COMP 515: Advanced Compilation for Vector and Parallel Processors

Prof. Krishna Palem
Prof. Vivek Sarkar
Department of Computer Science
Rice University
{palem,vsarkar}@rice.edu

https://wiki.rice.edu/confluence/display/PARPROG/COMP515

Coarse-Grain Parallelism (contd)

Chapter 6 of Allen and Kennedy

- **Acknowledgment: Slides from previous offerings of COMP 515 by Prof. Ken Kennedy**
  - **http://www.cs.rice.edu/~ken/comp515/**

Scheduling Parallel Loops

- Let σ_0 = serial scheduling overhead per processor
  - ➔ Contributes p * σ_0 overhead to parallel execution time for p processors
- N = number of iterations in parallel loop
- B = sequential execution time for one iteration
- Then, parallel execution is slower than serial execution if

$$\sigma_0 \geq (NB)/p$$

- Use of strip mining (chunking)
  - —Goal: select chunk size that achieves most efficient parallelism by trading off overhead with load imbalance
    - No load imbalance ➔ choose largest chunk size, C = N/p

Guided Self-Scheduling

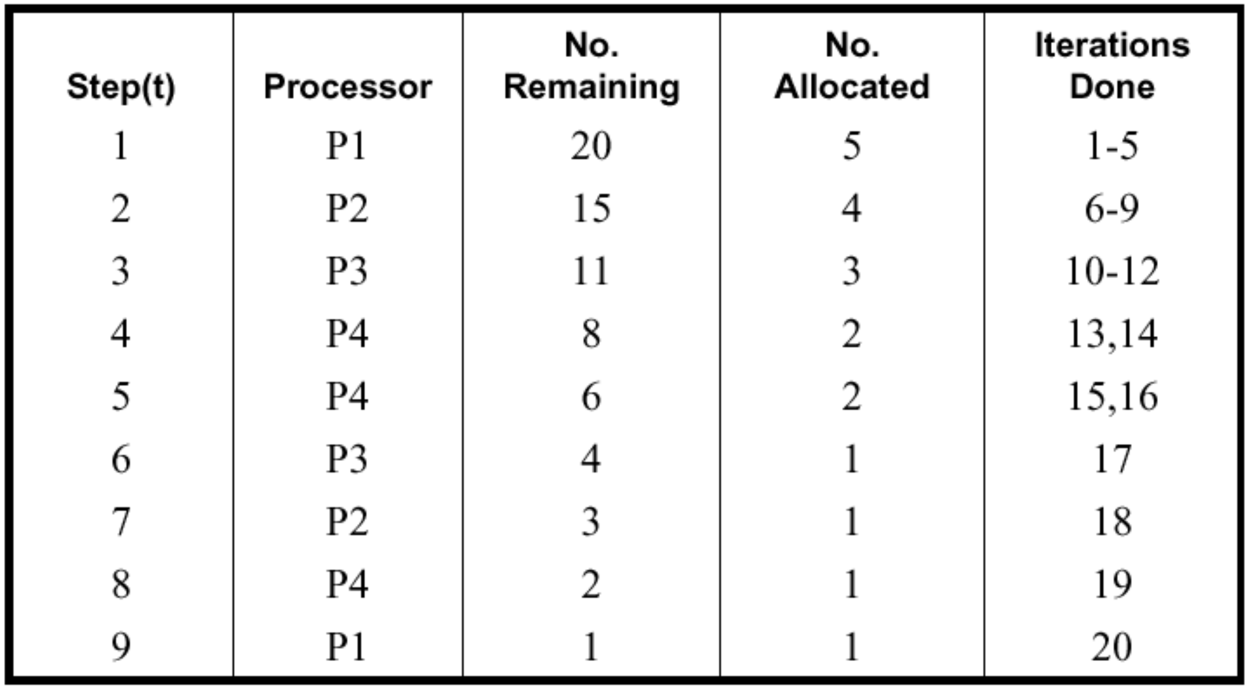
- Reduces synchronization overhead while still adapting to load imbalances
  - —Schedules groups of iterations unlike the bakery counter method
  - —Going from large to small chunks of work
- Keep all processors busy at all times
- Iterations dispensed at step t when N_t iterations remain:

$$x = \left\lceil \frac{N_t}{p} \right\rceil$$

- Alternatively, we can have GSS(k) that guarantees that all blocks handed out are of size k or greater

Guided Self-Scheduling

- *GSS(1)*



Step(t)	Processor	No. Remaining	No. Allocated	Iterations Done
1	P1	20	5	1-5
2	P2	15	4	6-9
3	P3	11	3	10-12
4	P4	8	2	13,14
5	P4	6	2	15,16
6	P3	4	1	17
7	P2	3	1	18
8	P4	2	1	19
9	P1	1	1	20

Chapter 6 Summary

- Coarse-Grained Parallelism
  - Privatization
  - Loop distribution
  - Loop alignment
  - Loop fusion
  - Loop interchange
  - Loop reversal
  - Loop skewing
  - Profitability-based methods
  - Pipeline parallelism
  - Scheduling

- Next: Handling Control Flow (Chapter 7)

Homework 2 solution

- No output dependence on A
- Output dependence on B with direction vector (=, <, >)
- Flow dependence on A with direction vector (<, *, <=)
- Flow dependence on B with direction vector (<, *, <)

Midterm Review

Chapters 1-6 of Allen and Kennedy book

Compiler Challenges for High Performance Architectures

Allen and Kennedy, Chapter 1

Bernstein's Conditions [1966]

- When is it safe to run two tasks R1 and R2 in parallel?
  - If none of the following holds:
    1. R1 writes into a memory location that R2 reads
    2. R2 writes into a memory location that R1 reads
    3. Both R1 and R2 write to the same memory location
- How can we convert this to loop parallelism?
  - Think of loop iterations as tasks
- Does this apply to sequential loops embedded in an explicitly parallel program?
  - Impact of memory model on ordering of read operations

Dependence: Theory and Practice

Allen and Kennedy, Chapter 2

Dependences

- Formally:

  There is a data dependence from statement S_1 to statement S_2 (S_2 depends on S_1) if:

  1. Both statements access the same memory location and at least one of them stores onto it, and
  2. There is a feasible run-time execution path from S_1 to S_2

Load Store Classification

- Quick review of dependences classified in terms of load-store order:

  1. True dependences (RAW hazard)
     - S_2 depends on S_1 is denoted by $S_1\ \delta\ S_2$
  2. Antidependence (WAR hazard)
     - S_2 depends on S_1 is denoted by $S_1\ \delta^{-1}\ S_2$
  3. Output dependence (WAW hazard)
     - S_2 depends on S_1 is denoted by $S_1\ \delta^{0}\ S_2$

Formal Definition of Loop Dependence

- Theorem 2.1 Loop Dependence:
  There exists a dependence from statements S_1 to statement S_2 in a common nest of loops if and only if there exist two iteration vectors i and j for the nest, such that
  (1) $i < j$ or $i = j$ and there is a path from S_1 to S_2 in the body of the loop,
  (2) statement S_1 accesses memory location M on iteration i and statement S_2 accesses location M on iteration j, and
  (3) one of these accesses is a write.

- Follows from the definition of dependence

Reordering Transformations

- A reordering transformation is any program transformation that merely changes the order of execution of the code, without adding or deleting any executions of any statement
- A reordering transformation does not eliminate dependences
- A reordering transformation preserves a dependence if it preserves the relative execution order of the source and sink of that dependence.
- Fundamental Theorem of Dependence:
  - Any reordering transformation that preserves every dependence in a program preserves the meaning of that program
  - Proof by contradiction. Theorem 2.2 in the book.
- A transformation is said to be valid or legal for the program to which it applies if it preserves all dependences in the program.

Distance & Direction Vectors

- Consider a dependence in a loop nest of n loops
  - Statement S_1 on iteration i is the source of the dependence
  - Statement S_2 on iteration j is the sink of the dependence

- The distance vector is a vector of length n, d(i,j) such that: $d(i,j)_k = j_k - i_k$
  - We normalize distance vectors for loops in which the index step size is not equal to 1 (but usually prefer to normalize the loops to step of +1 instead)

- The direction vector is a vector of length n, r(i,j) such that: $r(i,j)_k = j_k \; op_k \; i_k$, where op_k is a relational operator (<, >, =, <=, >=, !=, *)

Implausible Distance & Direction Vectors

- A distance vector is implausible if its leftmost nonzero element is negative i.e., if the vector is lexicographically less than the zero vector
- Likewise, a direction vector is implausible if its leftmost non "=" component is not "<"
- No dependence in a sequential program can have an implausible distance or direction vector as this would imply that the sink of the dependence occurs before the source.

Direction Vector Transformation

- Theorem 2.3. Direction Vector Transformation. Let T be a transformation that is applied to a loop nest and that does not rearrange the statements in the body of the loop. Then the transformation is valid if, after it is applied, none of the direction vectors for dependences with source and sink in the nest has a leftmost non- "=" component that is ">" i.e., none of the transformed direction vectors become implausible.

- Follows from Fundamental Theorem of Dependence:
  - All dependences exist
  - None of the dependences have been reversed

Loop-carried and Loop-independent Dependences

- If in a loop statement S_2 depends on S_1, then there are two possible ways of this dependence occurring:

1. S_1 and S_2 execute on different iterations
   - This is called a loop-carried dependence.
2. S_1 and S_2 execute on the same iteration
   - This is called a loop-independent dependence.

Loop-carried dependence

- Definition 2.11
- Statement S_2 has a loop-carried dependence on statement S_1 if and only if S_1 references location M on iteration i, S_2 references M on iteration j and d(i,j) > 0 i.e., D(i,j) contains a "<" as leftmost non "=" component and is lexicographically positive.

Example:

```
DO I = 1, N
S1        A(I+1) = F(I)
S2        F(I+1) = A(I)
ENDDO
```

Loop-carried dependence

- Level of a loop-carried dependence is the index of the leftmost non-“=” of D(i,j) for the dependence.

For instance:

```
DO I = 1, 10
   DO J = 1, 10
      DO K = 1, 10
S1       A(I, J, K+1) = A(I, J, K)
      ENDDO
   ENDDO
ENDDO
```

- Direction vector for S1 is (=, =, <)
- Level of the dependence is 3
- A level-k dependence between S_1 and S_2 is denoted by $S_1 \delta_k S_2$

Loop-carried Transformations

- Theorem 2.4 Any reordering transformation that (1) preserves the iteration order of the level-k loop, (2) does not interchange any loop at level < k to a level > k, and (3) does not interchange any loop at level > k to a position < k, must preserve all level-k dependences.
- Proof:
  - D(i, j) has a "<" in the kth position and "=" in positions 1 through k-1

  ⇒ Source and sink of dependence are in the same iteration of loops 1 through k-1

  ⇒ Cannot change the sense of the dependence by a reordering of iterations of those loops

- As a result of the theorem, powerful transformations can be applied

Loop-independent dependences

- Definition 2.15. Statement S_2 has a loop-independent dependence on statement S_1 if and only if there exist two iteration vectors i and j such that:
  1) Statement S_1 refers to memory location M on iteration i, S_2 refers to M on iteration j, and i = j.
  2) There is a control flow path from S_1 to S_2 within the iteration.

Example:

```
    DO I = 1, 10
S1      A(I) = ...
S2      ... = A(I)
    ENDDO
```

Loop-independent dependences

- Theorem 2.5. If there is a loop-independent dependence from S_1 to S_2, any reordering transformation that does not move statement instances between iterations and preserves the relative order of S_1 and S_2 in the loop body preserves that dependence.

- S_2 depends on S_1 with a loop independent dependence is denoted by $S_1 \, \delta_\infty \, S_2$
- Note that the direction vector will have entries that are all "=" for loop independent dependences

Simple Vectorization Algorithm

```
procedure vectorize (L, D)
// L is the maximal loop nest containing the statement.
// D is the dependence graph for statements in L.
find the set {$S_1$, $S_2$, ... , $S_m$} of maximal strongly-connected regions in the dependence
    graph D restricted to L (Tarjan);
construct $L_p$ from L by reducing each $S_i$ to a single node and compute $D_p$, the
    dependence graph naturally induced on $L_p$ by D;
let {$p_1$, $p_2$, ... , $p_m$} be the m nodes of $L_p$ numbered in an order consistent with $D_p$ (use
    topological sort);

    for i = 1 to m do begin
            if $p_i$ is a dependence cycle then
                generate a DO-loop nest around the statements in $p_i$;
            else
                directly rewrite $p_i$ in Fortran 90, vectorizing it with respect to every loop
                    containing it;
    end
end vectorize
```

Problems With Simple Vectorization

```
     DO I = 1, N
              DO J = 1, M
S1                     A(I+1,J) = A(I,J) + B
              ENDDO
     ENDDO
```

- Dependence from S_1 to itself with d(i, j) = (1,0)
- Key observation: Since dependence is at level 1, we can manipulate the other loop!
- Can be converted to:

```
     DO I = 1, N
S1       A(I+1,1:M) = A(I,1:M) + B
     ENDDO
```

- The simple algorithm does not capitalize on such opportunities

Advanced Vectorization Algorithm

```
procedure codegen(R, k, D);
// R is the region for which we must generate code.
// k is the minimum nesting level of possible parallel loops.
// D is the dependence graph among statements in R..
find the set {$S_1$, $S_2$, ... , $S_m$} of maximal strongly-connected
   regions in the dependence graph D restricted to R;
construct $R_p$ from R by reducing each $S_i$ to a single node and
   compute $D_p$, the dependence graph naturally induced on $R_p$ by D;
let {$p_1$, $p_2$, ... , $p_m$} be the m nodes of $R_p$ numbered in an order
   consistent with $D_p$ (use topological sort to do the numbering);
for i = 1 to m do begin
   if $p_i$ is cyclic then begin
      generate a level-k DO statement;
      let $D_i$ be the dependence graph consisting of all dependence edges in D that are at level
         k+1 or greater and are internal to $p_i$;
      codegen ($p_i$, k+1, $D_i$);
      generate the level-k ENDDO statement;
   end
   else
      generate a vector statement for $p_i$ in r($p_i$)-k+1 dimensions, where r ($p_i$) is the number of
         loops containing $p_i$;
end
```

Dependence Testing

Allen and Kennedy, Chapter 3

Simple Dependence Testing

```
DO $i_1$ = $L_1$, $U_1$, $S_1$
    DO $i_2$ = $L_2$, $U_2$, $S_2$
      ...
        DO $i_n$ = $L_n$, $U_n$, $S_n$
    $S_1$      $A(f_1(i_1,...,i_n),...,f_m(i_1,...,i_n))$ = ...
    $S_2$      ... = $A(g_1(i_1,...,i_n),...,g_m(i_1,...,i_n))$
        ENDDO
      ...
    ENDDO
ENDDO
```

- A dependence exists from S_1 to S_2 if and only if there exist values of α and β such that (1) α is lexicographically less than or equal to β and (2) the following system of dependence equations is satisfied:

  $f_i(\alpha) = g_i(\beta)$ for all i, $1 \leq i \leq m$

- Direct application of Loop Dependence Theorem

The General Problem

```
DO $i_1$ = $L_1$, $U_1$
  DO $i_2$ = $L_2$, $U_2$
             …
    DO $i_n$ = $L_n$, $U_n$
 $S_1$                  A($f_1$($i_1$,…,$i_n$),…,$f_m$($i_1$,…,$i_n$)) = …
 $S_2$                  … = A($g_1$($i_1$,…,$i_n$),…,$g_m$($i_1$,…,$i_n$))
    ENDDO
    ...
  ENDDO
ENDDO
```

Under what conditions is the following true for iterations α and β ?

$f_i(\alpha) = g_i(\beta)$ **for all** $i, 1 \leq i \leq m$

Note that the number of equations equals the rank of the array,

and the number of variables is twice the number of loops that enclose both array references (two iteration vectors)

Basics: Complexity

A subscript equation is said to be

- ZIV if it contains no index (zero index variable)
- SIV if it contains only one index (single index variable)
- MIV if it contains more than one index (multiple index variables)

For Example:

```
        A(5,I+1,j) = A(1,I,k) + C
                First subscript equation is ZIV
                Second subscript equation is SIV
                Third subscript equation is MIV
```

Dependence Testing: Overview

- Partition subscripts of a pair of array references into separable and coupled groups
- Classify each subscript as ZIV, SIV or MIV
- For each separable subscript apply single subscript test. If not done goto next step
- For each coupled group apply multiple subscript test
- If still not done, merge all direction vectors computed in the previous steps into a single set of direction vectors

Linear Diophantine Equations

- A basic result tells us that there are values for x_1,x_2,…,x_n,y_1,y_2,…,y_n so that

  $$a_1x_1 - b_1y_1 + \ldots + a_nx_n - b_ny_n = \gcd(a_1,\ldots,a_n,b_1,\ldots,b_n)$$

  What's more, gcd(a_1,…,a_n,b_1,…,b_n) is the smallest number this is true for.

- As a result, the equation has a solution iff gcd(a_1,…,a_n,b_1,…,b_n) divides b_0 - a_0
  - But the solution may not be in the region (loop iteration values) of interest

- Exercise: try this result on the A(4*i+2) & A(4*i+4) example

Real Solutions

- Unfortunately, the gcd test is less useful then it might seem.
- Useful technique is to show that the equation has no solutions in region of interest ==> explore real solutions for this purpose
- Solving h(x) = 0 is essentially an integer programming problem. Linear programming techniques are used as an approximation.
- Since the function is continuous, the Intermediate Value Theorem says that a solution exists iff:

$$\min_R h \le 0 \le \max_R h$$

Banerjee Inequality

- We need an easy way to calculate $\min_R$ h and $\max_R$ h.

- Definitions:

$$h_i^+ = \max_{Ri} h(x_i, y_i) \qquad a^+ = \begin{cases} a & a \geq 0 \\ 0 & a < 0 \end{cases}$$

$$h_i^- = \min_{Ri} h(x_i, y_i) \qquad a^- = \begin{cases} |a| & a < 0 \\ 0 & a \geq 0 \end{cases}$$

- a^+ and a^- are both >= 0 and are called the positive part and negative part of a, so that $a = a^+ - a^-$

Banerjee Inequality

- Theorem 3.3 (Banerjee). Let D be a direction vector, and h be a dependence function. h = 0 can be solved in the region R iff:

$$\sum_{i=1}^{n} H_i^-(D_i) \le b_0 - a_0 \le \sum_{i=1}^{n} H_i^+(D_i)$$

Proof: Immediate from Lemma 3.3 and the IMV.

Preliminary Transformations

Chapter 4 of Allen and Kennedy

Loop Normalization

- Transform loop so that
  - The new stride becomes +1 (more important)
  - The new lower bound becomes +1 (less important)
- To make dependence testing as simple as possible
- Serves as information gathering phase

Enhancing Fine-Grained Parallelism

Chapter 5 of Allen and Kennedy

Chapter 2’s Codegen

- Codegen: tries to find parallelism using transformations of loop distribution and statement reordering
- If we deal with loops containing cyclic dependences early on in the loop nest, we can potentially vectorize more loops
- Goal in Chapter 5: To explore other transformations to exploit parallelism

Loop Interchange: Safety

- Theorem 5.1 **Let D(i,j) be a direction vector for a dependence in a perfect nest of loops. Then the direction vector for the same dependence after a permutation of the loops in the nest is determined by applying the same permutation to the elements of D(i,j).**

```
DO I = 1, L
  DO J = 1, M
    DO K = 1, N
      A(I+1,J+1,K) = B(I,J,K)
    ENDDO
  ENDDO
ENDDO
```

Dependence: (<, <, =)

```
DO I = 1, L
  DO K = 1, N
    DO J = 1, M
      A(I+1,J+1,K) = B(I,J,K)
    ENDDO
  ENDDO
ENDDO
```

Dependence: (<, =, <)

Loop Interchange: Safety

- Theorem 5.2 **A permutation of the loops in a perfect nest is legal if and only if the direction matrix, after the same permutation is applied to its columns, has no ">" direction as the leftmost non-"=" direction in any row.**
- Follows from Theorem 5.1 and Theorem 2.3

Example:

$$\begin{array}{ccc} i & j & k \\ < & < & = \\ < & = & > \\ & & \end{array} \longrightarrow \begin{array}{ccc} j & k & i \\ < & = & < \\ = & > & < \\ & \bigcirc & \end{array}$$

Fully Permutable Loop Nest

- A contiguous set of $k \geq 1$ loops, $i_j,\ldots,i_{j+k-1}$ is fully permutable if all permutations of $i_j,\ldots,i_{j+k-1}$are legal

- Data dependence test: Loops $i_j,\ldots,i_{j+k-1}$ are fully permutable if for each dependence vector $(d_1,\ldots,d_n)$ carried at levels j … j +k-1, each of $d_j,\ldots,d_{j+k-1}$ is non-negative

- Fundamental result (to be discussed later in course): a set of k fully permutable loops can be transformed using only Interchange, Reversal and Skewing transformations into an equivalent set of k loops where k-1 of the loops are parallel

Scalar Expansion and its use in Removing Anti and Output Dependences

```
      DO I = 1, N
S1       T = A(I)
S2       A(I) = B(I)
S3       B(I) = T
     ENDDO
```

- Scalar Expansion:

```
     DO I = 1, N
S1       T$(I) = A(I)
S2       A(I) = B(I)
S3       B(I) = T$(I)
     ENDDO
     T = T$(N)
```

- leads to:

```
S1        T$(1:N) = A(1:N)
S2        A(1:N) = B(1:N)
S3        B(1:N) = T$(1:N)
           T = T$(N)
```

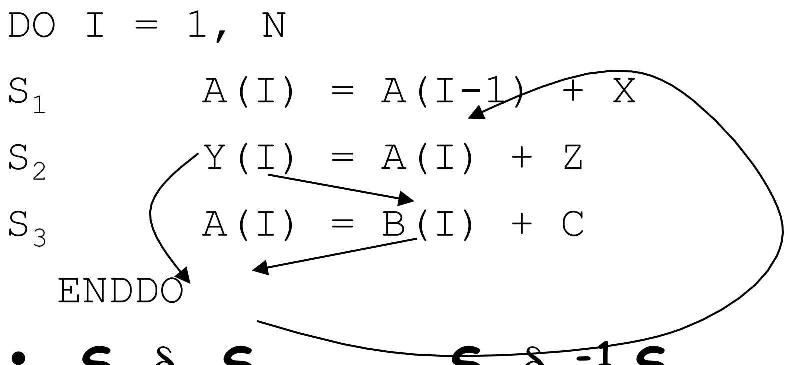
Scalar Renaming

- Renaming algorithm partitions all definitions and uses into equivalent classes, each of which can occupy different memory locations.
- Use the definition-use graph to:
  - Pick definition
  - Add all uses that the definition reaches to the equivalence class
  - Add all definitions that reach any of the uses…
  - ..until fixed point is reached
- Example:

```
    IF (…) THEN
S1      T = …
    ELSE
S2      T = …
    ENDIF
S3  … = T
S4    T = …
S5  … = T
```

```
IF (…) THEN
     T1 = …
ELSE
     T1 = …
ENDIF
… = T1
T2 = …
… = T2
```

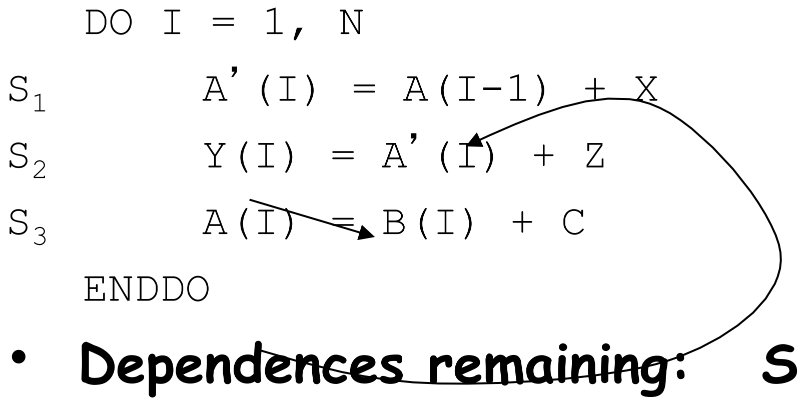
Array Renaming



```
DO I = 1, N
S1      A(I) = A(I-1) + X
S2      Y(I) = A(I) + Z
S3      A(I) = B(I) + C
  ENDDO
```

- $S_1\ \delta_\infty\ S_2$ $S_2\ \delta_\infty^{-1}\ S_3$ $S_3\ \delta_1\ S_1$ $S_1\ \delta_\infty^{0}\ S_3$

- **Rename A(I) to A'(I):**

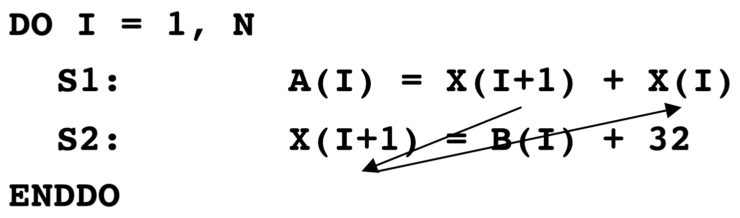


```
   DO I = 1, N
S1      A'(I) = A(I-1) + X
S2      Y(I) = A'(I) + Z
S3      A(I) = B(I) + C
   ENDDO
```

- **Dependences remaining:** $S_1\ \delta_\infty\ S_2$ **and** $S_3\ \delta_1\ S_1$

Node Splitting

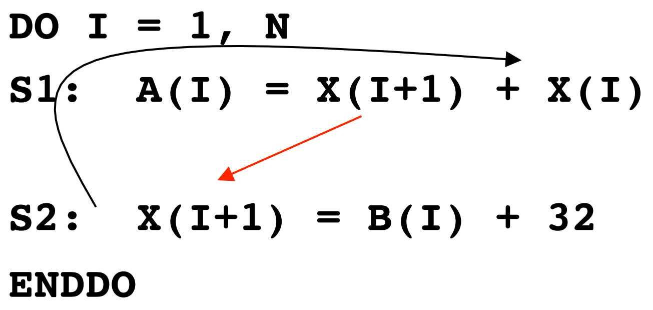
- Sometimes Renaming fails



```
DO I = 1, N
  S1:       A(I) = X(I+1) + X(I)
  S2:       X(I+1) = B(I) + 32
ENDDO
```

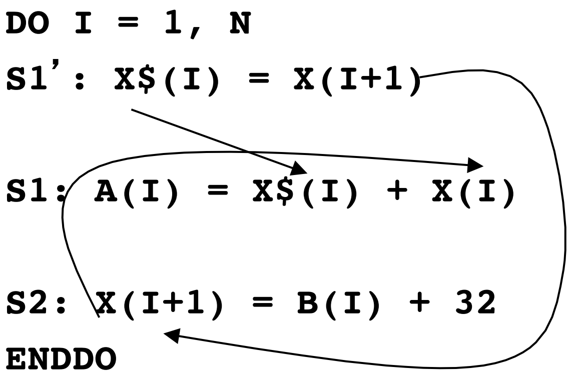
- Recurrence kept intact by renaming algorithm

Node Splitting



```
DO I = 1, N
S1:   A(I) = X(I+1) + X(I)
S2:   X(I+1) = B(I) + 32
ENDDO
```

- Break critical antidependence
- Make copy of read from which antidependence emanates



```
DO I = 1, N
S1’: X$(I) = X(I+1)
S1: A(I) = X$(I) + X(I)
S2: X(I+1) = B(I) + 32
ENDDO
```

- Recurrence broken
- Vectorized to

```
S1’:  X$(1:N) = X(2:N+1)
S2:   X(2:N+1) = B(1:N) + 32
S1:   A(1:N) = X$(1:N) + X(1:N)
```

Index-set Splitting

- Subdivide loop into different iteration ranges to achieve partial parallelization
  - Threshold Analysis [Strong SIV, Weak Crossing SIV]
  - Loop Peeling [Weak Zero SIV]
  - Section Based Splitting [Variation of loop peeling]

Threshold Analysis

```
DO I = 1, 20
     A(I+20) = A(I) + B
ENDDO
Vectorize to..
A(21:40) = A(1:20) + B
```

```
DO I = 1, 100
     A(I+20) = A(I) + B
ENDDO
Strip mine to..
DO I = 1, 100, 20
   DO i = I, I+19
     A(i+20) = A(i) + B
   ENDDO
ENDDO
```

Vectorize this

Loop Peeling

- **Source of dependence is a single iteration**

```
DO I = 1, N
     A(I) = A(I) + A(1)
ENDDO

Loop peeled to..
A(1) = A(1) + A(1)
DO I = 2, N
     A(I) = A(I) + A(1)
ENDDO

Vectorize to..
A(1) = A(1) + A(1)
A(2:N)= A(2:N) + A(1)
```

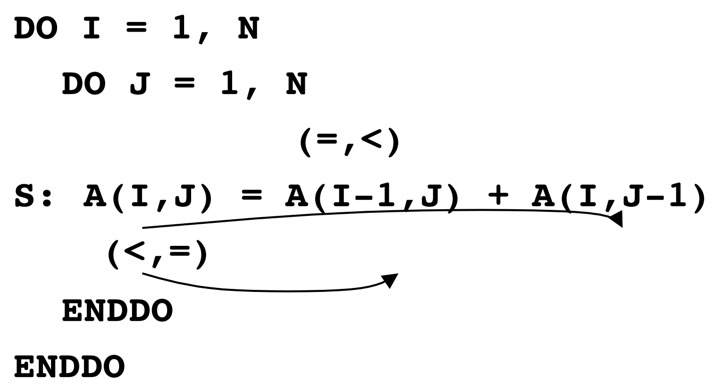
Run-time Symbolic Resolution

- “Breaking Conditions”

```
DO I = 1, N
   A(I+L) = A(I) + B(I)
ENDDO
Transformed to..
IF(L.LE.0 .OR. L.GT.N) THEN
   A(L+1:N+L) = A(1:N) + B(1:N)
ELSE
   DO I = 1, N
      A(I+L) = A(I) + B(I)
   ENDDO
ENDIF
```

Loop Skewing

- Reshape Iteration Space to uncover parallelism

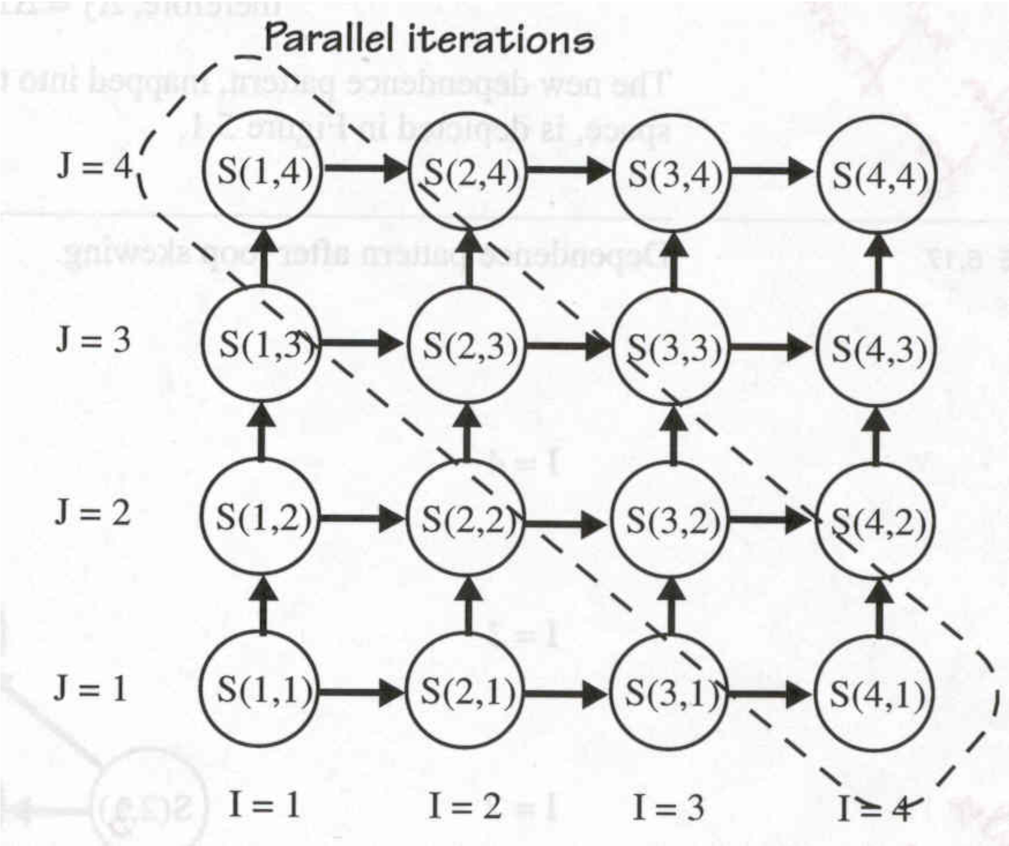


```
DO I = 1, N
  DO J = 1, N
            (=,<)
S: A(I,J) = A(I-1,J) + A(I,J-1)
    (<,=)
  ENDDO
ENDDO
```

Parallelism not apparent

Loop Skewing

- **Dependence Pattern before loop skewing**



Loop Skewing

- Do the following transformation called loop skewing

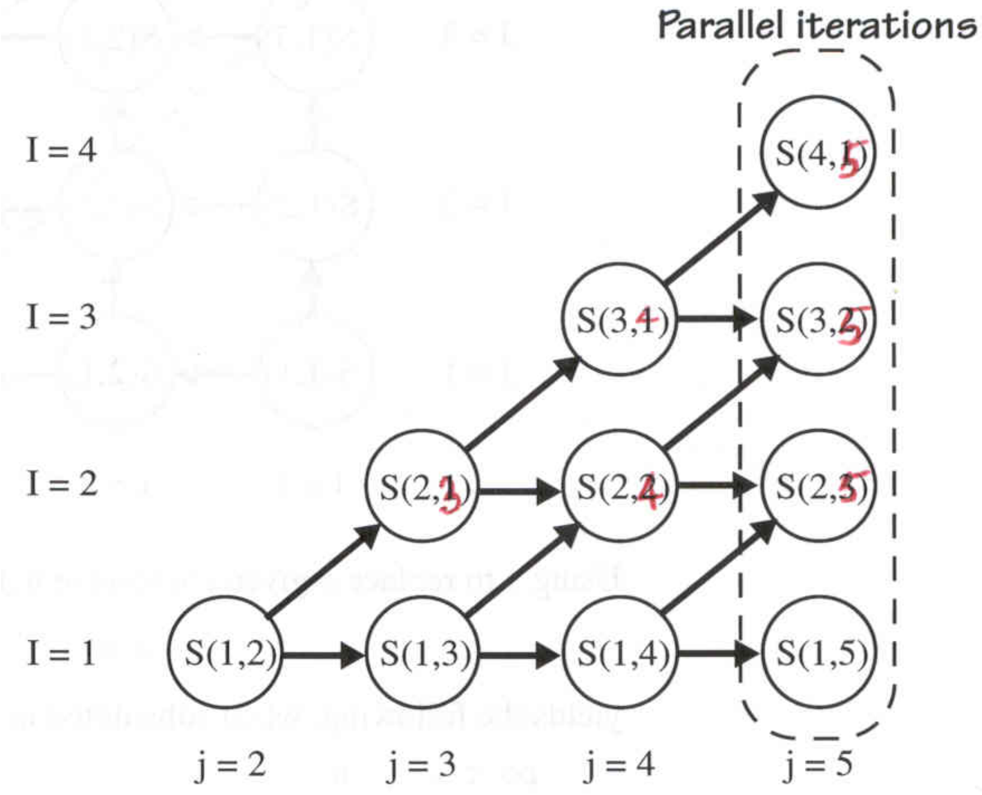
```
jj=J+I or J=jj-I

DO I = 1, N
  DO jj = I+1, I+N
   J = jj - I             (=,<)
S:  A(I,J) = A(I-1,J) + A(I,J-1)
     (<,<)
  ENDDO
ENDDO

Note: Direction Vector Changes, but statement body remains the same
(Examples in textbook usually copy propagate J=jj-I in all uses of J)
```

Loop Skewing

- **Dependence pattern after loop skewing**



Pipeline Parallelism with Strip Mining

```
POST (EV(1, 1))
DOACROSS I = 2, N-1
    K = 0
    DO J = 2, N-1, 2   ! CHUNK SIZE = 2
        K = K+1
        WAIT (EV(I-1,K))
        DO m = J, MIN(J+1, N-1)
            A(I, m) = .25 * (A(I-1, m) + A(I, m-1) + A(I+1, m) + A(I, m+1))
        ENDDO
        POST (EV(I, K+1))
    ENDDO
ENDDO
```

Midterm exam

- Take-home exam (3 hours)
  - Open book: textbook only, no other resources
  - Will be made available on Thursday, Oct 24th, and will be due in class on Tuesday, Oct 29th
  - Scope of exam is Chapters 1-6 of textbook