Comp 311
Functional Programming

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Announcements

- Homework 2 Available from Piazza (Due October 1)
- Two Sigma Info Session at Huff House, 4pm Today

Traversing Multiple Recursive Datatypes

Taking the First Few Elements

```
def take(n: Nat, xs: List): List = {
  // require n <= size(xs)
  (n,xs) match {
    case (Zero, xs) => Empty
    case (Next(m), Cons(y, ys)) => Cons(y, take(m, ys))
  }
}
```

Taking the First Few Elements

```
def take(n: Int, xs: List): List = {
  require ((n >= 0) && (n <= size(xs)))
  (n,xs) match {
    case (0, xs) => Empty
    case (n, Cons(y, ys)) => Cons(y, take(n-1, ys))
  }
}
```

Dropping the First Few Elements

```
def drop(n: Int, xs: List): List = {
  require (n <= size(xs))
  (n, xs) match {
    case (0, xs) => xs
    case (n, Cons(y, ys)) => drop(n-1, ys)
  }
}
```

Functional Update of a List

```
def update(xs: List, i: Nat, y: Int): List = {
  require (xs != Empty) // && i < size(xs)
  (xs, i) match {
    case (Cons(z, zs), Zero) => Cons(y, zs)
    case (Cons(z, zs), Next(j)) => Cons(z, update(zs,j,y))
  }
}
```

Functional Update of a List

```
def update(xs: List, i: Int, y: Int): List = {
  require ((i >= 0) && (i < size(xs)))
  assert (xs != Empty)

  (xs, i) match {
    case (Cons(z, zs), 0) => Cons(y, zs)
    case (Cons(z, zs), _) => Cons(z, update(zs,i-1,y))
  }
}
```

Design Abstraction

Our Function Templates Reveal Common Structure

```
def containsZero(xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => (n == 0) || containsZero(ys)
  }
}

def containsOne(xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => (n == 1) || containsOne(ys)
  }
}
```

Our Function Templates Reveal Common Structure

```
def contains(m: Int, xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => (n == m) || contains(m, ys)
  }
}
```

But Sometimes the Part We Want to Abstract Is a Function

```
def below(m: Int, xs: List): List = {
  xs match {
    case Empty => Empty
    case Cons(n, ys) => {
      if (n < m) Cons(n, below(m, ys))
      else below(m, ys)
    }
  }
}
```

But Sometimes the Part We Want to Abstract Is a Function

```
def above(m: Int, xs: List): List = {
  xs match {
    case Empty => Empty
    case Cons(n, ys) => {
      if (n > m) Cons(n, above(m, ys))
      else above(m, ys)
    }
  }
}
```

Taking Functions As Parameters

```
def filter(f: (Int)=>Boolean, xs: List): List = {
  xs match {
    case Empty => Empty
    case Cons(n, ys) => {
      if (f(n)) Cons(n, filter(f, ys))
      else filter(f, ys)
    }
  }
}
```

Passing Functions as Arguments

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter(((n: Int) => (n > 0)), xs) ↦*
Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter(((n: Int) => (n < 0)), xs) ↦*
Empty

filter(((n: Int) => (n < 3)), xs) ↦*
Cons(1,Cons(2,Empty))
```

Passing Functions as Arguments

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter((n: Int) => (n > 0), xs) ↦*
Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter((n: Int) => (n < 0), xs) ↦*
Empty

filter((n: Int) => (n < 3), xs) ↦*
Cons(1,Cons(2,Empty))
```

These are *function literals*

First-Class Functions

- Function literals are expressions with static arrow types that reduce to *function values*
- The value type of a function value is also an arrow type
- Function values are first-class values:
  - They are allowed to be passed as arguments
  - They are allowed to be returned as results

Simplifying Function Literals

- Parameter types on function literals are allowed to be elided whenever the types are clear from context

```
filter(((n: Int) => (n > 0)), xs)
```

can be written as

```
filter(((n) => (n > 0)), xs)
```

Simplifying Function Literals

- Parentheses around a single parameter is allowed to be omitted

```
filter((n) => (n > 0)), xs)
```

can be written as

```
filter(n => (n > 0), xs)
```

Simplifying Function Literals

- When a single parameter is used only once in the body of a function literal:
  - We can drop the parameter list
  - We simply write the body with an _ at the place where the parameter is used

For example,

```
((x: Int) => (x < 0))
```

becomes

```
_ < 0
```

Passing Functions as Arguments

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter(_ < 3, xs) ↦* Cons(1,Cons(2,Empty))
```

Mapping a Computation Over a List

```
def double(xs: List) = {
  xs match {
    case Empty => Empty
    case Cons(y,ys) => Cons(y * y, double(ys))
  }
}
```

We Might Express a Similar Computation Mathematically as a Comprehension

$$\{2x \mid x \in xs\}$$

Mapping a Computation Over a List

```
def negate(xs: List) = {
  xs match {
    case Empty => Empty
    case Cons(y,ys) => (-y, negate(ys))
  }
}
```

Negation as a Comprehension

$$\{-x \mid x \in xs\}$$

Mapping a Computation Over a List

```
def map(f: Int => Int, xs: List) = {
  xs match {
    case Empty => Empty
    case Cons(y,ys) => Cons(f(y), map(f,ys))
  }
}
```

Mapping a Computation Over a List

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

negate(xs) ↦*
Cons(-1,Cons(-2,Cons(-3,Cons(-4,Cons(-5,Cons(-6,Empty))))))

double(xs) ↦*
Cons(1,Cons(4,Cons(9,Cons(16,Cons(25,Cons(36,Empty))))))
```

Mapping a Computation Over a List

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

map(-_, xs) ↦*
Cons(-1,Cons(-2,Cons(-3,Cons(-4,Cons(-5,Cons(-6,Empty))))))

map(x => x * x, xs) ↦*
Cons(1,Cons(4,Cons(9,Cons(16,Cons(25,Cons(36,Empty))))))
```

Recall Our Sum Function Over Lists

```
def sum(xs: List): Int = {
  xs match {
    case Empty => 0
    case Cons(y,ys) => y + sum(ys)
  }
}
```

In Mathematics, We Might Write a Summation

$$\sum_{n \in xs} n$$

And Our Product Function Over Lists

```
def product(xs: List): Int = {
  xs match {
    case Empty => 1
    case Cons(y,ys) => y * sum(ys)
  }
}
```

In Mathematics, We Might Write a Summation

$$\prod_{n \in xs} n$$

We Abstract to a Reduction Function Over Lists

```
def reduce(base: Int, f: (Int, Int) => Int, xs: List): Int = {
  xs match {
    case Empty => base
    case Cons(y,ys) => f(y, reduce(base, f, ys))
  }
}
```

Example Reductions

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

reduce(0, (x,y) => x + y, xs) ↦* 21

reduce(1, (x,y) => x * y, xs) ↦* 720
```

Simplifying Function Literals

- When *each* parameter is used only once in the body of a function literal, and in the order in which they are passed:
  - We can drop the parameter list
  - We simply write the body with an _ at the place where each parameter is used

For example,

```
((x: Int, y: Int) => (x + y))
```

becomes

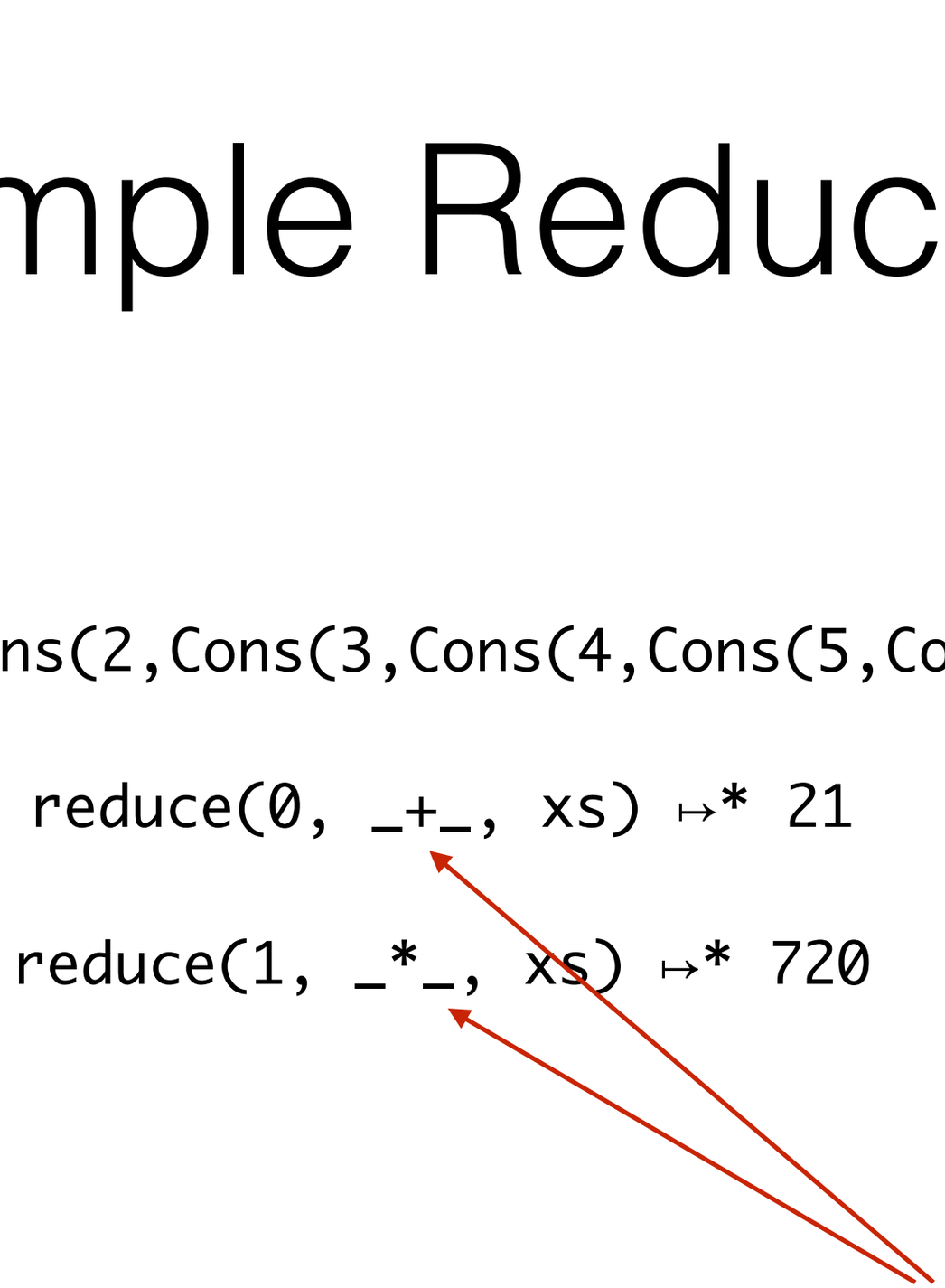
```
_ + _
```

Example Reductions

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

reduce(0, _+_, xs) ↦* 21

reduce(1, _*_, xs) ↦* 720
```



Note the multiple parameters

Combinations of Maps and Reductions

$$\sum_{n \in xs} n^2+1$$

Combinations of Maps and Reductions

```
reduce(0, _+_, map(x=> x*x + 1, xs))
```

↦

97

Summation

```
def square(x:Int) = x * x
```

```
def summation(f: Int => Int, xs: List) =
  reduce(0, _+_, map(f, xs))
```

Summation

```
summation(square(_)+1, xs) ↦ 97
```

More Syntactic Sugar

- Functions defined with `def` can be passed as arguments whenever an expression of a compatible function type is expected

- What constitutes a compatible function type?

Partially Applied Functions

- If we want to pass a function as an argument, but supply some of the arguments to the function overselves, we can wrap an application to the function in a function literal:

```
map(x=>x + 1, xs)
```

Partially Applied Functions

- If we want to pass a function as an argument, but supply some of the arguments to the function overselves, we can wrap an application to the function in a function literal:

```
map(x=>x + 1, xs)
```

which is equivalent to

```
map(_ + 1, xs)
```

Partially Applied Functions

- **Eta Expansion:** Wrapping a function in function literal that takes all of the arguments of f and immediately calls f with those arguments

```
(x:Int) => square(x)
```

is equivalent to

```
square
```

Mapping a Computation Over a List

We can use eta expansion to pass operators as arguments:

```
map(x => -x, xs)
```

Mapping a Computation Over a List

We can use eta expansion to pass operators as arguments:

```
map(-_, xs)
```

Recommended Viewing

- Guy L. Steele Jr., “Growing a Language”:

  https://www.youtube.com/watch?v=_ahvzDzKdB0