Consider the case when multiple threads call `insert()` and `remove()` methods concurrently for a single `BoundedBuffer` instance with `SIZE >= 1`.

1) Can you provide an example in which the wait set includes a thread waiting at line 2 in `insert()` and a thread waiting at line 11 in `remove()`, in slide 11? If not, why not?

No, only producer threads enter the wait set when the buffer is full, and only consumer threads enter the wait set when the buffer is empty.

2) How would the code behave if all wait/notify calls (lines 2, 6, 11, 15) were removed from the `insert()` and `remove()` methods in slide 11?

- `insert()` may overwrite existing elements when buffer is supposed to be full
- `remove()` may return undefined values when buffer is supposed to be empty
Concurrent Objects

- A concurrent object is an object that can correctly handle methods invoked concurrently by different tasks or threads
  - Also referred to as “thread-safe objects”
  - e.g., AtomicInteger, ConcurrentHashMap, BoundedBuffer, …

- For the discussion of linearizability, we will assume that the body of each method in a concurrent object is itself sequential
  - Assume that methods do not create child async tasks

- Implementations of methods can be serialized (e.g., using synchronized or object-based isolated statements) or can be concurrent (e.g., by using read-write modes in object-based isolation)

- A desirable goal is to develop implementations that are concurrent for performance while being as close to the semantics of the serial version as possible
Correctness of a Concurrent Object

• Consider a simple FIFO (First In, First Out) queue as a canonical example of a concurrent object
  — Method `q.enq(o)` inserts object `o` at the tail of the queue
    – Assume that there is unbounded space available for all `enq()` operations to succeed
  — Method `q.deq()` removes and returns the item at the head of the queue.
    – Throws `EmptyException` if the queue is empty.

• What does it mean for a concurrent object like a FIFO queue to be correct?
  — What is a concurrent FIFO queue?
  — FIFO implies a strict temporal order
  — Concurrent implies an ambiguous temporal order
Linearization: identifying a sequential order of concurrent method calls

```
q.deq():x
isolated-wait/begin
enq(x)
isolated-end

Task T1

q.enq(x)
isolated-wait/begin
deq
isolated-end

Task T2

"Linearizability" -- sequence of enq() and deq() calls is consistent with sequential execution
```

Source: http://www.elsevierdirect.com/companions/9780123705914/Lecture%20Slides/03~Chapter_03.ppt
Informal definition of Linearizability

• Assume that each method call takes effect “instantaneously” at some point in time between its invocation and return.

• An execution (schedule) is linearizable if we can choose one set of instantaneous points that is consistent with a sequential execution in which methods are executed at those points
  • It’s okay if some other set of instantaneous points is not linearizable

• A concurrent object is linearizable if all its executions are linearizable
  • Linearizability is a “black box” test based on the object’s behavior, not its internals
Example 1

Task T1

$q\text{.enq}(x)$

Source: http://www.elsevierdirect.com/companions/9780123705914/Lecture%20Slides/03~Chapter_03.ppt
Example 1 (contd)

Task T1
q.enq(x)

Task T2
q.enq(y)

Source: http://www.elsevierdirect.com/companions/9780123705914/Lecture%20Slides/03~Chapter_03.ppt
Example 1 (contd)

Task T1

q.enq(x)

q.enq(y)

Task T2

q.deq():x

Source: http://www.elsevierdirect.com/companions/9780123705914/Lecture%20Slides/03~Chapter_03.ppt
Example 1 (contd)

Task T1

Task T2

Source: http://www.elsevierdirect.com/companions/9780123705914/Lecture%20Slides/03~Chapter_03.ppt
Example 1 (contd)

Task T1

(1) q.enq(x)
(2) q.enq(y)

Task T2

(3) q.deq():x
(4) q.deq():y

linearizable

Source: http://www.elsevierdirect.com/companions/9780123705914/Lecture%20Slides/03~Chapter_03.ppt
Example 2: is this execution linearizable?

Source: [http://www.elsevierdirect.com/companions/9780123705914/Lecture%20Slides/03~Chapter_03.ppt](http://www.elsevierdirect.com/companions/9780123705914/Lecture%20Slides/03~Chapter_03.ppt)
Example 3

Is this execution linearizable? How many possible linearizations does it have?

(linearizable) (two possible linearizations)
Example 4: execution of an isolated implementation of FIFO queue q

Is this a linearizable execution?

<table>
<thead>
<tr>
<th>Time</th>
<th>Task A</th>
<th>Task B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Invoke q.enq(x)</td>
<td>Invoke q.enq(y)</td>
</tr>
<tr>
<td>1</td>
<td>Work on q.enq(x)</td>
<td>Work on q.enq(y)</td>
</tr>
<tr>
<td>2</td>
<td>Work on q.enq(x)</td>
<td>Work on q.enq(y)</td>
</tr>
<tr>
<td>3</td>
<td>Return from q.enq(x)</td>
<td>Return from q.enq(y)</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>Invoke q.deq()</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>Return x from q.deq()</td>
</tr>
</tbody>
</table>

Yes! Can be linearized as “q.enq(x) ; q.enq(y) ; q.deq():x”.
Example 5: execution of a concurrent implementation of a FIFO queue q

Is this a linearizable execution?

<table>
<thead>
<tr>
<th>Time</th>
<th>Task A</th>
<th>Task B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Invoke q.enq(x)</td>
<td>Invoke q.enq(y)</td>
</tr>
<tr>
<td>1</td>
<td>Work on q.enq(x)</td>
<td>Return from q.enq(y)</td>
</tr>
<tr>
<td>2</td>
<td>Work on q.enq(x)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Return from q.enq(x)</td>
<td>Invoke q.deq()</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>Return x from q.deq()</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Yes! Can be linearized as “q.enq(x) ; q.enq(y) ; q.deq():x”.
Linearizability of Concurrent Objects (Summary)

Concurrent object

- A concurrent object is an object that can correctly handle methods invoked in parallel by different tasks or threads
  - Examples: concurrent queue, AtomicInteger

Linearizability

- Assume that each method call takes effect "instantaneously" at some distinct point in time between its invocation and return.
- An execution is linearizable if we can choose instantaneous points that are consistent with a sequential execution in which methods are executed at those points.
- An object is linearizable if all its possible executions are linearizable.