Comp 311
Functional Programming

Nick Vrvilo, Two Sigma Investments
Robert "Corky" Cartwright, Rice University

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Family Trees

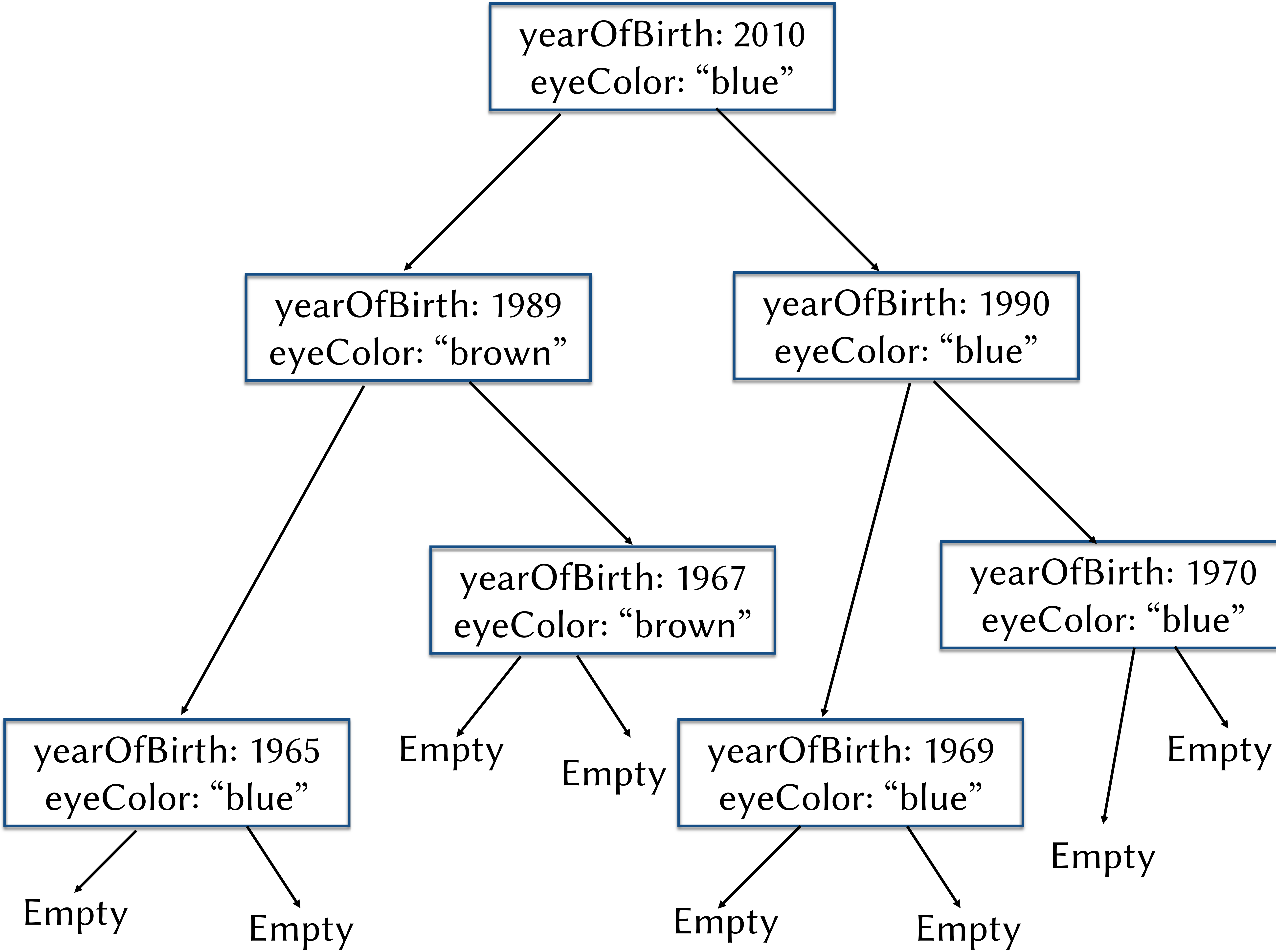
```
TreeNode ::= Empty
           | Child(TreeNode,
                   TreeNode,
                   Int,
                   String)
```

Family Trees

```
abstract class TreeNode

case object EmptyNode extends TreeNode

case class Child(mother: TreeNode,
                 father: TreeNode,
                 yearOfBirth: Int,
                 eyeColor: String)
extends TreeNode
```


yearOfBirth: 2010
eyeColor: "blue"
yearOfBirth: 1989
eyeColor: "brown"
yearOfBirth: 1990
eyeColor: "blue"
yearOfBirth: 1967
eyeColor: "brown"
yearOfBirth: 1970
eyeColor: "blue"
yearOfBirth: 1965
eyeColor: "blue"
Empty
Empty
yearOfBirth: 1969
eyeColor: "blue"
Empty
Empty
Empty
Empty
Empty
Empty
Empty

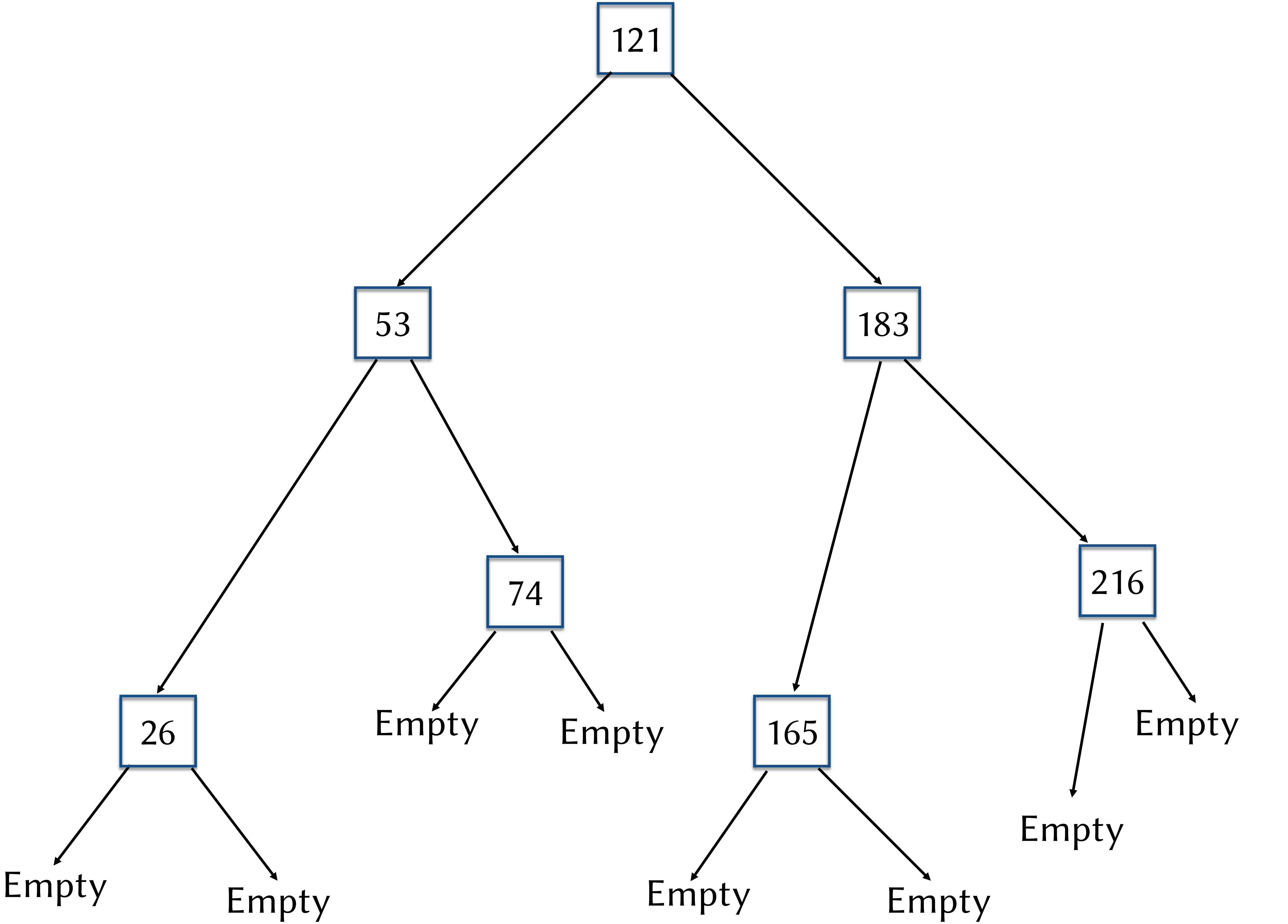
Family Trees

```
def hasBlueEyedAncestor(t: TreeNode): Boolean = {
  t match {
    case EmptyNode => false
    case Child(m,f,b,e) => ((e == "Blue") ||
                            hasBlueEyedAncestor(m) ||
                            hasBlueEyedAncestor(f))
  }
}
```

Binary Search Trees

Binary Search Trees

- We define trees containing only Ints
- To help us find elements quickly, we abide by the following invariant:
  - At a given node containing value *n*:
    - All values in the left subtree are less than *n*
    - All values in the right subtree are greater than *n*


121
53
183
26
74
165
216
Empty
Empty
Empty
Empty
Empty
Empty
Empty
Empty

Binary Search Trees

```
abstract class BinarySearchTree {
  def contains(n: Int): Boolean
  def insert(n: Int): BinarySearchTree
}
```

Binary Search Trees

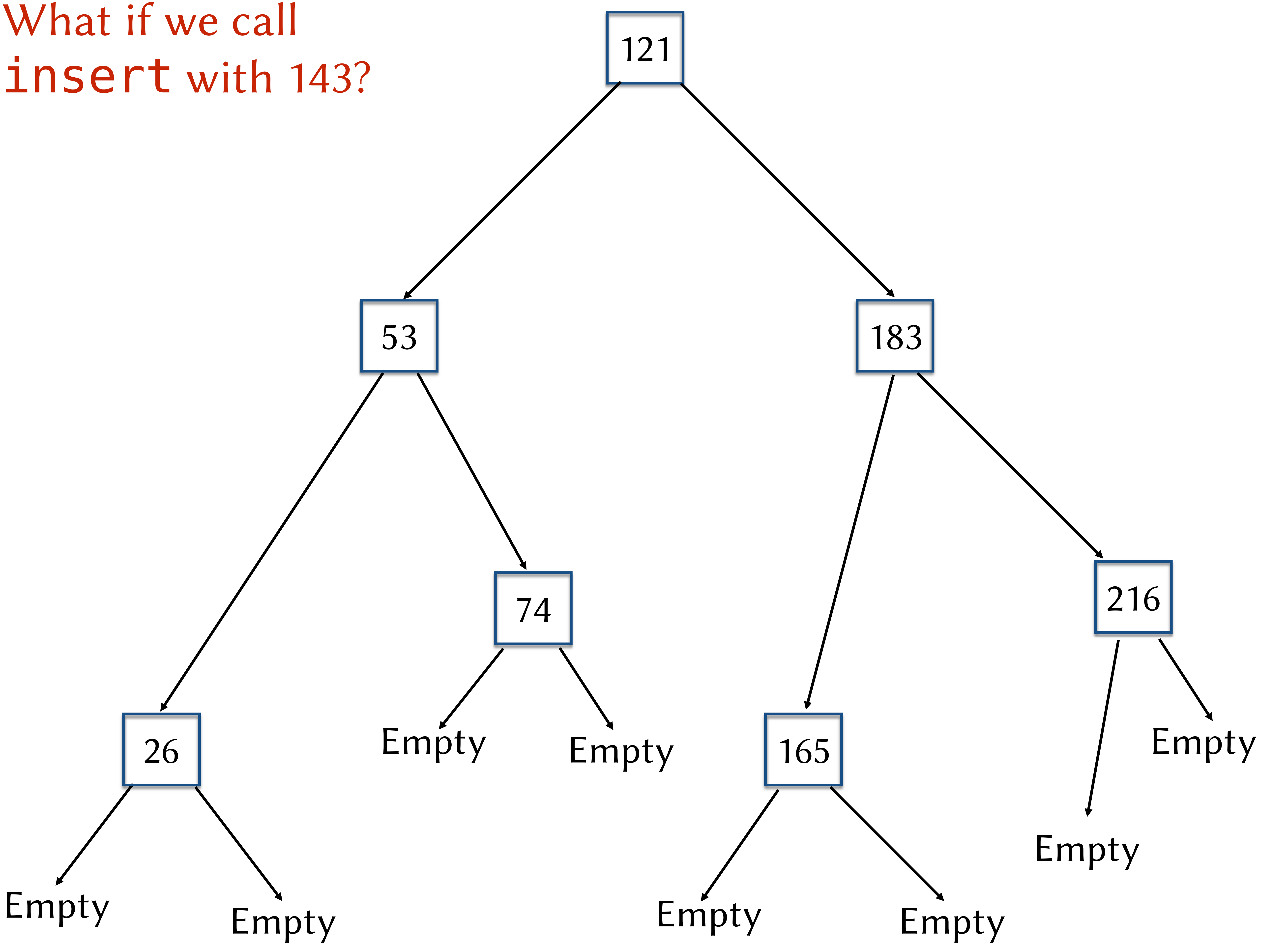
```
case object EmptyTree extends BinarySearchTree {
  def contains(n: Int) = false
  def insert(n: Int) = ConsTree(n, EmptyTree, EmptyTree)
}
```

Binary Search Trees

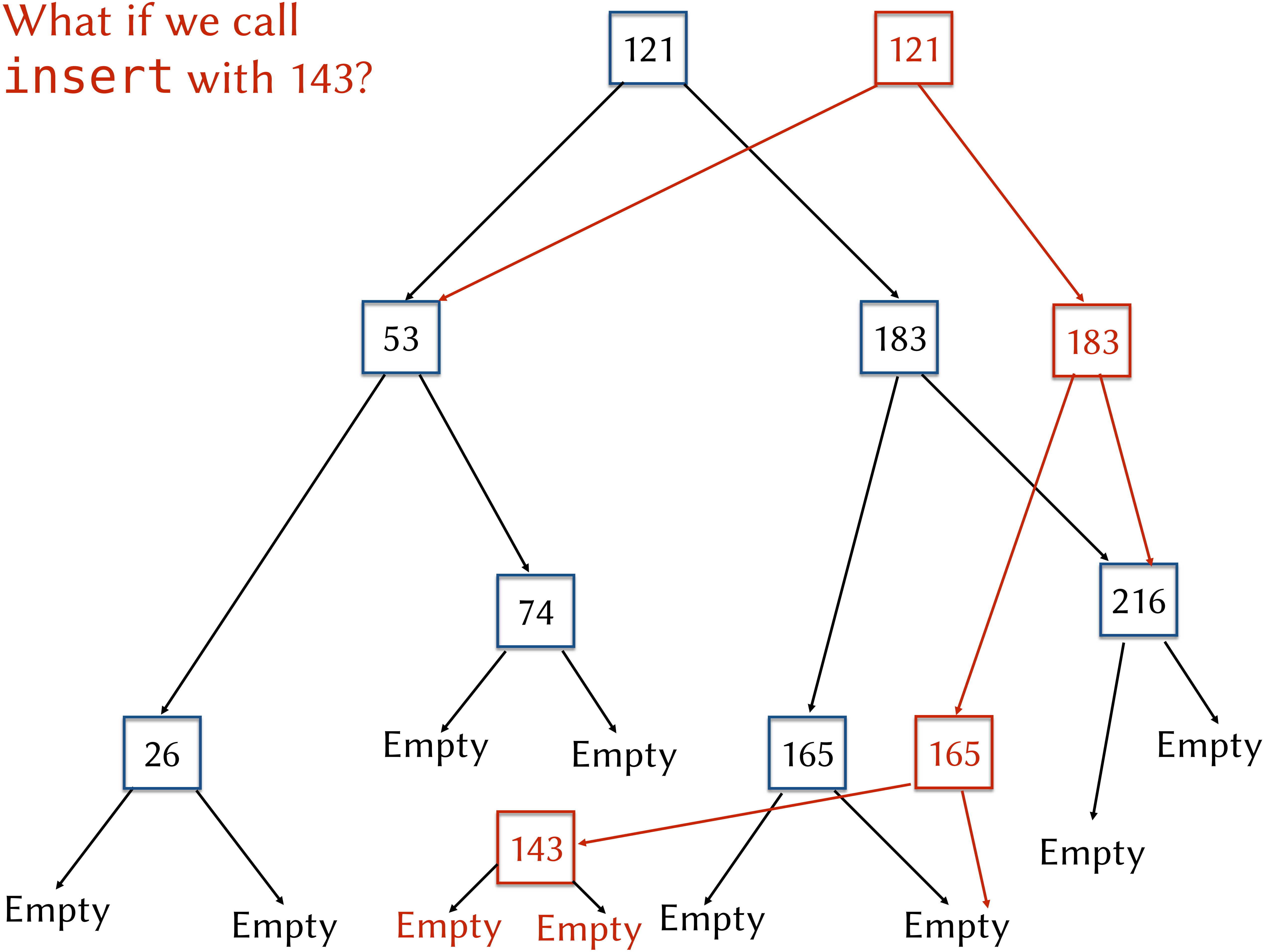
```
case class ConsTree(m: Int,
                    left: BinarySearchTree,
                    right: BinarySearchTree)
extends BinarySearchTree {

  def contains(n: Int): Boolean = {
    if (n < m) left.contains(n)
    else if (n > m) right.contains(n)
    else true // n == m
  }
  def insert(n: Int) = {
    if (n < m) ConsTree(m, left.insert(n), right)
    else if (n > m) ConsTree(m, left, right.insert(n))
    else this // n == m
  }
}
```

What if we call `insert` with 143?



What if we call `insert` with 143?



Traversing Multiple Recursive Datatypes

Taking the First Few Elements

```
def take(n: Nat, xs: List): List = {
  // require n <= size(xs)
  (n,xs) match {
    case (Zero, xs) => Empty
    case (Next(m), Cons(y, ys)) => Cons(y, take(m, ys))
  }
}
```

Taking the First Few Elements

```
def take(n: Int, xs: List): List = {
  require ((n >= 0) && (n <= size(xs)))
  (n,xs) match {
    case (0, xs) => Empty
    case (n, Cons(y, ys)) => Cons(y, take(n-1, ys))
  }
}
```

Dropping the First Few Elements

```
def drop(n: Int, xs: List): List = {
  require ((n >= 0) && (n <= size(xs)))
  (n, xs) match {
    case (0, xs) => xs
    case (n, Cons(y, ys)) => drop(n-1, ys)
  }
}
```

Functional Update of a List

```
def update(xs: List, i: Nat, y: Int): List = {
  require (xs != Empty) // && i < size(xs)

  (xs, i) match {
    case (Cons(z, zs), Zero) => Cons(y, zs)
    case (Cons(z, zs), Next(j)) => Cons(z, update(zs,j,y))
  }
}
```

Functional Update of a List

```
def update(xs: List, i: Int, y: Int): List = {
  require ((i >= 0) && (i < size(xs)))
  assert (xs != Empty) // implied by requirements

  (xs, i) match {
    case (Cons(z, zs), 0) => Cons(y, zs)
    case (Cons(z, zs), _) => Cons(z, update(zs,i-1,y))
  }
}
```

Design Abstraction

Our Function Templates Reveal Common Structure

```
def containsZero(xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => (n == 0) || containsZero(ys)
  }
}

def containsOne(xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => (n == 1) || containsOne(ys)
  }
}
```

Our Function Templates Reveal Common Structure

```
def contains(m: Int, xs: List): Boolean = {
  xs match {
    case Empty => false
    case Cons(n, ys) => (n == m) || contains(m, ys)
  }
}
```

But Sometimes the Part We Want to Abstract Is a Function

```
def below(m: Int, xs: List): List = {
  xs match {
    case Empty => Empty
    case Cons(n, ys) => {
      if (n < m) Cons(n, below(m, ys))
      else below(m, ys)
    }
  }
}
```

But Sometimes the Part We Want to Abstract Is a Function

```
def above(m: Int, xs: List): List = {
  xs match {
    case Empty => Empty
    case Cons(n, ys) => {
      if (n > m) Cons(n, above(m, ys))
      else above(m, ys)
    }
  }
}
```

Taking Functions As Parameters

```
def filter(f: (Int)=>Boolean, xs: List): List = {
  xs match {
    case Empty => Empty
    case Cons(n, ys) => {
      if (f(n)) Cons(n, filter(f, ys))
      else filter(f, ys)
    }
  }
}
```

Passing Functions as Arguments

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter(((n: Int) => (n > 0)), xs) ↦*
Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter(((n: Int) => (n < 0)), xs) ↦*
Empty

filter(((n: Int) => (n < 3)), xs) ↦*
Cons(1,Cons(2,Empty))
```

Passing Functions as Arguments

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter(((n: Int) => (n > 0)), xs) ↦*
Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter(((n: Int) => (n < 0)), xs) ↦*
Empty

filter(((n: Int) => (n < 3)), xs) ↦*
Cons(1,Cons(2,Empty))
```

These are *function literals*

First-Class Functions

- Function literals are expressions with static arrow types that reduce to *function values*
- The value type of a function value is also an arrow type
- Function values are first-class values:
  - They are allowed to be passed as arguments
  - They are allowed to be returned as results

Simplifying Function Literals

- Parameter types on function literals are allowed to be elided whenever the types are clear from context

```
filter(((n: Int) => (n > 0)), xs)
```

can be written as

```
filter(((n) => (n > 0)), xs)
```

Simplifying Function Literals

- Parentheses around a single parameter is allowed to be omitted

```
filter(((n) => (n > 0)), xs)
```

can be written as

```
filter(n => (n > 0), xs)
```

Simplifying Function Literals

- When a single parameter is used only once in the body of a function literal:
  - We can drop the parameter list
  - We simply write the body with an _ at the place where the parameter is used

For example,

```
((x: Int) => (x < 0))
```

becomes

```
_ < 0
```

Passing Function Literals As Arguments

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

filter(_ < 3, xs) ↦* Cons(1,Cons(2,Empty))
```

Guidelines On Using Function Literals

- Function literals are well-suited to situations in which:
  - The function is only used once
  - The function is not recursive
  - The function does not constitute a key concept in the problem domain

Comprehensions

$$\{2x \mid x \in xs\}$$

Mapping a Computation Over a List

```
def double(xs: List) = {
  xs match {
    case Empty => Empty
    case Cons(y,ys) => Cons(2 * y, double(ys))
  }
}
```

Mapping a Computation Over a List

```
def negate(xs: List) = {
  xs match {
    case Empty => Empty
    case Cons(y,ys) => (-y, negate(ys))
  }
}
```

Negation as a Comprehension

$$\{-x \mid x \in xs\}$$

Generalizing a Mapping Computation

```
def map(f: Int => Int, xs: List): List = {
  xs match {
    case Empty => Empty
    case Cons(y,ys) => Cons(f(y), map(f,ys))
  }
}
```

Mapping a Computation Over a List

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

negate(xs) ↦*
Cons(-1,Cons(-2,Cons(-3,Cons(-4,Cons(-5,Cons(-6,Empty))))))

double(xs) ↦*
Cons(1,Cons(4,Cons(9,Cons(16,Cons(25,Cons(36,Empty))))))
```

Mapping a Computation Over a List

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

map(-_, xs) ↦*
Cons(-1,Cons(-2,Cons(-3,Cons(-4,Cons(-5,Cons(-6,Empty))))))

map(x => 2 * x, xs) ↦*
Cons(1,Cons(4,Cons(6,Cons(8,Cons(10,Cons(12,Empty))))))
```

Recall Our Sum Function Over Lists

```
def sum(xs: List): Int = {
  xs match {
    case Empty => 0
    case Cons(y,ys) => y + sum(ys)
  }
}
```

In Mathematics, We Might Write this as a Summation

$$\sum_{x \in xs} x$$

And Our Product Function Over Lists

```
def product(xs: List): Int = {
  xs match {
    case Empty => 1
    case Cons(y,ys) => y * product(ys)
  }
}
```

In Mathematics, We Might Write this as a Product

$$\prod_{x \in xs} x$$

We Abstract to a Reduction Function Over Lists

```scala
def reduce(base: Int, f: (Int, Int) => Int, xs: List): Int = {
  xs match {
    case Empty => base
    case Cons(y,ys) => f(y, reduce(base, f, ys))
  }
}
```

Example Reductions

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

reduce(0, (x,y) => x + y, xs) ↦* 21

reduce(1, (x,y) => x * y, xs) ↦* 720
```

Min and Max

```
def max(xs: List) = {
  reduce(Int.MinValue, (x,y) => if (x > y) x else y, xs)
}

def min(xs: List) = {
  reduce(Int.MaxValue, (x,y) => if (x < y) x else y, xs)
}
```

Simplifying Function Literals

- When *each* parameter is used only once in the body of a function literal, and in the order in which they are passed:
    - We can drop the parameter list
    - We simply write the body with an _ at the place where each parameter is used

For example,

```
((x: Int, y: Int) => (x + y))
```

becomes

```
_ + _
```

Example Reductions

```
val xs = Cons(1,Cons(2,Cons(3,Cons(4,Cons(5,Cons(6,Empty))))))

reduce(0, _+_, xs) ↦* 21

reduce(1, _*_, xs) ↦* 720
```

Note the multiple parameters

Combinations of Maps and Reductions

$$\sum_{x \in xs} x^2 + 1$$

Combinations of Maps and Reductions

```
reduce(0, _+_, map(x => x*x + 1, xs))
```

Summation

```
def summation(xs: List, f: Int => Int) =
    reduce(0, _+_, map(f, xs))
```

Summation

```
def square(x: Int) = x * x

summation(xs, square(_)+1)
```