Comp 311
Functional Programming

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October 2, 2018

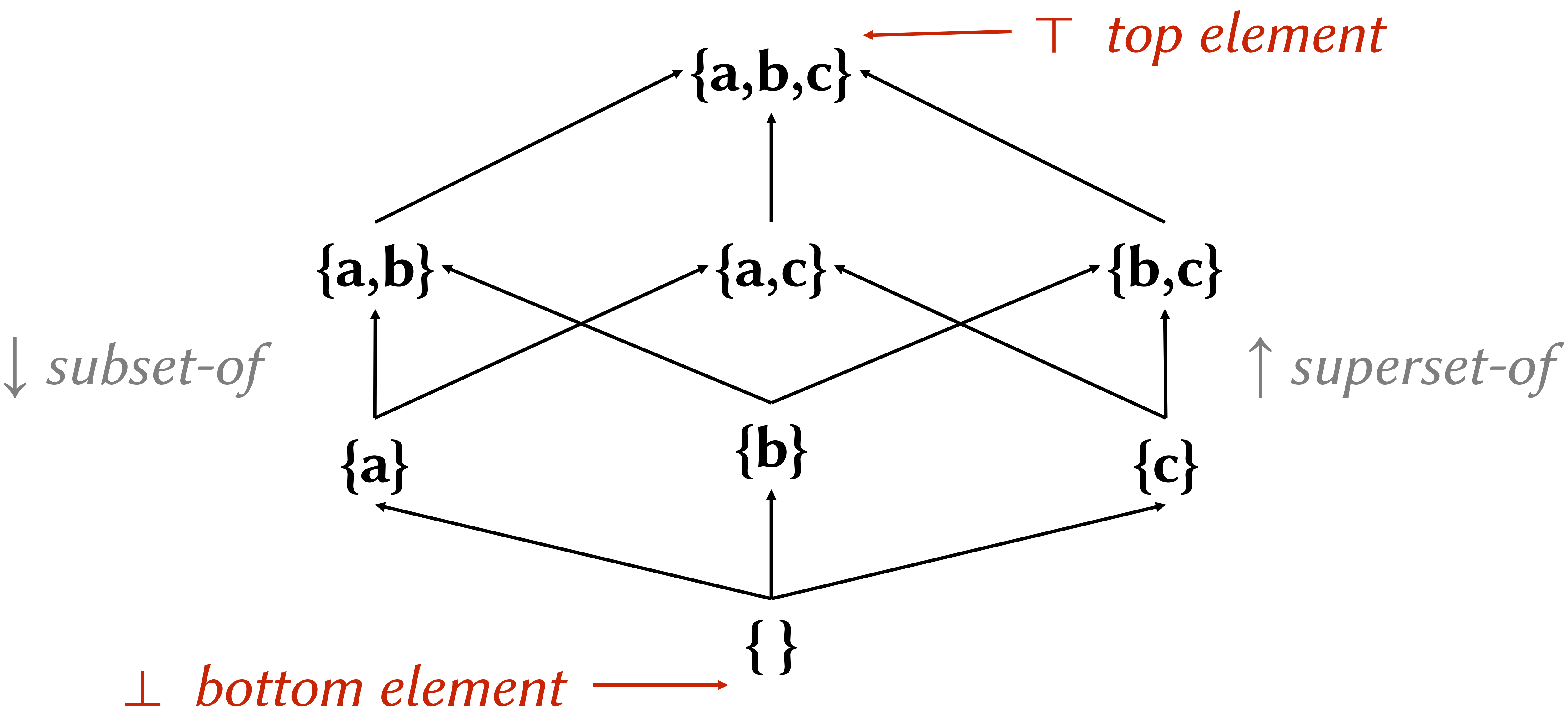
Scala Type Hierarchy

Type Hierarchies

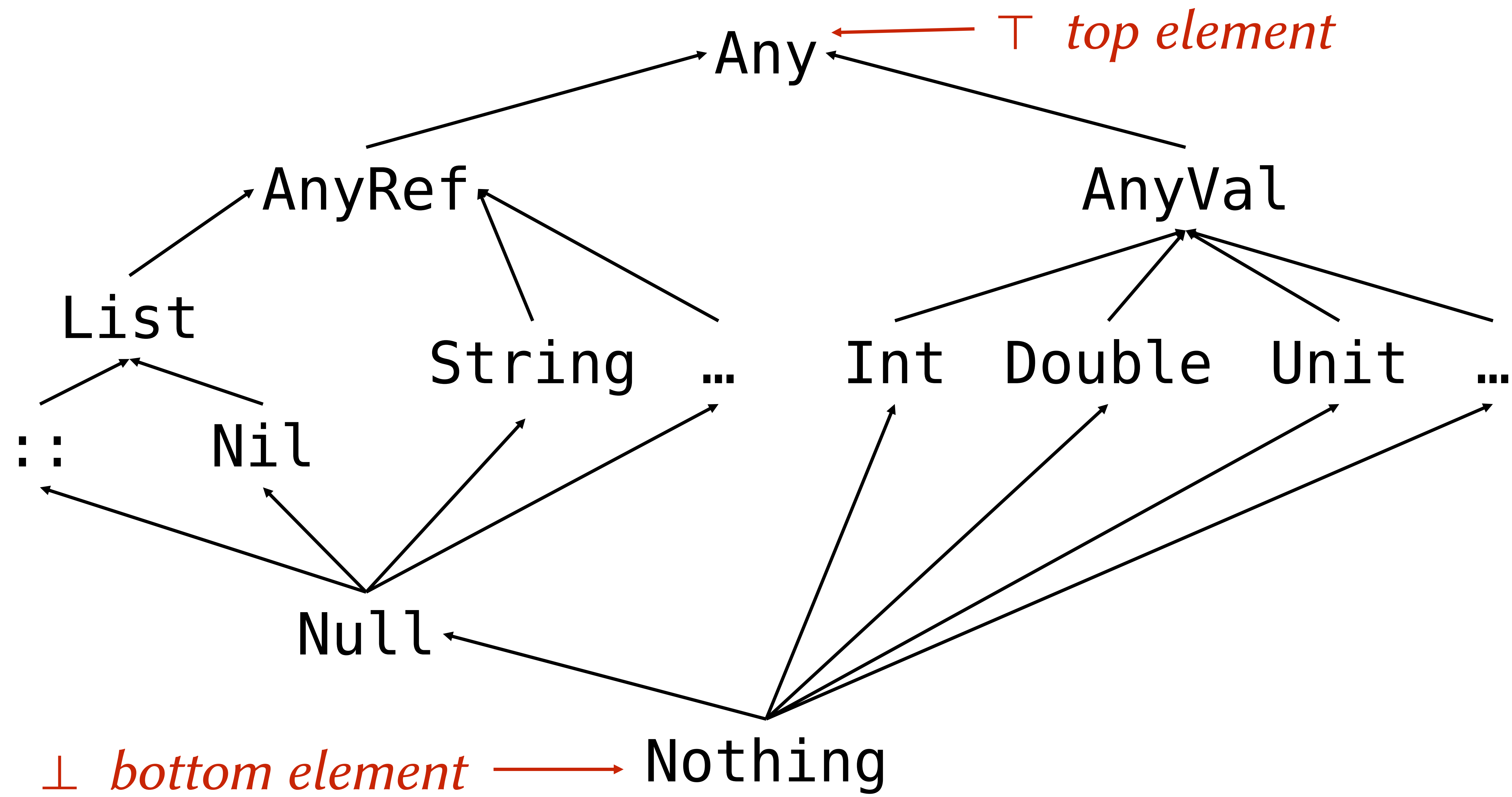
Inheritance (subclass / superclass relationships) form a *complete lattice* in the Scala type system:

- Each pair of classes has exactly one:
  - *Least upper-bound*
  - *Greatest lower-bound*
- The same applies to all value types

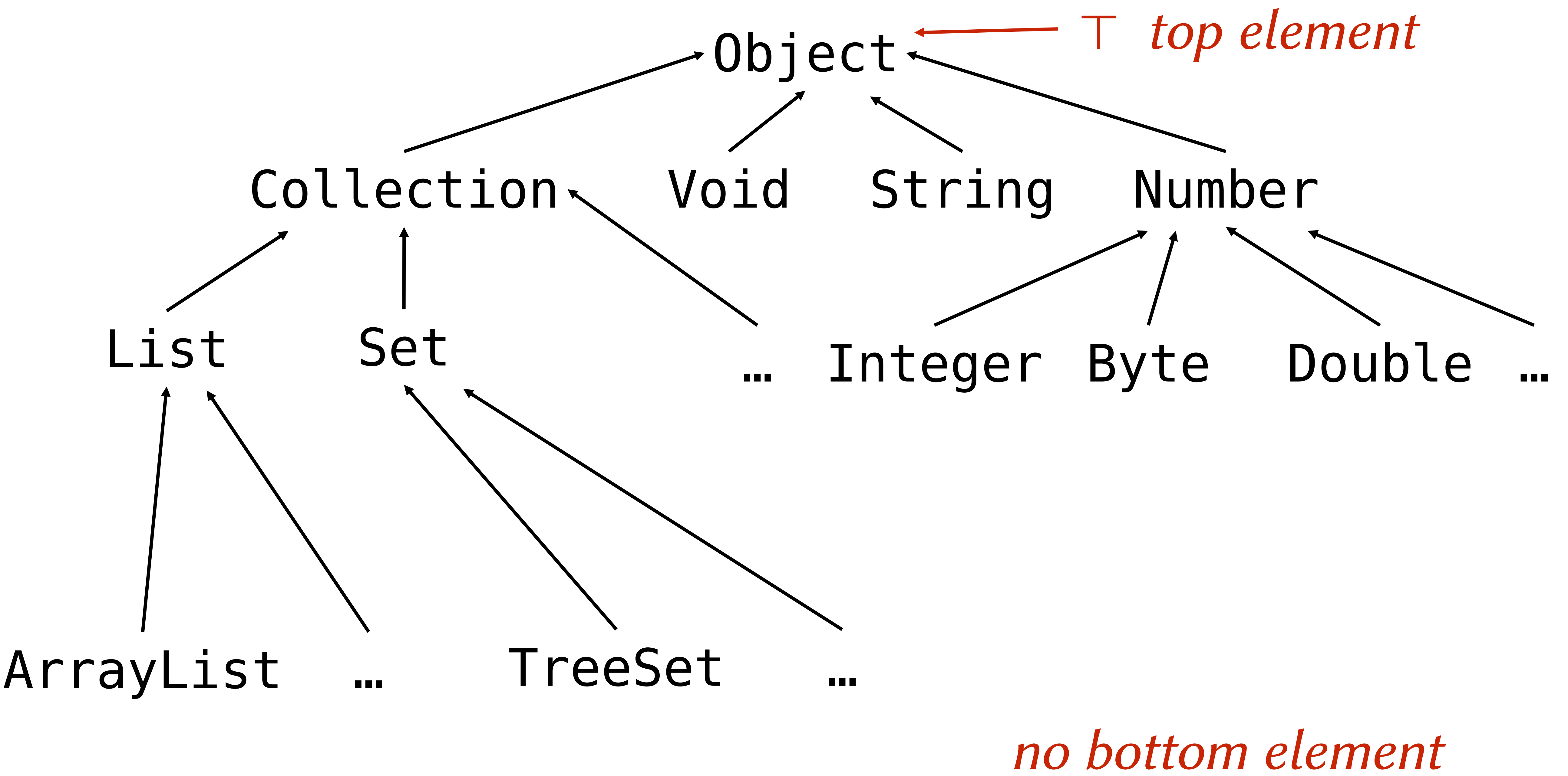
Hasse Diagrams



Scala Type Lattice



Java Type Semi-Lattice



Variance of Parametric Types

Covariance

In general, we say that a parametric type `C` is covariant with respect to its type parameter `S` if:

```
S <: T
```

implies

```
C[S] <: C[T]
```

Contravariance

In general, we say that a parametric type `C` is contravariant with respect to its type parameter `S` if:

```
S <: T
```

implies

```
C[T] <: C[S]
```

Invariance

In general, we say that a parametric type C is invariant with respect to its type parameter S if:

```
S <: T
```

implies neither

```
C[S] <: C[T]
```

nor

```
C[T] <: C[S]
```

Syntax for Variance

Syntactically:

- Covariant type parameter declarations are annotated with a *plus* sign.
- Contravariant type parameter declarations are annotated with a *minus* sign.
- Invariant type parameter declarations are not annotated with an extra symbol.

```
case class X[+A,-B,C]
```

Image by Andrey Tyukin: https://stackoverflow.com/a/19739576/1427124

See also: https://www.cs.rice.edu/~javaplt/nv4/pecs/

Generic Functions

Parametric Functions

- Just as we can add type parameters to a class definition, we can also add them to a function definition

- The type parameters are in scope in the header and body of the function

Map Revisited

```
abstract class List[+T] {
  …
  def map[U](f: T => U): List[U]
}
```

Is this occurrence of T acceptable?
Does this definition of `map` still work as expected given covariance-enabled subtyping?

We Consider Specific Instantiations

```
abstract class List[Any] {
  …
  def map[U](f: Any => U): List[U]
}
abstract class List[String] {
  …
  def map[U](f: String => U): List[U]
}
```

List[String] is an acceptable subtype of List[Any]
only if the function argument `f: Any ⇒ U`
works when passed to the `map` *method of List[String].*

We Consider Specific Instantiations

Scala-style hash-code

```
val xs: List[Any] =
  List[String]("a", "b", "c")

xs map[Int] { x: Any => x.## }
// ↦ List[Int](97, 98, 99)
```

List[String] is an acceptable subtype of List[Any]
only if the function argument `f: Any ⇒ U`
works when passed to the `map` *method of List[String].*

Generalizing Our Rules

- In our example, type parameter `T` occurs as the parameter of an arrow type:
  - `(String => U) >: (Any => U)`, provided:
    - `String <: Any`
    - `U <: U`
  - Consistent with `List[String] <: List[Any]`

An Example of How We Might Use Contravariant Type Parameters

```
abstract class Function1[-S,+T] {
  def apply(x:S): T
}
```

Map Revisited

```
case object Empty extends List[Nothing] {
  …
  def map[U](f: Nothing => U) = Empty
}
```

Map Revisited

```
case class Cons[+T](head: T, tail: List[T])
extends List[T] {
  …
  def map[U](f: T => U) =
    Cons(f(head), tail.map(f))
}
```

Syntactic Sugar: Currying

- Scala provides special syntax for defining a function that immediately returns another function:

```
def f(x_0: T_0, … ,x_N: T_N) = (y_0: U_0, …,y_M: U_M) => expr
```

can be rewritten as:

```
def f (x_0: T_0, … ,x_N: T_N)(y_0: U_0, …,y_M: U_M) = expr
```

- Defining a function in this way is called "currying", after the computer scientist Haskell Curry

Folding Revisited

```
abstract class List[+T] {
  …
  def foldLeft[S](x: S)(f: (S, T) => S): S
  def foldRight[S](x: S)(f: (T, S) => S): S
}
```

Note that these functions are curried