Comp 311
Functional Programming

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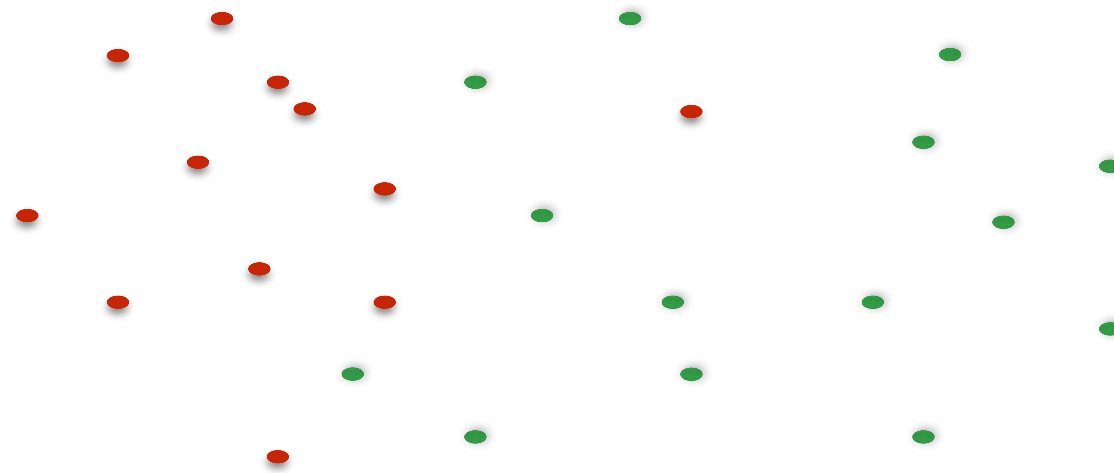
Machine Learning With Spark

- Given a collection of examples with various attributes and a label, we wish to predict the labels for new examples:

```
<height, weight, age, systolic bp, diastolic bp>: medicine?

<170 cm, 72 kg, 52, 120, 80>: YES

<150 cm, 60 kg, 34 years, 130, 70> : NO

…
```

Machine Learning With Spark

- We can view the examples as vectors in a high-dimensional vector space
- The problem of labeling yes/no can be solved by finding the best hyperplane that divides the given examples according to their labels
- This new hyperplane can be used to predict labels for new examples



Machine Learning With Spark

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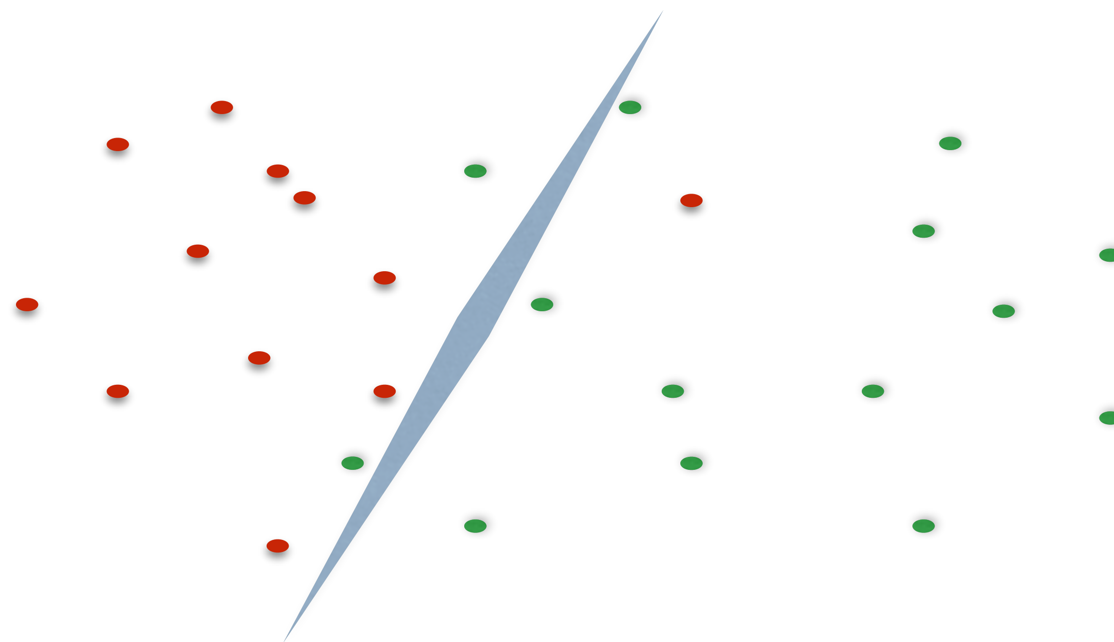

Logistic Regression With Spark

```
val points = spark.textFile(…).map(parsePoint).cache()

var w = Vector.random(D) // current separating plane

for (i <- 1 to ITERATIONS) {
  val gradient = points.map(p =>
    (1 / (1 + exp(-p.y*(w dot p.x))) - 1) * p.y * p.x
  ).reduce(_ + _)

  w -= gradient
}

println("Final separating plane: " + w)
```

Homework 6

```scala
def tactic(state: ProofState => (PartialProof, ProofState)) =
  StateAction[ProofState, PartialProof](state)
```

```
def orElimTactic(f: Formula) = tactic {
  (proofState: ProofState) => {
    (f, proofState) match {
      case (p \/ q, ((gamma :- r) :: goals)) =>
        def partialProof(proofs: List[Sequent]) = {
          proofs match {
            case (proofA :: proofB :: proofC :: Nil) =>
              orElim(proofA, proofB, proofC)
            case _ => throw new ProofError("orElim applied to " + proofs)
          }
        }
        (partialProof,
         (gamma :- p \/ q) :: (gamma + p :- r) :: (gamma + q :- r) :: goals)
      case _ => throw TacticError("orElimTactic applied to " + proofState)
    }
  }
}
```

```
def impliesElimTactic(p: Formula) = tactic {
  (proofState: ProofState) => {
    proofState match {
      case ((gamma :- q) :: goals) =>
        def partialProof(proofs: List[Sequent]) = {
          proofs match {
            case (proofA :: proofB :: Nil) =>
              impliesElim(proofA, proofB)
            case _ => throw new ProofError("impliesElim applied to " + proofs)
          }
        }
        (partialProof, ((gamma :- (p -> q)) :: (gamma :- p) :: goals))
      case _ => throw TacticError("impliesElimTactic applied to " + proofState)
    }
  }
}
```

Summary and Conclusion

Functional Programming

- A style of programming involving no mutation
- A style of programming in which computations are represented by passing functions as arguments and returning them as results
- A style of programming in which the entirety of a computation is determined by explicit input and output values

The Substitution Model

- The behavior of purely functional programs can be understood via the Substitution Model of Evaluation
- Having a rigorous model allows us to reason more precisely about the behavior of programs

Static Types and Values

- There is a deep connection between computation of static types and computation of values
- We can think of a computation as having both a static and dynamic evaluation

Design Recipes

- Categorizing solution based on templates
- Test-driven development

Abstract and Recursively Defined Datatypes

- Immutable data is often well characterized by this framework
- Language syntax is defined by the same framework
- Every abstract datatype can be thought of as characterizing its own language

Pattern Matching

- Dramatically improves the conciseness of computations over abstract datatypes

Variance in Types

- Immutable datatypes naturally lead to more expressive type relationships
  - Arrow types
  - Immutable collections

Programming Principles

- Keep It Simple
- Don’t Repeat Yourself

First-Class Functions

- Enables significant reduction in code repetition
- Leads to new ways of thinking about and organizing computations
  - map
  - reduce
  - filter
  - flatMap

Monads

- An overarching pattern of organization for many computations can be found via the concept of monads
- For-expressions are the syntax of monadic computation

Monads

- Monads express many computing constructs
  - Collections
  - Options
  - Purely functional state

Continuations

- A unifying construct for control in computation
  - Exceptions
  - Concurrency
  - Pre-emption

The Environment Model

- Facilitates reasoning about mutual recursion
- Facilitates reasoning about state
- Facilitates reasoning about types
- Facilitates reasoning about logics

Alternative Approaches to Computation

- Lexical vs Dynamic Scoping
  - Dynamic scoping is incompatible with static types
- Call-by-Value vs Call-by-Name
  - Call-by-Name is in often a better fit for functional programming
- Traits
  - Composable units of computation

Generative Recursion

- Not all computations can be expressed as structural traversals of abstract datatypes
  - *But many can!*
- Sometimes either domain knowledge or deep algorithmic insights is needed
  - Quicksort, Heaps, Red-Black Trees

Accumulators and Tail Recursion

- There is far more to accumulators than just simple tail recursion
  - Determine the *meaning* of your accumulator variables, and respect that meaning
  - Distinct approaches to accumulator meaning lead to significantly different results
    - Tree traversal, peg solitaire

State and Functional Programming

- Streams
  - Call-By-Name-Style lists
  - Often an effective alternative to stateful computation
- Memoization
  - Using state to improve performance of stateless computation
- The State Monad
  - Encapsulate and replay stateful computation, providing initial state as input

The Curry-Howard Isomorphism

- There is a deep connection between types and logical propositions
- There is a deep connection between programs and logical proofs

Some Scala Libraries of Note

- Parser combinators
- Actors for concurrency

Tactical Theorem Proving

- One of the driving problems driving the development of functional programming
- Term-rewriting systems
  - A unifying concept for computer science
  - Appear in types, models of computation, logic
- Tactics and the state monad

Functional Distributed Computing

- MapReduce
- Apache Spark and RDDs
- Distributed machine learning

A Functional Programming Renaissance

- Multiple trends are driving software design toward functional programming
  - Multicore processors
  - Big data

Take Advantage of the Work of the Past

- The Substitution Model
- Lexical scoping
- Closures
- Streams
- Monads
- Memoization
- Continuations

Consider Comp 411

- Application of functional programming to defining programming languages
- Abstract datatypes to define syntax
- Type checkers to define static semantics
- Interpreters to define dynamic semantics (via a meta-language)
- Incrementally enhance understanding by removing available features from the meta-language

Adopt a Sense of Coding Craftsmanship

- Keep It Simple
- Don’t Repeat Yourself
- Test from the start!

Teach a Class

- There's no better way to learn a subject