COMP 322: Fundamentals of Parallel Programming

Lecture 8: Parallel Quicksort

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Announcements

- Homework 3 is due by 5pm on Monday, Feb 7th
  - This is a programming assignment with abstract performance metrics
  - To prepare for HW3, please make sure that you can compile and run the programs from Lab 2 on your own, using the –perf option. In case of problems, please send email to comp322-staff @ mailman.rice.edu
- We have requested 24-hour access to Ryon building and Ryon 102 lab for all students enrolled in COMP 322
- Preferred naming convention for homework folders in clear is hw_?? e.g. hw_3
  - Please try and use this convention in the future

Acknowledgments for Today's Lecture

- Reference [2]: C.A.R. Hoare. "Algorithm 63: partition". Commun. ACM, 4:321-, July 1961. http://doi.acm.org/10.1145/366622.366642
- Reference [3]: C.A.R. Hoare. "Algorithm 64: Quicksort". Commun. ACM, 4:321-, July 1961. http://doi.acm.org/10.1145/366622.366644
- COMP 322 Lecture 8 handout
- Quicksort example figure from "Introduction to Parallel Computing", 2nd Edition, Ananth Grama, Anshul Gupta, George Karypis, Vipin Kumar, Addison-Wesley, 2003
- Max Grossman for HJ code

Quicksort

- Classical sequential sorting algorithm introduced by C.A.R. Hoare in 1961 [3]
- Some reasons why Quicksort is still in use today:
  - Simple to implement
  - Worst case $O(n^2)$ execution time, but executes in O(n log n) time in practice (with high probability)
  - "In place'' sorting algorithm -- does not need allocation of a second copy of the array.
  - Exemplar of divide-and-conquer paradigm

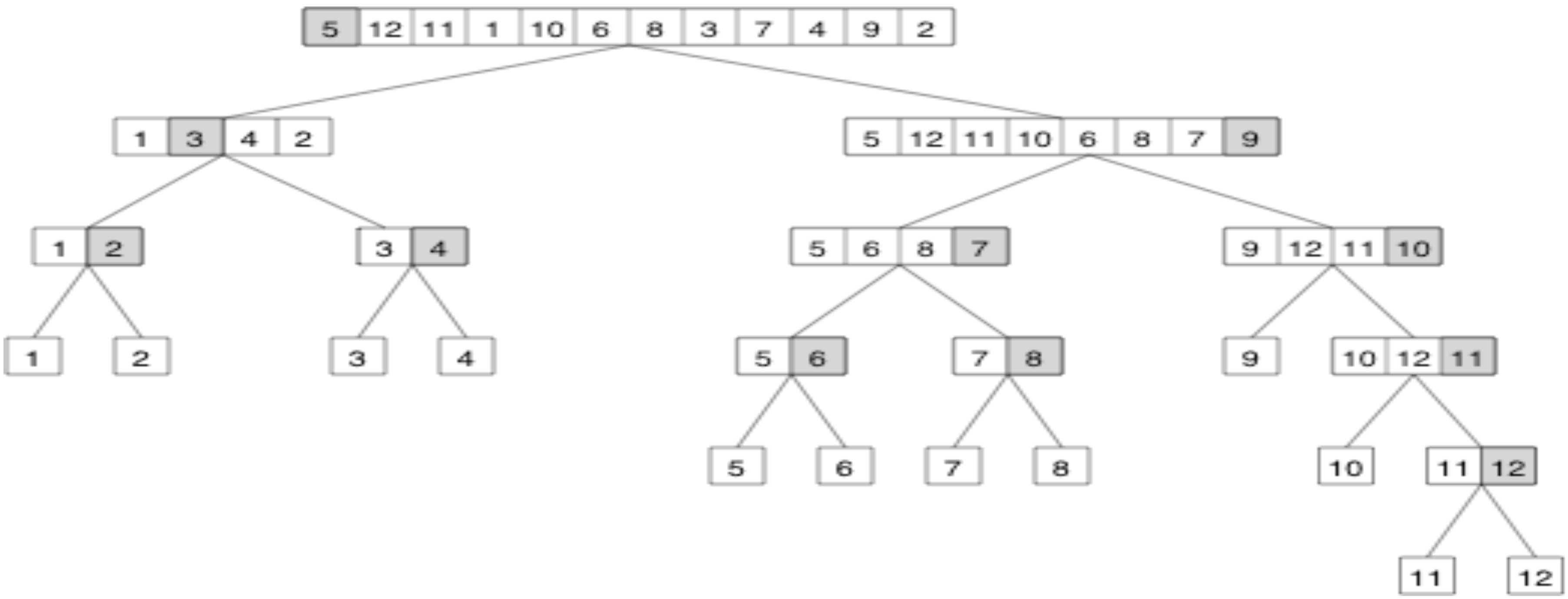
Original description of Quicksort algorithm (Reference [3], 1961)

```
procedure   quicksort (A,M,N);   value M,N;
                array A;   integer M,N;
comment  Quicksort is a very fast and convenient method of
sorting an array in the random-access store of a computer. The
entire contents of the store may be sorted, since no extra space is
required. The average number of comparisons made is 2(M−N) ln
(N−M), and the average number of exchanges is one sixth this
amount. Suitable refinements of this method will be desirable for
its implementation on any actual computer;
begin        integer I,J;
             if M < N then begin partition (A,M,N,I,J);
                                 quicksort (A,M,J);
                                 quicksort (A, I, N)
                           end
end          quicksort
```

Sequential HJ implementation of Quicksort (Listing 1)

```
static void quicksort(int[] A, int M, int N) {
  if (M < N) {
    // partition() selects a pivot element in A[M…N]
    // to partition A[M…N] into A[M…J] and A[I…N]
    point p = partition(A, M, N);
    int I=p.get(0); int J=p.get(1);
    quicksort(A, M, J);
    quicksort(A, I, N);
  }
} //quicksort
```

Example Execution of Quicksort algorithm



Pivot element (can be selected randomly, or as median of three fixed elements, or by any other approach)

Original description of partition() [2]

comment I and J are output variables, and A is the array (with subscript bounds M:N) which is operated upon by this procedure. Partition takes the value X of a random element of the array A, and rearranges the values of the elements of the array in such a way that there exist integers I and J with the following properties:

$$M \leqq J < I \leqq N \text{ provided } M < N$$
$$A[R] \leqq X \text{ for } M \leqq R \leqq J$$
$$A[R] = X \text{ for } J < R < I$$
$$A[R] \geqq X \text{ for } I \leqq R \leqq N$$

The procedure uses an integer procedure random (M,N) which chooses equiprobably a random integer F between M and N, and also a procedure exchange, which exchanges the values of its two parameters;

Original code for partition() [2] -- see Listing 1 for HJ code

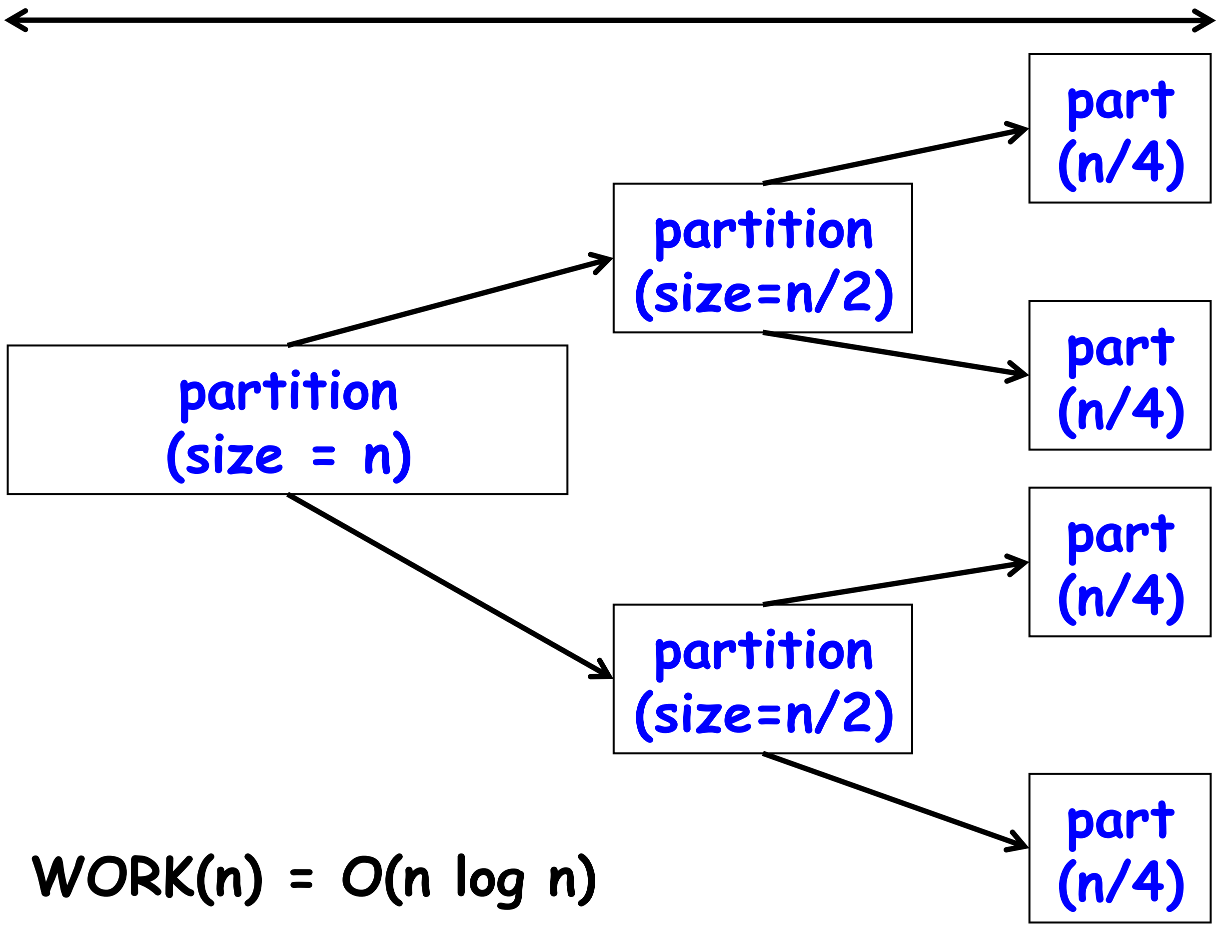
```
begin    real X;  integer F;
         F := random (M,N);  X := A[F];
         I := M;  J := N;
up:      for I := I step 1 until N do
                   if X < A [I] then go to down;
         I := N;
down:    for J := J  step −1 until M do
                   if A[J]<X then go to change;
         J := M;
change:  if I < J then begin exchange (A[I], A[J]);
                             I := I + 1; J := J − 1;
                             go to up
                       end
else     if I < F then begin exchange (A[I], A[F]);
                             I := I + 1
                       end
else     if F < J then  begin exchange (A[F], A[J]);
                             J := J − 1
                       end;
end      partition
```

Two Opportunities in Parallelizing Quicksort

```
procedure Quicksort(S) {
   if S contains at most one element then return S
   else {
       choose an element a randomly from S;
       // Opportunity: Parallelize partitoning
       let S1, S2 and S3 be the sequences of elements in S less
       than, equal to, and greater than a, respectively;
       // Opportunity: Parallelize recursive calls
       return (Quicksort(S1) followed by S2 followed by
               Quicksort(S3))
    } // else
} // procedure
```

Approach 1: sequential partition, parallel calls

O(log n) depth of calls to partition()



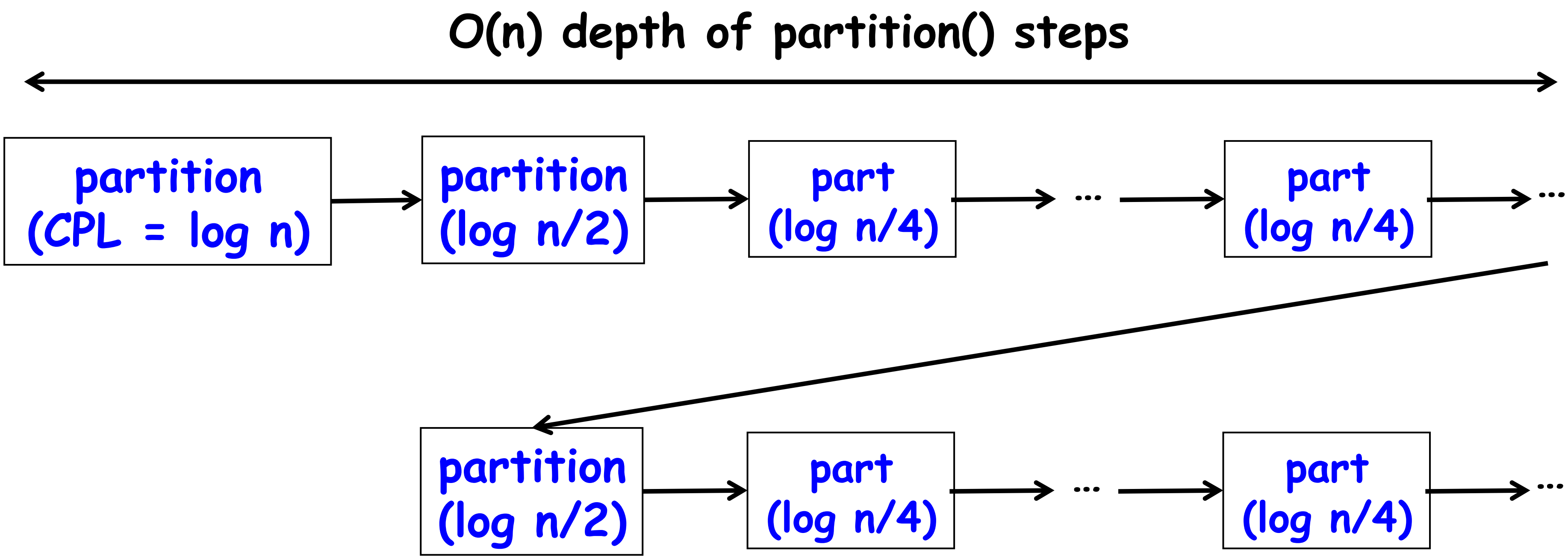
WORK(n) = O(n log n)

CPL(n) = O(n) + O(n/2) + O(n/4) + … = O(n)

Parallel HJ implementation of Quicksort for Approach 1 (Listing 2)

```
static void quicksort(int[] A, int M, int N) {
  if (M < N) {
    // partition() selects a pivot element in A[M…N]
    // to partition A[M…N] into A[M…J] and A[I…N]
    point p = partition(A, M, N);
    int I=p.get(0); int J=p.get(1);
    async quicksort(A, M, J);
    async quicksort(A, I, N);
  }
} //quicksort
```

Approach 2: Parallel partition, sequential calls



WORK(n) = O(n log n)

CPL(n) = log(n) + 2 log(n/2) + 4 log(n/4) + … = O(n)

Parallel HJ implementation of partition() for Approach 2 (Listing 3)

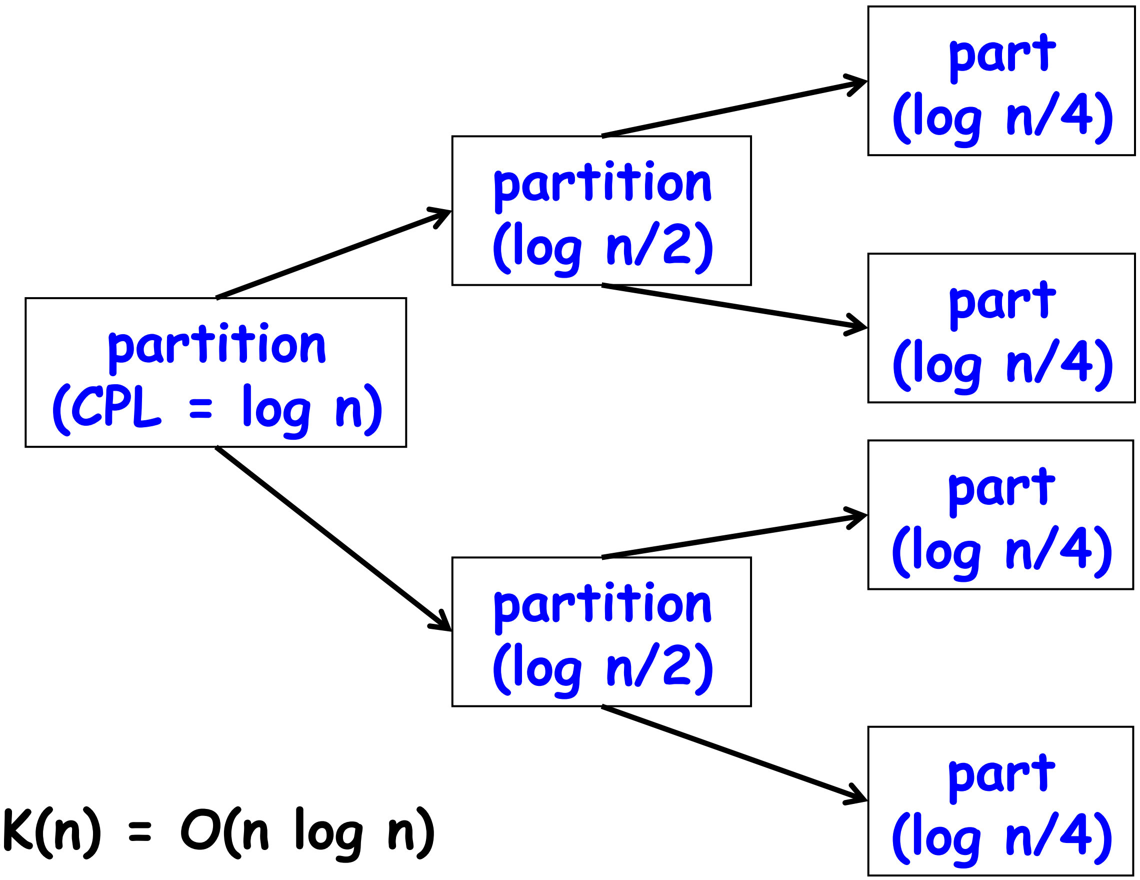
```
static point partition(int[] A, int M, int N) {
  int I, J;
  final int pivot = M + new java.util.Random().nextInt(N-M+1);
  final int[] buffer = new int[N-M+1];
  final int[] lt = new int[N-M+1];
  final int[] gt = new int[N-M+1];
  final int[] eq = new int[N-M+1];
  forall(point [k] : [0:N-M]) {
    lt[k] = (A[M+k] < A[pivot] ? 1 : 0);
    eq[k] = (A[M+k] == A[pivot] ? 1 : 0);
    gt[k] = (A[M+k] > A[pivot] ? 1 : 0);
    buffer[k] = A[M+k];
  }
```

Parallel HJ implementation of partition() for Approach 2 (Listing 3)

```
final int ltCount = computePrefixSums(lt);
final int eqCount = computePrefixSums(eq);
final int gtCount = computePrefixSums(gt);
forall(point [k] : [0:N-M]) {
 if(ltCount[k]==1) A[M+lt[k]-1] = buffer[k];
 else if(eqCount[k]==1) A[M+ltCount+eq[k]-1] = buffer[k];
 else A[M+ltCount+eqCount+gt[k]-1] = buffer[k];
}
if(M+ltCount == M) return [M+ltCount+eqCount, M+ltCount];
else if(M+ltCount == N) return [M+ltCount, M+ltCount-1];
else return [M+ltCount+eqCount, M+ltCount-1];
} // partition
```

Approach 3: parallel partition, parallel calls

O(log n) depth of calls to partition()



WORK(n) = O(n log n)

CPL(n) = O(log n) + O(log n/2) + O(log n/4) + … = $O(\log^2 n)$