Comp 311
Functional Programming

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Announcements

- Homework 1 will be assigned next Thursday
- Watch “Working Hard to Keep it Simple” available on the course website

Including Constant Definitions

- We can include constant definitions within functions by using `val`
- We refer to expressions prefixed with a sequence of constant definitions as compound expressions

Place After The Requires Clause and Before the "Result" Expression

```
def cost(ticketPrice: Int) = {
  require (ticketPrice >= 0 & ticketPrice <= 1000)

  val fixedCost = 18000
  val perAttendeeCost = 4

  fixedCost + perAttendeeCost * attendance(ticketPrice)
} ensuring (_ >= 0)
```

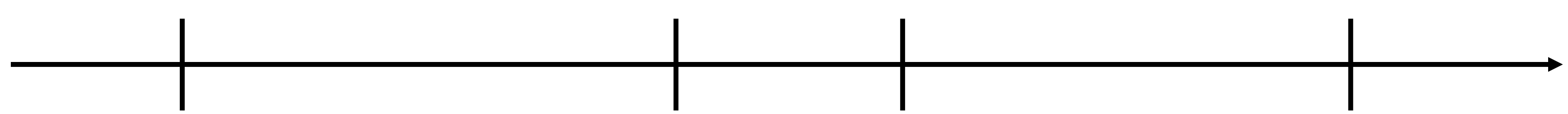
To Reduce A Compound Expression

- First compute the value of each constant definition, top to bottom
- Then reduce the result expression, substituting each occurrence of a constant name with its computed value

Conditional Functions On Ranges

Conditional Functions On Ranges

- Often a computation falls into distinct cases depending on which of a finite set of ranges a value falls into
  - In such cases, it can help to break the number line into distinct regions that we must handle separately:

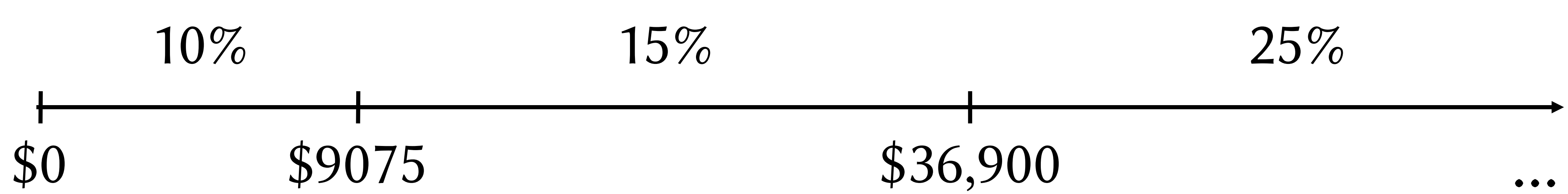


Designing Conditional Functions

- Example: Graduated Income Tax (Single Filer):
    - Up to $9,075: 10%
    - $9,075 to $36,900: 15%
    - $36,901 to $89,350: 25%
    - $89,351 to 186,350: 28%
    - $186,351 to $405,100: 33%
    - $405,101 to $406,750: 35%
    - $405,751 or more: 39.6%
- We follow the Design Recipe

Graduated Income Tax: Data Analysis and Definition

- We use `Int`s to denote US$ values and tax percentages
- Both income and tax should be non-negative
- We break the number line into the relevant intervals



Contract

```
/**
 * Given an income in U.S. Dollars,
 * returns the dollar value of tax
 * owed for a single tax payer, using
 * 2014-2015 IRS tax brackets.
 */
def incomeTax(income: Int) = {
  require(income >= 0)
  ...
} ensuring (_ >= 0)
```

Function Application Examples

We should develop at least one example per case, as well as borderline cases

```
100 = incomeTax(1000)

907 = incomeTax(9075)

907 + 138 = incomeTax(10000)

...
```

Our Function Template for Conditional Functions

```
/**
 * Given an income in U.S. Dollars,
 * returns the dollar value of tax
 * owed for a single tax payer, using
 * 2014-2015 IRS tax brackets.
 */
def incomeTax(income: Int): Int = {
  require(income >= 0)

  if (income <= cutoff0) {
   ...
  } else if (income <= cutoff1) {
   ...
  } else if (income <= cutoff2) {
   ...
  } else if (income <= cutoff3) {
   ...
  } else if (income <= cutoff4) {
   ...
  } else if (income <= cutoff5) {
   ...
  } else if (income <= cutoff6) {
   ...
  } else { // income > cutoff6
   ...
  }
} ensuring (_ >= 0)
```

Defining Our Constant Values in One Place

```
val bracket0 = 0
val cutoff0 = 0

val bracket1 = 100
val cutoff1 = 9075

val bracket2 = 150
val cutoff2 = 36900

val bracket3 = 250
val cutoff3 = 89350

val bracket4 = 280
val cutoff4 = 186350

val bracket5 = 330
val cutoff5 = 405100

val bracket6 = 350
val cutoff6 = 406750

val bracket7 = 396
val cutoff7 = Int.MaxValue
```

As We Fill In Cases, We Find a Common Pattern

```
/**
 * Given:
 *   an income in U.S. Dollars
 *   the next lowest cutoff in U.S. Dollars
 *   a tax percentage for the bracket above the cutoff
 * Returns the income tax due for the given income
 */
def incomeTaxForBracket(income: Int, cutoff: Int, bracket: Int): Int = {
  require(income >= 0)
  (income - cutoff) * bracket / divisor + incomeTax(cutoff)
} ensuring (_ >= 0)
```

And Now We Call This New Function to Fill in the The Income Tax Function Template

```scala
/**
 * Given an income in U.S. Dollars, returns the dollar value of tax
 * owed for a single tax payer, using 2014-2015 IRS tax brackets.
 */
def incomeTax(income: Int): Int = {
  require(income >= 0)

  if (income <= cutoff0) {
    bracket0
  } else if (income <= cutoff1) {
    incomeTaxForBracket(income, cutoff0, bracket1)
  } else if (income <= cutoff2) {
    incomeTaxForBracket(income, cutoff1, bracket2)
  } else if (income <= cutoff3) {
    incomeTaxForBracket(income, cutoff2, bracket3)
  } else if (income <= cutoff4) {
    incomeTaxForBracket(income, cutoff3, bracket4)
  } else if (income <= cutoff5) {
    incomeTaxForBracket(income, cutoff4, bracket5)
  } else if (income <= cutoff6) {
    incomeTaxForBracket(income, cutoff5, bracket6)
  } else { // income > cutoff6
    incomeTaxForBracket(income, cutoff6, bracket7)
  }
} ensuring (_ >= 0)
```

Remarks On Conditional Functions

- The clauses in a conditional function need not all have the same form
- Avoid factoring out code into a helper function until there is more than one place to call the helper
- There is more we can factor out in this example, but first we will need more powerful language features (stay tuned)

Conditional Functions On Point Values

Conditional Functions On Point Values

- Often the cases on a conditional function must test for equality rather than whether values fall in a range
    - This is especially common with String values
    - What about Boolean values?
    - Double values should not be tested this way (why?)

Example: Days in a Month

Given the name of a month, we want to return the number of days

Data Analysis and Definition

We use `String`s to denote months and `Int`s for the number of days

Contract

- We state the preconditions in documentation:

```
/**
 * Given a string identifying a month,
 * with the first (and only the first) letter capitalized,
 * returns the number of days in that month
 * for an ordinary year (non-leap) year.
 */
def days(month: String): Int = {
  ...
} ensuring (_ <= 31)
. ensuring (0 < _)
```

- How can we improve the precondition? What data types would we want?

A Function Template for Conditional Functions on Point Values

```scala
/**
 * Given a string identifying a month,
 * with the first (and only the first) letter capitalized,
 * returns the number of days in that month
 * for an ordinary year (non-leap) year.
 */
def days(month: String): Int = {
  month match {
    case ... => ...
    ...
  }
} ensuring (_ <= 31)
. ensuring (0 < _)
```

Syntax for Match

```
expr0 match {
  case Pattern1 => expr1
    ...
  case PatternN => exprN
}
```

Primitive Value Patterns

A primitive value pattern is either:

- A primitive value
- A free parameter
- The special “don’t care” pattern: _

Matching a Primitive Value With a Pattern

A primitive value *v* matches:

- Itself
- A free parameter
- The special "don't care" pattern _
  - _ should only be used as the final clause of a match (why?)

Meaning of a Match Expression

- To reduce a match expression:

```
expr₀ match {
  case Pattern₁ => expr₁
    ...
  case Patternₙ => exprₙ
}
```

  - Reduce $\mathbf{expr_0}$ to a value **v**

  - Find the first pattern **k** matching **v** (if it exists) and reduce to $\mathbf{expr_k}$ (replacing all occurrences of **k** with **v** if **k** is a free parameter)

- Failure to match a pattern results in a new form of exceptional condition

Using Match for Point Value Matching

```
/**
 * Given a string identifying a month,
 * with the first (and only the first) letter capitalized,
 * returns the number of days in that month
 * for an ordinary year (non-leap) year.
 */
def days(month: String): Int = {
  month match {
    case "January" => 31
    case "February" => 28
    case "March" => 31
    case "April" => 30
    case "May" => 31
    case "June" => 30
    case "July" => 31
    case "August" => 31
    case "September" => 30
    case "October" => 31
    case "November" => 30
    case "December" => 31
  }
} ensuring (_ <= 31)
```

Reducing Match

```
days(“September”)
↦

“September” match {
    case "January" => 31
    case "February" => 28
    case "March" => 31
    case "April" => 30
    case "May" => 31
    case "June" => 30
    case "July" => 31
    case "August" => 31
    case "September" => 30
    case "October" => 31
    case "November" => 30
    case "December" => 31
  }
} ensuring (_ <= 31)
↦

30
```

A Match With a Free Parameter

```
def plural(word: String): String =
  word match {
    case "deer" => "deer"
    case "fish" => "fish"
    case "mouse" => "mice"
    case x => x + "s"
  }
```

Compound Datatypes

Compound Datatypes

- Although many computations can be performed on primitive data types, it is often useful to combine data into larger structures
- We call all data of this form *compound data*
- The two simplest compound datatypes in Core Scala are tuples and arrays

Tuple Values

- A tuple value contains a sequence of values

$$\texttt{(v}_1\texttt{, ..., v}_N\texttt{)}$$

- There is one empty tuple `()`
- Tuples of length one do not exist (why?)
- The value type of a tuple is simply the tuple of the corresponding value types

$$\texttt{(T}_1\texttt{, ..., T}_N\texttt{)}$$

Tuple Types

- The empty tuple has the special type `Unit`
- The static type of a tuple expression:

$$(e_1, \ldots e_N)$$

is

$$(T_1, \ldots, T_N)$$

where

$$e_1 : T_1, \ldots e_N : T_N$$

Tuple Types

- Tuple types allow us to combine data of distinct types. For example:

```
(Int, Boolean, String)
```

- However, tuple types restrict the length of any corresponding tuple value

Accessing Tuple Elements

- We can access the ***k*th** element of an expression **e** with static type **(T_1, ..., T_N)** using the syntax:

$$\texttt{e.\_}k$$

- The static type of this expression is T_k
- Note that tuples are 1-indexed
- Example:

```
(1,2,3)._2 ↦ 2
```

Accessing Tuple Elements

- We can access the elements of a tuple using match expressions
  - We add the following syntactic form to our definition of patterns

$$\texttt{(Pattern}_1\texttt{, ... , Pattern}_N\texttt{)}$$

- We call this new syntactic form a *tuple pattern*

Accessing Tuple Elements

- A tuple matches a tuple pattern iff each element of the tuple matches a corresponding element of the tuple pattern, and vice versa (bijection)
- Does `(x,y,z)` match `(1,2)`?

Income Tax Revisited

```
def incomeTaxForBracketCutoff(income: Int, bracketCutoff: (Int, Int)) = {
  require(income >= 0)

  bracketCutoff match {
    case (bracket, cutoff) => {
      (income - cutoff) * bracket /
        divisor + incomeTax(cutoff)
    }
  }
} ensuring (_ >= 0)
```

Tuple Types and Arrow Types

- We can now view every arrow type as taking exactly one parameter:
- Example:

```
(Int, String, Boolean) → Int
```

Tuple Types and Arrow Types

- We can also use tuple types to denote that a function returns “multiple values”:
- Example:

```
(Int, String, Boolean) → (Int, Double)
```

Array Values

- An array is a sequence of values all of the same value type

```
Array(1,2,3)
```

Array Types

- If the elements of an array value are of type T then the array is of type Array[T]

- If the expressions e_1, ... , e_N are of static type T then the expression

```
Array(e1, ..., eN)
```

- has static type

```
Array[T]
```

Array Types

- Array types require that all elements of an array share a common type
- However, array types match array values of any length
- Contrast with tuple types

Accessing Array Values

- We can access the `kth` element of an expression of type `Array[T]` with the syntax:

```
expr(k)
```

- The static type of this expression is `T`
- Note that arrays are zero-indexed
- Example:

```
Array(1,2,3)(2) ↦ 3
```

Accessing Array Elements

- We can access the elements of an array using match expressions
    - We add the following syntactic form to our definition of patterns:

$$\texttt{Array(Pattern}_1\texttt{, ... , Pattern}_N\texttt{)}$$

- We call this new syntactic form an *array pattern*

Accessing Array Elements

An array matches an array pattern iff each element of the array matches a corresponding element of the array pattern, and vice versa

Accessing Array Elements

```
def sumOfSquares(coordinates: Array[Int]) = {
  coordinates match {
    case Array(x,y,z) => x*x + y*y + z*z
  }
}
```

Structural Data

Structural Data

- Tuples and arrays allow us to combine multiple primitive values into a single data value
- However,
  - They do not allow us to attach names to the constituent elements
  - They do not allow us to distinguish elements of conceptually distinct datatypes

Case Classes

- We can think of a case class as a tuple with its own *type* and *accessors* for its elements

Case Classes

```
case class Coordinate(x: Int, y: Int)
```

Simple Syntax for Case Classes

`case class Name(field1: Type`$_1$`, ..., field`$_N$`: Type`$_N$`)`

Creating Instances of a Case Class

- We construct new instances of a case class

```
case class C(field₁: Type₁, ..., fieldN: TypeN)
```

- with the syntax

$$\texttt{C(expr}_1\texttt{, ..., expr}_N\texttt{)}$$

- To reduce this expression, reduce each argument $\texttt{expr}_K$ to a value $\texttt{v}_K$, forming the value $\texttt{C(v}_1\texttt{, ..., v}_N\texttt{)}$
- If the types of $\texttt{expr}_1\texttt{,...,expr}_N$ match the types of the corresponding fields, then this expression has type `C`

Accessing Fields of a Case Class

- Given a case class:
  `case class C(field₁: Type₁, ..., field_N: Type_N)`

- We can access field with name $\texttt{field}_K$ of an instance $\texttt{C(v}_1\texttt{, ..., v}_N\texttt{)}$ with the expression syntax:

$$\texttt{C(v}_1\texttt{,...,v}_N\texttt{).field}_K$$

- The static type of this expression is $\texttt{Type}_K$

Accessing Fields of a Case Class

```
def magnitude(coordinate: Coordinate) = {
  coordinate.x * coordinate.x +
  coordinate.y * coordinate.y
}
```

Accessing Class Elements

- We can access the elements of a case class instance using match expressions
  - For each case class, we add the following syntactic form to our definition of patterns

$$\texttt{C(Pattern}_1\texttt{, ... , Pattern}_N\texttt{)}$$

- We call this new syntactic form a *class pattern*

Accessing Case Class Elements

- An instance of a case class `C(v`$_1$`, ..., v`$_N$`)` matches a class pattern `C(P`$_1$`, ..., P`$_N$`)` iff
  - The class name is identical to the class pattern name
  - Each element of the instance matches a corresponding element of the class pattern

Accessing Case Class Elements

```
def magnitude(coordinate: Coordinate) =
  coordinate match {
    case Coordinate(x,y) => x*x + y*y
  }
```