nat  
  def *(n: Nat): Nat  
}
```


Defining The Natural Numbers in Scala

```
case object Zero extends Nat {  
  def +(n: Nat) = n  
  def *(n: Nat) = Zero  
}
```

```
case class Next(n: Nat) extends Nat {  
  def +(m: Nat) = Next(n + m)  
  def *(m: Nat) = m + (n * m)  
}
```

Defining The Natural Numbers in Scala

```
case object Zero extends Nat {  
  def +(n: Nat) = n  
  def *(n: Nat) = Zero  
}
```

Again we have natural recursion: base case, recursion, combination

```
case class Next(n: Nat) extends Nat {  
  def +(m: Nat) = Next(n + m)  
  def *(m: Nat) = m + (n * m)  
}
```

Example Reduction

(3 + 2)

Next(Next(Next(Zero)) + Next(Next(Zero))) \mapsto
Next(Next(Next(Zero)) + Next(Next(Zero))) \mapsto
Next(Next(Next(Zero) + Next(Next(Zero)))) \mapsto
Next(Next(Next(Zero + Next(Next(Zero)))))) \mapsto
Next(Next(Next(Next(Next(Zero))))))

Factorial

```
def factorial(n: Nat): Nat = {  
  n match {  
    case Zero => Next(Zero)  
    case Next(m) => n * factorial(m)  
  }  
}
```

Transferring The Pattern To Ints

```
def factorial(n: Int): Int = {  
  require (n >= 0)  
  
  if (n == 0) 1  
  else n * factorial(n - 1)  
  
} ensuring (_ > 0)
```

Combining Via Auxiliary Functions

Combining Via Auxiliary Functions

- As our examples with natural numbers shows, it is often necessary to define the combining operation of a natural recursion as an auxiliary function
- We can apply this insight to lists and use our template to cover yet more cases

Sorting Lists

```
def sort(xs: List): List = {  
  xs match {  
    case Empty => Empty  
    case Cons(n, ys) => insert(n, sort(ys))  
  }  
}
```

We need to explain how to
insert into a sorted list

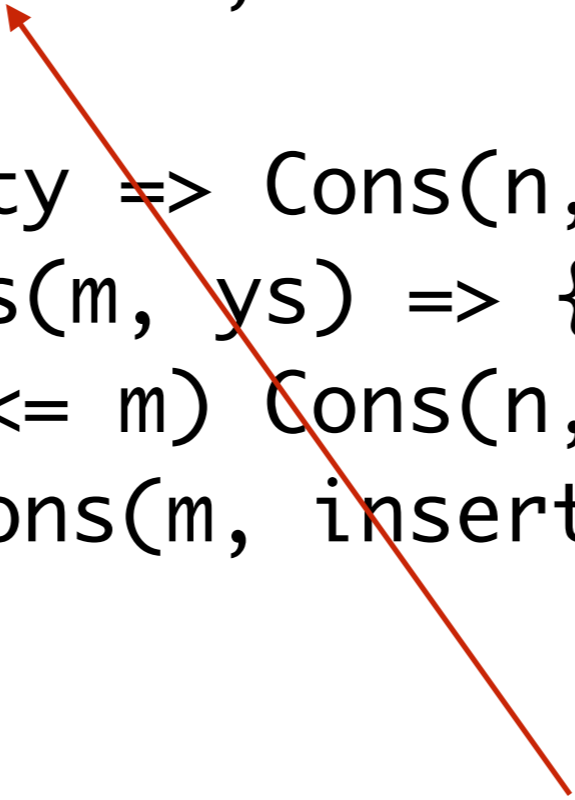


Insertion

```
def insert(n: Int, xs: List): List = {  
  xs match {  
    case Empty => Cons(n, Empty)  
    case Cons(m, ys) => {  
      if (n <= m) Cons(n, xs)  
      else Cons(m, insert(n, ys))  
    }  
  }  
}
```

Insertion

```
def insert(n: Int, xs: List): List = {  
  xs match {  
    case Empty => Cons(n, Empty)  
    case Cons(m, ys) => {  
      if (n <= m) Cons(n, xs)  
      else Cons(m, insert(n, ys))  
    }  
  }  
}
```



This parameter is not traversed,
but is used for combination and comparison
Other functions follow this pattern.

Appending Two Lists

```
abstract class List {  
  /**  
   * Returns a new list with the elements of  
   * this list appended to the given list.  
   */  
  def ++(ys: List): List  
}
```

Appending Two Lists

```
case object Empty extends List {  
  def ++(ys: List) = ys  
}
```

Appending Two Lists

```
case class Cons(first: Int, rest: List) extends List {  
  def ++(ys: List) = Cons(first, rest ++ ys)  
}
```