COMP 322: Fundamentals of Parallel Programming

Lecture 6: Memoization

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Worksheet #5: Computation Graphs for Async-Finish and Future Constructs

1) Can you write pseudocode with `async-finish` constructs that generates a Computation Graph with the same ordering constraints as the graph on the right? If so, provide a sketch of the program.

No. Finish cannot be used to ensure that D waits for both B and C, while E waits only for C.

2) Can you write pseudocode with `future async-get` constructs that generates a Computation Graph with the same ordering constraints as the graph on the right? If so, provide a sketch of the program.

Yes, see program sketch with void futures. A dummy return value can also be used.
1. HjFuture<Void> A = future(() -> {
  return "A"; });
2. HjFuture<Void> B = future(() -> {
  A.get(); return "B"; });
3. HjFuture<Void> C = future(() -> {
  A.get(); return "C"; });
4. HjFuture<Void> D = future(() -> {
  // Order of B.get() & C.get() doesn't matter
  B.get(); C.get(); return "D"; });
5. HjFuture<Void> E = future(() -> {
  C.get(); return "E"; });
6. HjFuture<Void> F = future(() -> {
  D.get(); E.get(); return "F"; });
7. F.get();
Background: Functional Programming

• Eliminate side-effects
  • emphasizes functions whose results that depend only on their inputs and not on any other program state
  • calling a function, f(x), twice with the same value for the argument x will produce the same result both times

Helpful Link: http://en.wikipedia.org/wiki/Functional_programming
Example: Binomial Coefficient

• The coefficient of the $x^k$ term in the polynomial expansion of the binomial power $(1 + x)^n$
• Number of sets of $k$ items that can be chosen from $n$ items
• Indexed by $n$ and $k$
  • written as $C(n, k)$
  • read as “n choose k”
• Factorial Formula: $C(n, k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$
• Recursive Formula
  $C(n, k) = C(n - 1, k - 1) + C(n - 1, k)$
  Base cases: $C(n, n) = C(n, 0) = C(0, k) = 1$

Example: Binomial Coefficient
(Recursive Sequential version)

1. int choose(int N, int K) {
2.     if (N == 0 || K == 0 || N == K) {
3.         return 1;
4.     }
5.     int left = choose (N-1, K - 1);
6.     int right = choose (N-1, K);
7.     return left + right;
8. }


Example: Binomial Coefficient (Parallel Recursive Pseudocode)

1. int choose(int N, int K) {
2.     if (N == 0 || K == 0 || N == K) {
3.         return 1;
4.     }
5.     future<int> left =
6.         future { return choose (N-1, K-1); }
7.     future<int> right =
8.         future { return choose (N-1, K); }
9.     return left.get() + right.get();
10. }

- Use of futures supports incremental parallelization with low developer effort
What inefficiencies do you see in the recursive Binomial Coefficient algorithm?

C(4, 2) = 6

C(3, 1) = 3

C(3, 2) = 3

C(2, 0)

C(2, 1) = 2

C(1, 0)

C(1, 1)

C(2, 2)
Memoization

• Memoization - saving and reusing previously computed values of a function rather than recomputing them
  • A optimization technique with space-time tradeoff
• A function can only be memoized if it is referentially transparent, i.e. functional
• Related to caching
  • memoized function "remembers" the results corresponding to some set of specific inputs
  • memoized function populates its cache of results transparently on the fly, as needed, rather than in advance

Helpful Link: http://en.wikipedia.org/wiki/Memoization
Pascal’s Triangle is an example of Memoization

\[ C(n, k) = C(n - 1, k - 1) + C(n - 1, k) \]
Example: Binomial Coefficient (sequential memoized version)

1. final Map<Pair<Int, Int>, Int> cache = new ...;

2. int choose(int N, int K) {
3.     Pair<Int, Int> key = Pair.factory(N, K);
4.     if (cache.contains(key)) {
5.         return cache.get(key);
6.     }
7.     if (N == 0 || K == 0 || N == K) {
8.         return 1;
9.     }
10.    int left = choose (N - 1, K - 1);
11.    int right = choose (N - 1, K);
12.    int result = left + right;
13.    cache.put(key, result);
14.    return result;
15. }
Example: Binomial Coefficient
(parallel memoized version w/ futures)

1. final Map<Pair<Int, Int>, Int> cache = new ...;
2. int choose(final int N, final int K) {
3.     final Pair<Int, Int> key = Pair.factory(N, K);
4.     if (cache.contains(key)) {
5.         return cache.get(key);
6.     }
7.     if (N == 0 || K == 0 || N == K) {
8.         return 1;
9.     }
10.    future<int> left = future { return choose(N - 1, K - 1); }
11.    future<int> right = future { return choose(N - 1, K); }
12.    int result = left.get() + right.get();
13.    cache.put(key, result);
14.    return result;
15. }

• Assumes availability of a “thread-safe” cache library, e.g., ConcurrentHashMap
Example: Binomial Coefficient (parallel memoized version w/ futures - better)

1. final Map<Pair<Int, Int>, future<Int>> cache = new ...;
2. int choose(final int N, final int K) {
    final Pair<Int, Int> key = Pair.factory(N, K);
    if (cache.contains(key)) {
        return cache.get(key).get();
    }
    future<Int> f = future {
        if (N == 0 || K == 0 || N == K) return 1;
        future<int> left = future { return choose (N-1, K-1); }
        future<int> right = future { return choose (N-1, K); }
        return left.get() + right.get();
    }
    cache.put(key, f);
    return f.get();
}

• Assumes availability of a “thread-safe” cache library, e.g., ConcurrentHashMap