Comp 311
Functional Programming

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November 14, 2017

Review:
Pseudo-Random Number Generation

- There are many approaches to generating a pseudo-random stream of `Int` values

- One common approach is to define a *linear congruential generator (LCG)*:

$$X_{n+1} = (aX_n + c) \bmod m$$

- The pseudo-random numbers are the elements of this recurrence

Linear Congruential Generators

- LCGs can produce generators capable of passing formal tests for randomness
- The quality of the results is highly dependent on the initial values selected
- Poor statistical properties
- Not well suited for cryptographic purposes

A Linear Congruent Generator (C++11 minstd_rand)

```
def makeRandomGenerator(): () => Int = {
  val a = 48271
  val b = 0
  val m = Int.MaxValue
  var seed = 2

  def inner() = {
    seed = (a*seed + b) % m
    seed
  }
  inner
}
```

A Linear Congruent Generator (C++11 minstd_rand)

```
val g = makeRandomGenerator()<E> ↦
val g =
< def inner() = {
        seed = (a*seed + b) % m
        seed
  } ,
  val a = 48271
  val b = 0
  val m = Int.MaxValue
  var seed = 2 >
```

```
g()<E> ↦
< def inner() = {
      seed = (a*seed + b) % m
      seed
  } ,
  val a = 48271
  val b = 0
  val m = Int.MaxValue
  var seed = 3 >()<E> ↦
```

```
seed = (a*seed + b) % m
seed,
< val a = 48271
  val b = 0
  val m = Int.MaxValue
  var seed = 3 >
↦
seed = (48271*2 + 0) % Int.MaxValue
seed,
< val a = 48271
  val b = 0
  val m = Int.MaxValue
  var seed = 3 >
↦
```

```
seed, <val a = 48271
       val b = 0
       val m = Int.MaxValue
       var seed = 96542>
↦
96542
```

And now the environment closing over generator g binds seed to 96542.

Purely Functional State

Rolling a Die

- Suppose we want to implement a function that simulates the rolling of a six-sided die
- The result of calling the function should be a random number from 1 to 6

Rolling a Die

```scala
def rollDie: Int = {
  val rng = new scala.util.Random
  rng.nextInt(6) + 1
}
```

The call to nextInt will return a value from 0 to 5, not 1 to 6..

Stateful Programs and Debugging

- Because of the state encapsulated in our random number generator:
  - Repeatability of testing is hard
  - Bugs are difficult to reduce
- We would like to use effects when necessary without losing the benefits of referential transparency

Purely Functional Random Number Generation

```
trait RandomNumberGenerator {
  def nextInt: (Int, RandomNumberGenerator)
}
```

Purely Functional Random Number Generation

```
case class SimpleRNG(seed: Int) extends RandomNumberGenerator {
  val a = 48271
  val b = 0
  val m = Int.MaxValue

  def nextInt: (Int, RandomNumberGenerator) = {
    val newSeed = (a*seed + b) % m
    val newRNG = SimpleRNG(newSeed)
    (newSeed, newRNG)
  }
}
```

Threading State Through a Sequence of Statements

```
val rng = SimpleRNG(3)
val (n, rng2) = rng.nextInt
(n + 1, rng2)
```

Transforming Stateful APIs to Functional APIs

```
trait Foo {
  private var s: State = MyState
  def bar: Bar
  def baz: Int
}
```

becomes

```
trait Foo {
  def bar: (Bar, FooState)
  def baz: (Int, FooState)
}
```

A Better API for State Actions

- Explicitly threading state from one function application to the next is tedious and error prone
- We would like to define combinators that pass the state from one application to the next automatically
- For now, we consider the state of our program to be defined entirely by the state of our random number generator

A Dream

```
val rng = SimpleRNG(3)

veryHelpfulFunction(
  val n = rng.nextInt,
  n + 1
)
```

A Dream

```
val rng = SimpleRNG(3)

veryHelpfulFunction {
  val n = rng.nextInt,
  n + 1
}
```

A Dream

```
val rng = SimpleRNG(3)

veryHelpfulFunction {
  val n = rng.nextInt,
  n + 1
}
↦
(4, rng1)
```

A More Realistic Dream

```
val rng = SimpleRNG(3)

veryHelpfulFunction {
  rng.nextInt,
  (n: Int) => n + 1
}
↦
(4, rng1)
```

A More Realistic Dream

```
val rng = SimpleRNG(3)

def run = veryHelpfulFunction {
  _.nextInt,
  (n: Int) => n + 1
}

run(rng)
↦
(4, rng1)
```

Defining a Type Alias for State Actions

```scala
type StateAction[+A] =
  RandomNumberGenerator => (A, RandomNumberGenerator)
```

A Simple State Action

```
val nextInt: StateAction[Int] = _.nextInt
```

Transforming State Actions With the Map Combinator

```
def veryUsefulFunction[A,B](action: StateAction[A],
                            f: A => B): StateAction[B] =
    state => {
      val (a, state2) = action(state)
      (f(a), state2)
    }
```

Transforming State Actions With the Map Combinator

```
def map[A,B](action: StateAction[A],
             f: A => B): StateAction[B] =
    state => {
      val (a, state2) = action(state)
      (f(a), state2)
    }
```

Transforming State Actions With the Map Combinator

```
case class StateAction[S,+A](run: S => (A,S))
    extends Function1[S,(A,S)] {
  def apply(s:S) = run(s)

  def map[B](f: A => B): StateAction[S,B] =
    StateAction { s: S =>
      val (a, s2) = run(s)
      (f(a), s2)
    }
}
```

Reformulating nextInt as a State Action

```
val nextInt =
  StateAction {
    (rng: RandomNumberGenerator) => rng.nextInt
  }
```

A Simple State Action

```scala
val nextInt = StateAction(_.nextInt)
```

A More Realistic Dream

```
val rng = SimpleRNG(6)

def run = rng.nextInt.map {
  (n: Int) => n + 1
}
```

A More Realistic Dream

```
val rng = SimpleRNG(6)

def run = {
  for {
    n <- rng.nextInt
  }
  yield n + 1
}
```

A More Realistic Dream

```
val rng = SimpleRNG(6)

def run = {
  for {
    n <- _.nextInt
  }
  yield n + 1
}

run(rng)
```

A “Compound” State Action

```
def nonNegativeInt = {
  for {
    n <- _.nextInt
  } yield {
    if (n == Int.MinValue) 0
    else if (n < 0) -n
    else n
  }
}
```

Using Map

```
def nonNegativeEven: StateAction[Int] =
    for {
      i <- nonNegativeInt
    }
    yield i - (i % 2)
```

Random Non-Negative Numbers in a Range (Attempt 1)

```
// INCORRECT
def nonNegativeLessThan(n: Int): StateAction[Int] =
    for {
      i <- nonNegativeInt
    }
    yield i % n
```

This definition skews the results because Int.MaxValue might not be divisible by n.

Random Non-Negative Numbers in a Range (Attempt 2)

```
// INCORRECT
def nonNegativeLessThan(n: Int): StateAction[Int] =
  for {
    i <- nonNegativeInt
  } yield
      val mod = i % n
      if (i + (n - 1) - mod >= 0) mod
      else nonNegativeLessThan(n)
  }
```

But this version does not pass type checking!

Random Non-Negative Numbers in a Range (Attempt 2)

- The problem with our Attempt 2 is that the recursive call to `nonNegativeLessThan` produces a `StateAction[Int]`
- Our map combinator expects an Int result from the mapped function, not a `StateAction[Int]`
- To get a better idea as to how to define `nonNegativeLessThan`, let us try defining it without combinators

Random Non-Negative Numbers in a Range (Attempt 3)

```
def nonNegativeLessThan(n: Int): StateAction[Int] = { rng =>
  val (i, rng2) = nonNegativeInt(rng)
  val mod = i % n
  if (i + (n - 1) - mod >= 0) (mod, rng2)
  else nonNegativeLessThan(n)(rng2)
}
```

This version works, but now we are back to threading state explicitly.

We need a new combinator.

StateAction with FlatMap

```
case class StateAction[S,+A](run: S => (A,S))
extends Function1[S,(A,S)] {
  def apply(s:S) = run(s)

  def map[B](f: A => B): StateAction[S,B] = StateAction { s =>
    val (a, s2) = run(s)
    (f(a), s2)
  }

  def flatMap[B](f: A => StateAction[S,B]): StateAction[S,B] =
    StateAction { s =>
      val (a, s2) = run(s)
      f(a)(s2)
    }
}
```

Every Partial Application of the StateAction Type Defines a Monad

```
type RNGStateAction[A] =
  StateAction[RandomNumberGenerator, A]
```

Random Non-Negative Numbers in a Range (Attempt 4)

```
def nonNegativeLessThan(n: Int): StateAction[Int] = {
  nonNegativeInt.flatMap { i =>
    val mod = i % n
    if (i + (n - 1) - mod >= 0) (mod, _)
    else nonNegativeLessThan(n)
  }
}
```

We have almost completely eliminated state threading, except for one underscore.

Random Non-Negative Numbers in a Range (Attempt 4)

- We now have the inverse of our earlier problem:
  - Our flatMap combinator expects a `StateAction[Int]` result from the mapped function, not an Int
- We can address this problem by wrapping part of the flatMapped function in an application of the unit constructor for `StateActions`

A “No-Op” Abstraction Over State Actions

```
def unit[A](a: A): StateAction[A] =
    rng => (a, rng)

def rngUnit[A](a: A): RngStateAction[A] =
    StateAction(s => (a, s))
```

Random Non-Negative Numbers in a Range (Attempt 5)

```
def nonNegativeLessThan4point5(n: Int):
StateAction[RandomNumberGenerator,Int] = {
  nonNegativeInt.flatMap { i =>
    val result = i % n
    if (i + (n - 1) - result >= 0) unit(result)
    else nonNegativeLessThan5(n)
  }
}
```

Random Non-Negative Numbers in a Range (Attempt 5)

```
def nonNegativeLessThan4point5(n: Int):
StateAction[RandomNumberGenerator,Int] = {
  nonNegativeInt.flatMap { i =>
    val result = i % n
    if (i + (n - 1) - result >= 0) unit(result)
    else nonNegativeLessThan5(n)
  } map (j => j)
}
```

A trailing map of the identity function defines an equivalent function.

Using For-Expression Syntax

```
def nonNegativeLessThan(n: Int): RngStateAction[Int] = {
  for {
    i <- nonNegativeInt
    result <- {
      val randN = i % n
      if (i + (n - 1) - randN >= 0) unit(randN)
      else nonNegativeLessThan(n)
    }
  }
  yield result
}
```

A General StateAction Class

```
case class StateAction[S,+A](run: S => (A,S))
extends Function1[S,(A,S)] {
  def apply(s:S) = run(s)

  def map[B](f: A => B): StateAction[S,B] = StateAction { s =>
    val (a, s2) = run(s)
    (f(a), s2)
  }

  def flatMap[B](f: A => StateAction[S,B]): StateAction[S,B] =
    StateAction { s =>
      val (a, s2) = run(s)
      f(a)(s2)
    }
}
```

The map method similarly applies the operation f and pairs the result with the updated state.

This is the key right here! The flatMap method does the work of threading the updated state.

Revisiting RollDie

```
def rollDie: StateAction[Int] = nonNegativeLessThan(6)
```

Revisiting RollDie

```
def rollDie: StateAction[Int] =
  map(nonNegativeLessThan(6))(_ + 1)
```

Revisiting RollDie

```
def rollDie =
  for {
    i <- nonNegativeLessThan(6)
  }
  yield (i + 1)
```