Comp 311
Functional Programming

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Announcements

- Homework 4 assigned today
- Halite-II contest is open! https://halite.io
  - Write a bot in Scala and get some extra credit!
  - Up to 25% of a project grade
  - Details will be posted to Piazza
- Extra credit in excess of 100% of projects grade will be curved down after the 100% threshold

How to Decide Between Structural and Generative Recursion

- Structural recursion is typically:
  - Easier to design
  - Easier to understand
- Generative recursion can be faster (sometimes!)

How to Decide Between Structural and Generative Recursion

- As a general guideline:
  - Start with structural recursion
  - If it turns out to be too slow:
    - Explore generatively recursive approaches

Strategies for Generative Recursion

Binary Search

- The strategy of searching over a sequence by breaking in half and searching over just one of them
- Our search for blue-eyed ancestors falls into this category
- We could also use binary search for root finding
- Newton's Method could be viewed as an optimization on binary search for root finding

Divide and Conquer

- The strategy of breaking a problem into smaller sub-problems of the same type
- Unlike *binary search*, you process *all* of the sub-pieces
- Quicksort falls into this category

Quicksort

```
def quickSort(xs: List[Int]): List[Int] = {
  xs match {
    case Nil => Nil
    case x :: xs => {
      val (smaller, larger) = separate(xs, x)
      quickSort(smaller) ++
      List(x) ++
      quickSort(larger)
    }
  }
}
```

Quicksort

```
def quickSort(xs: List[Int]): List[Int] = {
  xs match {
    case Nil => Nil
    case x :: xs => {
      val (smaller, larger) = separate(xs, x)
      quickSort(smaller) ++
      List(x) ++
      quickSort(larger)
    }
  }
}
```

Trivially solvable

Quicksort

```scala
def quickSort(xs: List[Int]): List[Int] = {
  xs match {
    case Nil => Nil
    case x :: xs => {
      val (smaller, larger) = separate(xs, x)
      quickSort(smaller) ++
      List(x) ++
      quickSort(larger)
    }
  }
}
```

Sub-problems

Quicksort

```
def quickSort(xs: List[Int]): List[Int] = {
  xs match {
    case Nil => Nil
    case x :: xs => {
      val (smaller, larger) = separate(xs, x)
      quickSort(smaller) ++
      List(x) ++
      quickSort(larger)
    }
  }
}
```

Combination

Separate

```
def separate(xs: List[Int], x: Int): (List[Int], List[Int]) = {
  xs match {
    case Nil => (Nil, Nil)
    case y :: ys => {
      val (smaller, larger) = separate(ys, x)
      if (y < x) (y :: smaller, larger)
      else (smaller, y :: larger)
    }
  }
}
```

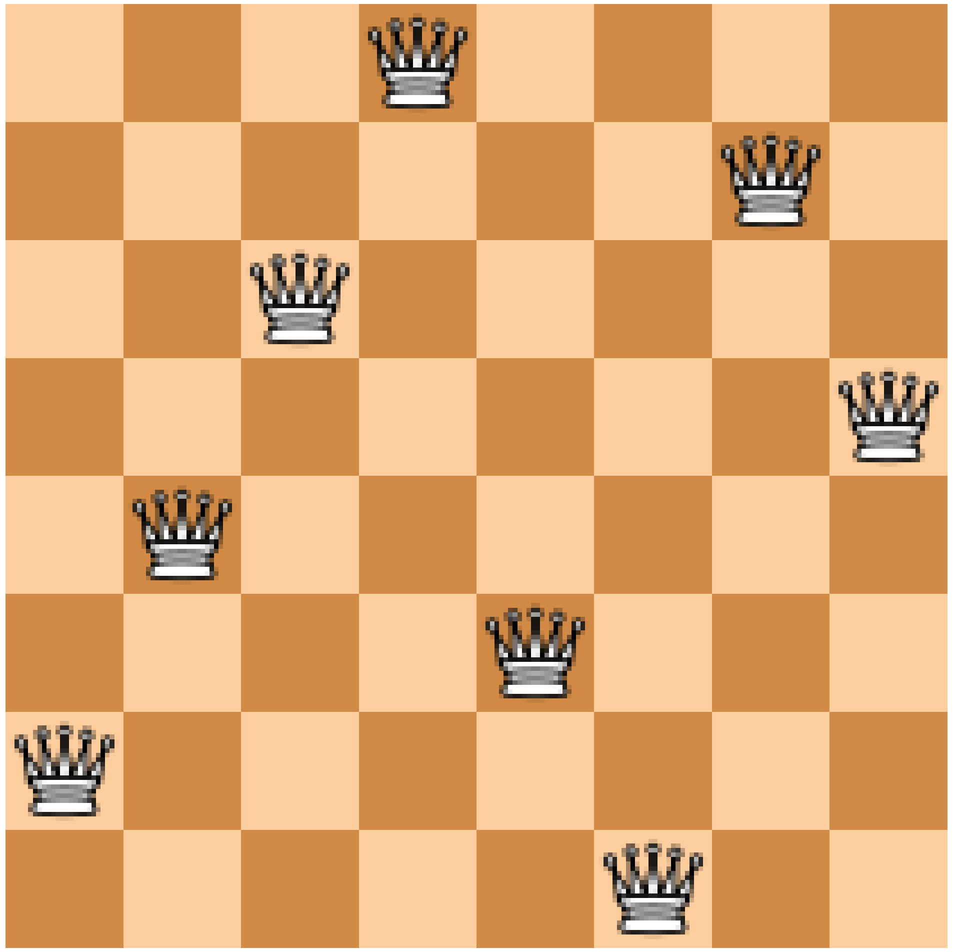
Description and Termination Argument

```
/**
 * Recurs on two sublists of the given list:
 *   All elements smaller than a given “pivot”
 *   All elements at least as large as the pivot
 * Appends the recursive solutions.
 * Because each sublist is strictly smaller
 * (the pivot was extracted from the list),
 * we eventually recur on an empty list.
 */
def quickSort(xs: List[Int]): List[Int] = {
  …
}
```

Backtracking Algorithms

N-Queens

- Place 8 Queens on an 8x8 chessboard such that none can attack any other
- Generalizable to NxN boards



Graph Algorithms

- Many problems can be expressed as traversals or computations over graphs
  - Travel planning
  - Circuit design
  - Social networks
  - etc.

Graph Algorithms

- We consider the problem of finding a path from one vertex to another in a graph

Data Analysis and Design

- We model graphs as Maps of Strings to Lists of Strings

```
case class Graph(elements: (String, List[String])*)
extends Function1[String, List[String]] {
  val _elements = Map(elements:_*)
  def apply(s: String) = _elements(s)
}
```

Data Analysis and Design

- We model graphs as Maps of Strings to Lists of Strings

```
val sampleGraph =
  new Graph ("A" -> List("E", "B"),
             "B" -> List("A"),
             "C" -> List("D"),
             "D" -> List(),
             "E" -> List("C", "F"),
             "F" -> List("A", "G"),
             "G" -> List())
```

What is a Trivially Solvable Problem?

- If the start and end vertices are identical

How Do We Generate Sub-Problems?

- Find nodes connected to start and recur

How Do We Relate the Solutions?

- We need only find one solution; no need to combine multiple solutions

Contract Attempt 1

```
/**
 * Create a path from start to finish in G
 */
def findRoute(start: String, end: String,
              graph: Graph): List[String]
```

But what if there is no path?

Options

- Often the result of a computation is that no satisfactory value could be found
  - Lookup in a table with a key that does not exist
  - Attempting to find a path that does not exist

Scala Options

```
abstract class Option[+A] {…}

object None extends Option[Nothing] {…}

class Some[+A](val contained: A) extends Option[A] {
  …
}
```

Options Are Monads!

```
abstract class Option[+A] {
  def flatMap[B](f: (A) ⇒ Option[B]): Option[B]
  def map[B](f: (A) ⇒ B): Option[B]
  def withFilter(p: (A) ⇒ Boolean):
    FilterMonadic[A, collection.Iterable[A]]
}
```

Contract Attempt 2

```
/**
 * Create a path from start to finish in G, if
 * it exists.
 */
def findRoute(start: String, end: String,
              graph: Graph):
             Option[List[String]]
```

Reduce to Backtracking Cases

```
def findRoute(start: String, end: String,
              graph: Graph): Option[List[String]] = {
  if (start == end) Some(List(end))
  else for (route <- routeFromOrigins(graph(start), end, graph))
         yield start :: route
}
```

Recursive Sub-Problems

```
def routeFromOrigins(origins: List[String], destination: String,
                     graph: Graph): Option[List[String]] = {
  origins match {
    case Nil => None
    case origin :: origins => {
      findRoute(origin, destination, graph) match {
        case None => routeFromOrigins(origins, destination,graph)
        case Some(route) => Some(route)
      }
    }
  }
}
```

Termination

- `routeFromOrigins` is structurally recursive:
  - It terminates provided that findRoute terminates
- But `findRoute` terminates only if there are no cycles in the graph it traverses