COMP 322: Fundamentals of Parallel Programming

Lecture 9: Memoization

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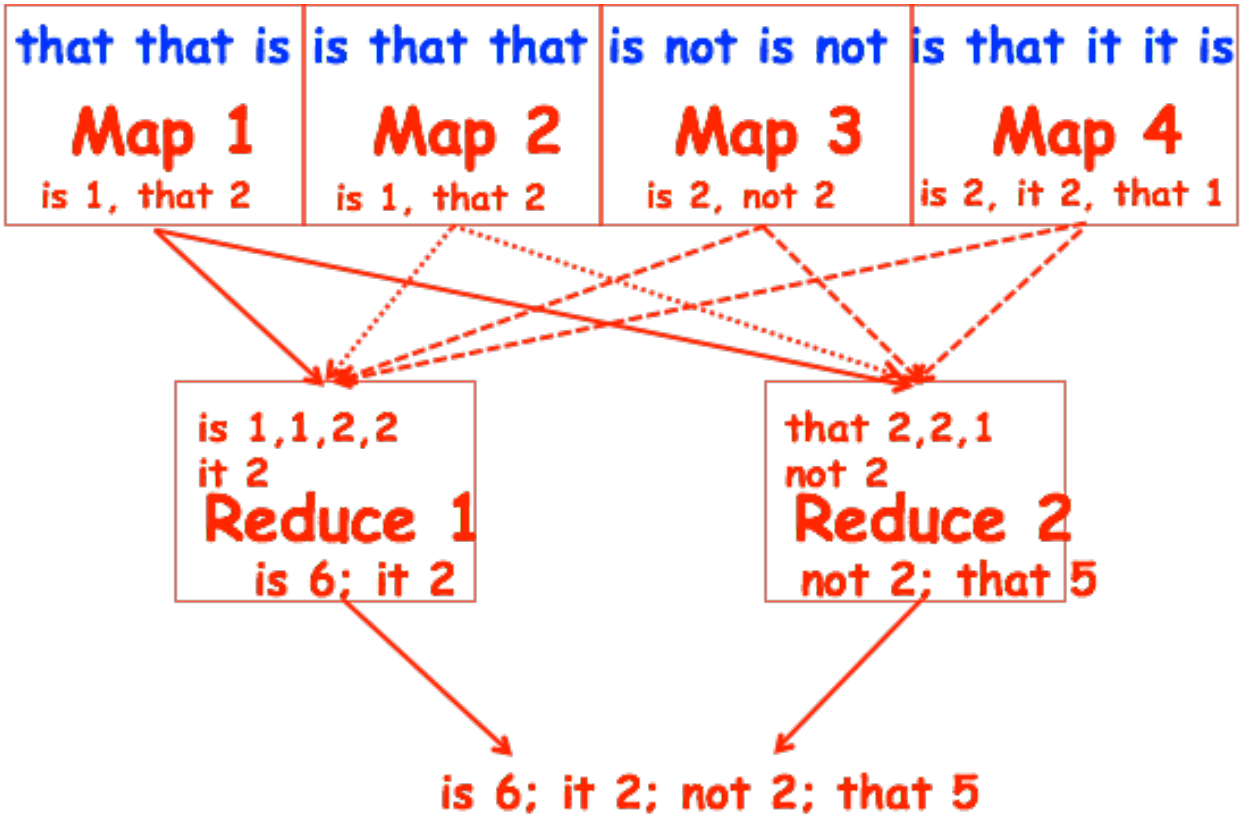
https://wiki.rice.edu/confluence/display/PARPROG/COMP322

Worksheet #8 solution: Analysis of Map Reduce Example

Analyze the total WORK and CPL for the Map reduce example in the previous slide, under the following assumptions:

- Assume that each Map step has WORK = number of input words, and CPL=1
- Assume that each Reduce step has WORK = number of input word-count pairs, and CPL = log2(# occcurences for input word with largest # pairs)



WORK/CPL for all Map steps:

- WORK = 15
- CPL = 1

WORK/CPL for Reduce 1 step:

- WORK = 5
- CPL = log2(4) = 2

WORK/CPL for Reduce 2 step:

- WORK = 4
- CPL = log2(3) = 2 (round up)

Total WORK and CPL

- WORK = 15+5+4 = 24
- CPL = 1 + 2 = 3

Background: Functional Programming

- Eliminate side-effects
  - emphasizes functions whose results that depend only on their inputs and not on any other program state
  - calling a function, f(x), twice with the same value for the argument x will produce the same result both times

Example: Binomial Coefficient

- The coefficient of the x^k term in the polynomial expansion of the binomial power $(1 + x)^n$
- Number of sets of k items that can be chosen from n items
- Indexed by *n* and *k*
  - written as C(n, k)
  - read as "n choose k"
- Factorial Formula: C(n, k) = $\left(\frac{n!}{k!(n-k)!} \right)$
- Recursive Formula

  C(n, k) = C(n – 1, k – 1) + C(n – 1, k)

  Base cases: C(n, n) = C(n, 0) = C(0, k) = 1

Example: Binomial Coefficient (Recursive Sequential version)

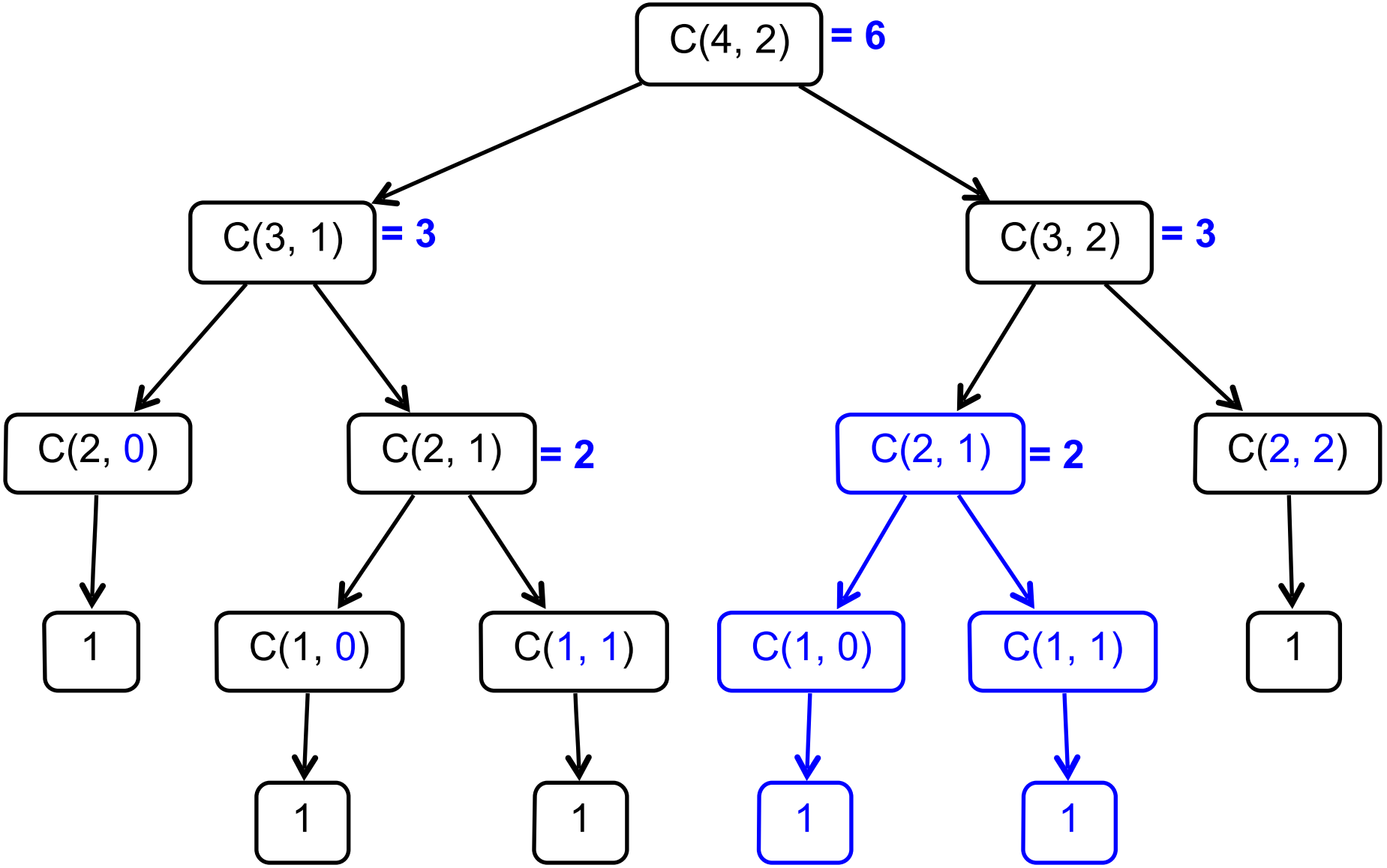
```
1. int choose(final int N, final int K) {
2.     if (N == 0 || K == 0 || N == K) {
3.         return 1;
4.     }
5.     final int left  = choose (N-1, K - 1);
6.     final int right = choose (N-1, K);
7.     return left + right;
8. }
```


Example: Binomial Coefficient (Parallel Recursive Pseudocode)

```
1. int choose(final int N, final int K) {
2.     if (N == 0 || K == 0 || N == K) {
3.         return 1;
4.     }
5.     final future<int> left  =
6.                 future { return choose (N-1, K-1); }
7.     final future<int> right =
8.                  future { return choose (N-1, K); }
7.     return left.get() + right.get();
8. }
```

- Use of futures supports incremental parallelization with low developer effort

What inefficiencies do you see in the recursive Binomial Coefficient algorithm?

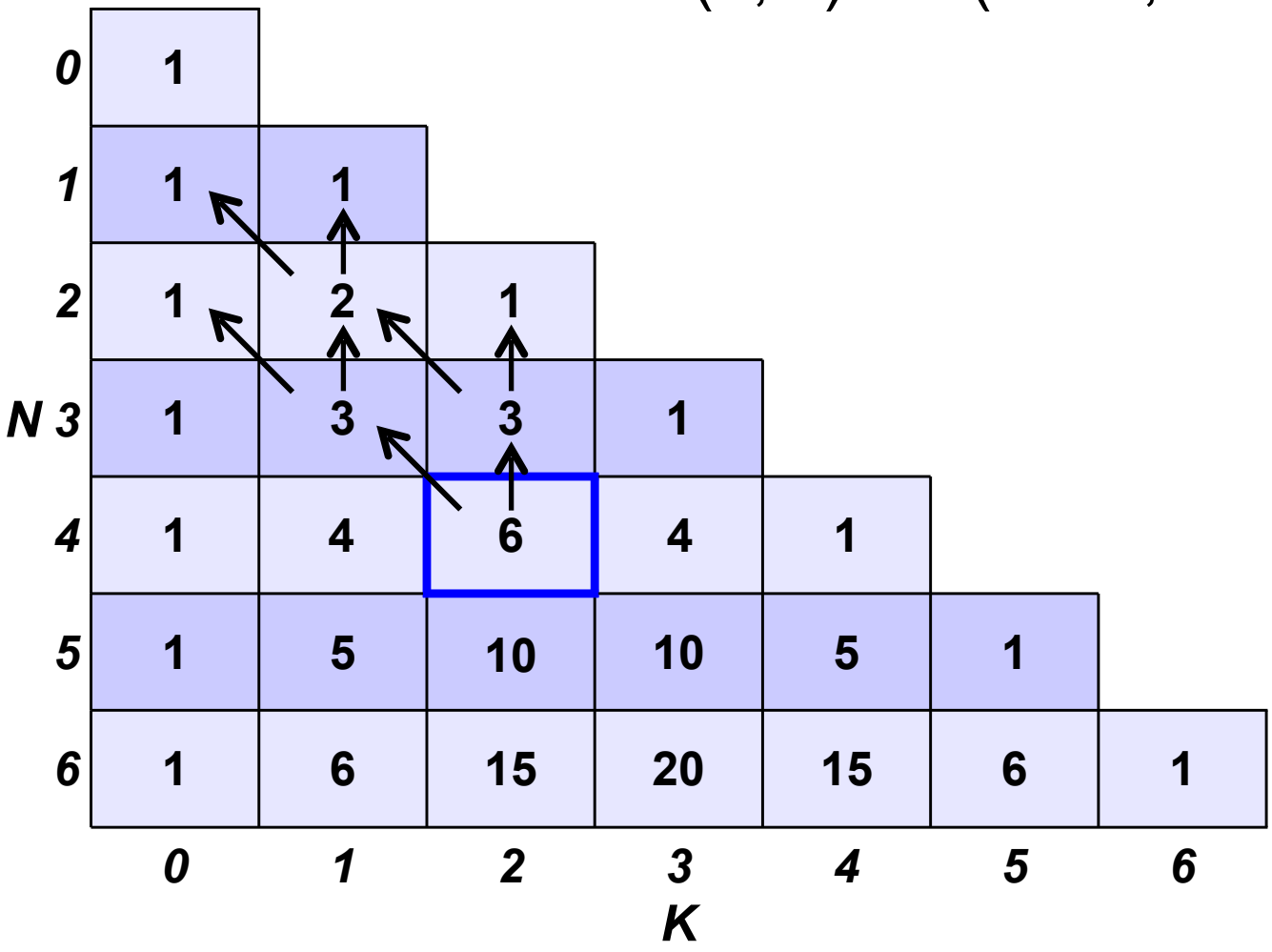


Memoization

- Memoization is the idea of saving and reusing previously computed values of a function rather than recomputing them
  - A space-time tradeoff
- A function can only be memoized if it is *referentially transparent*, i.e. functional
- Related to caching
  - memoized function "remembers" the results corresponding to some set of specific inputs
  - memoized function populates its cache of results transparently on the fly, as needed, rather than in advance

Pascal's Triangle is an exmaple of Memoization

$$C(n, k) = C(n - 1, k - 1) + C(n - 1, k)$$



Example: Binomial Coefficient (sequential memoized version)

```
1.  final Map<Pair<Int, Int>, Int> cache = new ...;

2.  int choose(final int N, final int K) {
3.      final Pair<Int, Int> key = Pair.factory(N, K);
4.      if (cache.contains(key)) {
5.          return cache.get(key);
6.      }
7.      if (N == 0 || K == 0 || N == K) {
8.          return 1;
9.      }
10.     final int left  = choose (N - 1, K - 1);
11.     final int right = choose (N - 1, K);
12.     final int result = left + right;
13.     cache.put(key, result);
14.     return result;
15. }
```


Example: Binomial Coefficient (parallel memoized version w/ futures)

```
1.  final Map<Pair<Int, Int>, future<Int>> cache = new ...;
2.  int choose(final int N, final int K) {
3.      final Pair<Int, Int> key = Pair.factory(N, K);
4.      if (cache.contains(key)) {
5.          return cache.get(key).get();
6.      }
7.      future<Int> f = future {
7.        if (N == 0 || K == 0 || N == K) return 1;
8.        int left = future { return choose (N-1, K-1); }
9.        int right = future { return choose (N-1, K); }
12.       return left.get() + right.get();
13.     }
14.     cache.put(key, f);
15.     return f.get();
16. }
```

- Assumes availability of a "thread-safe" cache library, e.g., ConcurrentHashMap

References

- Topic 2.2 Lecture & Demonstration
- Recursion and Memoization: http://zoo.cs.yale.edu/classes/cs201/Spring_2014/topics/topic-memoization.pdf
- Memoization: http://en.wikipedia.org/wiki/Memoization
- Functional Programming: http://en.wikipedia.org/wiki/Functional_programming
- Binomial coefficient: http://en.wikipedia.org/wiki/Binomial_coefficient