Comp 311
Functional Programming

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Homework 0

- Please follow these instructions for checking out your **turnin** repository as soon as possible:
  - Follow the instructions under Homework Submission Guide at the Course Website
  - Submit a **hw_0** folder with a single file **HelloWorld.txt** and a single line of text, **Hello, world!**
  - This submission is not for credit
  - We will let you know if we have not received your submission
  - You will be responsible for successfully submitting your **hw_1** assignment using **turnin**
  - Please bring problems to our attention as soon as possible

A New Paradigm

- Set aside what you've learned about programming
- The style we will practice might seem unfamiliar at first
- Initially, the material will seem quite basic
    - We will build a solid foundation that will enable us to explore advanced topics

A New Paradigm

- We will re-examine many things we’ve (partially) learned
  - Often in life, the way forward is to rethink our assumptions
  - Later, we can integrate what we’ve learned into our larger body of knowledge

Our first exposure to pure computation: Arithmetic

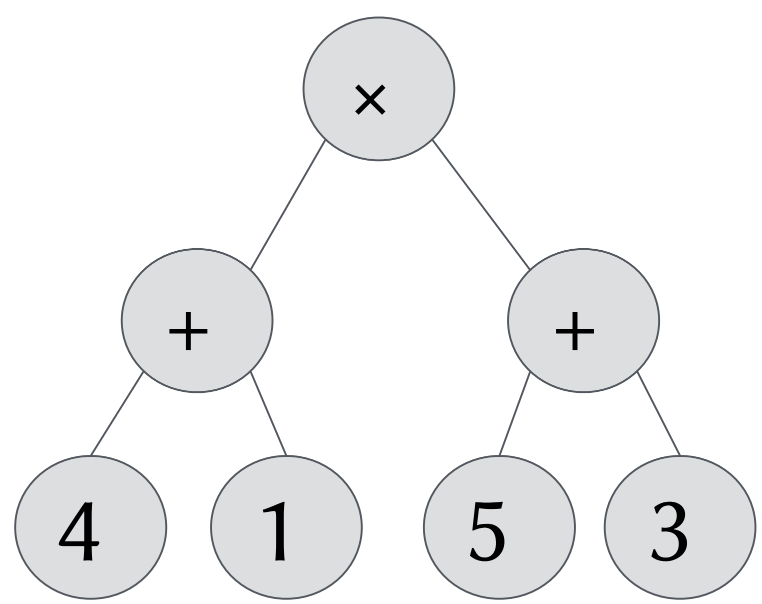
$$4 + 5 = 9$$

$$4 + 5 \mapsto 9$$

expressions are **reduced** to **values**

Critical Intuition

- Reduction rules (although typically written using conventional [*concrete*] syntax) work on *abstract syntax trees* (ASTs).
- Every expression in conventional (concrete) syntax corresponds to an abstract syntax tree.
- Example: $(4 + 1) \times (5 + 3)$



Critical Intuition II

- Tree structure is typically encoded in concrete syntax using parentheses
- Example:
  normal function application notation, *e.g.*,
  prod(sum(3,1), sum(5,3))
- Expressions with parentheses are hard for humans to read so common mathematical notation heavily relies on infix notation for binary operators and precedence conventions, *e.g.*,
  $2 + 3 \times 6$ *vs.* $2 \times 3 + 6$
- Thinking about syntax in terms of ASTs simplifies reduction rules

Expressions are Reduced to Values

- Rules for a fixed set of operators:
  - $4 + 5 \longmapsto 9$
  - $4 - 5 \longmapsto -1$
  - $4 \times 5 \longmapsto 20$
  - $9 / 3 \longmapsto 3$
  - $4^2 \longmapsto 16$
  - $\sqrt{4} \longmapsto 2$

Expressions are Reduced to Values

To reduce an operator applied to expressions, first reduce the subexpressions, left to right:

$$(4 + 1) \times (5 + 3) \mapsto$$

$$5 \times (5 + 3) \mapsto$$

$$5 \times 8 \mapsto$$

$$40$$

Expressions are Reduced to Values

A precedence is defined on operators to help us decide what to reduce next:

$$4 + 1 \times 5 + 3 \mapsto$$

$$4 + 5 + 3 \mapsto$$

$$9 + 3 \mapsto$$

$$12$$

New Operations Often Introduce New Types of Values

- $4 + 5 \longmapsto 9$
- $4 - 5 \longmapsto -1$
- $4 \times 5 \longmapsto 20$
- $4 / 5 \longmapsto 0.8$
- $4^2 \longmapsto 16$
- $\sqrt{-1} \longmapsto i$

Old Operations on New Types of Values Often Introduce Yet More New Types of Values

$$1 + i$$

So, what are types?

Values Have
Value Types

Definition: A *value type* is a *name* for a collection of values with common properties.

Values Have
Value Types

- Examples of value types:
  - Natural numbers
  - Integers
  - Floating point numbers
  - *And many more*

Expressions Have **Static Types**

Definition (Attempt 1): A *static type* is an assertion that an expression reduces to a value with a particular *value type.*

Expressions Have **Static Types**

$$4 + 5 : \mathbf{N} \mapsto 9 : \mathbf{N}$$

Static Type

Value Type

Rules for Static Types

- If an expression is a value, its static type is its value type

$$5 \mathbin{:} \mathbf{N}$$

- With each operator, there are "if-then" rules stating the required static types of the operands, and the static type of the application:

Integer Addition: If the operands to + are of type N then the application is of type N

Expressions Have **Static Types**

Definition (Attempt 1): A *static type* is an *assertion* that an expression reduces to a value with a particular *value type.*

Not quite.

Expressions Have **Static Types**

$$16 \,/\, 20 \mathbin{:} \mathbf{Q} \mapsto 0.8 \mathbin{:} \mathbf{Q}$$

So far, so good...

Expressions Have **Static Types**

$$16\ /\ 0 : \mathbf{Q} \mapsto ?$$

Expressions Have **Static Types**

Definition (Attempt 2): A *static type* is an *assertion* that either an expression reduces to a value with a particular *value type*, or one of a well-defined set of exceptional events occurs.

Why Static Types?

- Using our rules, we can determine whether an expression has a static type.

- If it does, we say the expression is *well-typed,* and we know that proceeding with our computation is *type safe:*

    Either our computation will finish with a value of the determined value type, or one of a well-defined exceptional events will occur.

What Constitutes the Set of Well-Defined Exceptional Events in Arithmetic?

- A “division by zero” error
- What else?

What are the Well-Defined Exceptional Events in Arithmetic?

- A “division by zero” error
- What if we run out of paper?
  - Or pencil lead? Or erasers?
- What if we run out of time?

What Constitutes the Set of Well-Defined Exceptional Events in Arithmetic?

- A “division by zero” error
- We run out of some finite resource

Our second exposure to pure computation: Algebra

Now, We Learn How to Define Our Own Operators (a.k.a. functions)

$$f(x) = 2x + 1$$

$$f(x, y) = x^2 + y^2$$

And We Learn How to Compute With Them

$$\frac{f(x) = 2x + 1}{\begin{array}{c} f(3 + 2) \mapsto \\ f(5) \mapsto \\ (2 \times 5) + 1 \mapsto \\ 10 + 1 \mapsto \\ 11 \end{array}}$$

The Substitution Rule of Computation

- To reduce an application of a function to a set of arguments:
  - Reduce the arguments, left to right
  - Reduce the body of the function, with each parameter replaced by the corresponding argument

Using the Substitution Rule

$$\frac{f(x, y) = x^2 + y^2}{\begin{array}{c} f(4 - 5, 3 + 1) \mapsto \\ f(-1, 3 + 1) \mapsto \\ f(-1, 4) \mapsto \\ -1^2 + 4^2 \mapsto \\ 1 + 16 \mapsto \\ 17 \end{array}}$$

What About Types?

- Eventually, we learn that our functions need to include rules indicating the required types of their arguments, and the types of applications
- You might have seen notation like this in a math class:

$$f\colon \boldsymbol{Z} \longrightarrow \boldsymbol{Z}$$

Typing Rules for Functions

$$f\colon \boldsymbol{Z} \to \boldsymbol{Z}$$

What does this rule mean?

Typing Rules for Functions

$$f\colon \boldsymbol{Z} \to \boldsymbol{Z}$$

- We can interpret the arrow as denoting data flow:

The function f consumes arguments with value type $\boldsymbol{Z}$ *and produces values with value type* $\boldsymbol{Z}$

(or one of a well-defined set of exceptional events occurs).

Typing Rules for Functions

$$f\colon \boldsymbol{Z} \to \boldsymbol{Z}$$

- We can also interpret the arrow as logical implication:

If f is applied to an argument expression with static type $\boldsymbol{Z}$ *then the application expression has static type* $\boldsymbol{Z}$*.*

What are The Exceptional Events in Algebra?

- A “division by zero” error
- We run out of some finite resource
- What else?

The Substitution Rule Allows for Computations that Never Finish

$$f\colon \mathbf{Z} \times \mathbf{Z} \longrightarrow \mathbf{Z}$$

$$\frac{\mathrm{f}(x, y) = \mathrm{f}(x, y)}{\begin{array}{c} \mathrm{f}(4 - 5, 3 + 1) \longmapsto \\ \mathrm{f}(-1, 3 + 1) \longmapsto \\ \mathrm{f}(-1, 4) \longmapsto \\ \mathrm{f}(-1, 4) \longmapsto \\ \cdots \end{array}}$$

The Substitution Rule Allows for Computations that Keep Getting Larger

$$f\colon \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$$

$$\frac{\mathrm{f}(x, y) = \mathrm{f}(\mathrm{f}(x, y), \mathrm{f}(x, y))}{\mathrm{f}(4 - 5, 3 + 1) \mapsto}$$

$$\mathrm{f}(-1, 3 + 1) \mapsto$$

$$\mathrm{f}(-1, 4) \mapsto$$

$$\mathrm{f}(\mathrm{f}(-1, 4), \mathrm{f}(-1, 4)) \mapsto$$

$$\mathrm{f}(\mathrm{f}(\mathrm{f}(-1, 4), \mathrm{f}(-1, 4)), \mathrm{f}(\mathrm{f}(-1, 4), \mathrm{f}(-1, 4))) \mapsto$$

...

But We Need at Least Limited Recursion to Define Common Algebraic Constructs

$$!: \mathbf{N} \longrightarrow \mathbf{N}$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n(n-1)! & \text{if } n > 0 \end{cases}$$

What are The Exceptional Events in Algebra?

- A “division by zero” error
- We run out of some finite resource
- The computation never stops (unbounded time)
- The computation keeps getting larger (unbounded space)

A new exposure to pure computation: Core Scala

Core Scala

- We will continue to use algebra as our model of computation
- We will switch to Scala syntax
- We will introduce new value types

Value Types in Core Scala

Int: -3, -2, -1, 0, 1, 2, 3

Double: 1.414, 2.718, 3.14, ∞

Boolean: false, true

String: "Hello, world!"

Primitive Operators on Ints and Doubles in Core Scala

Algebraic operators:

$e + e'$ $\quad$ $e - e'$ $\quad$ $e * e'$ $\quad$ e / e'

- For each operator:
  - If both arguments to an application of an operator are of type Int then the application is of type Int
  - If both arguments to an application of an operator are of type Double then the application is of type Double

Primitive Operators on Ints and Doubles in Core Scala

Comparison operators:

$$e == e' \quad e <= e' \quad e >= e' \quad e \mathrel{!}= e'$$

$$e > e' \quad e < e'$$

- For each operator:
  - If both arguments to an application of an operator are of type Int then the application is of type Boolean
  - If both arguments to an application of an operator are of type Double then the application is of type Boolean

Some Primitive Operators on Booleans in Core Scala

Conjunction, Disjunction:

$$e \mathbin{\&} e' \qquad e \mid e'$$

- In both cases:
  - If both arguments to an application are of type Boolean then the application is of type Boolean

More Primitive Operators on Booleans in Core Scala

Negation:

$$!e$$

- If the argument to an application is of type Boolean then the application is of type Boolean

Yet More Primitive Operators on Booleans in Core Scala

Conditional Expressions:

$$\text{if (e) e}' \text{ else e}''$$

- If the first argument is of type Boolean and the second and third argument are of the same type T then the application is of type T

Primitive Operators on Strings in Core Scala

String Concatenation:

$$e + e'$$

- If both arguments are of type String then the application is of type String

An Example Function Definition in Core Scala

```scala
def square(x: Double) = x * x
```

Syntax for Defining Functions

```
def fnName(arg0: type0, …, argk: typek): returnType =
  expr
```

- If there is no recursion, we may elide the return type:

```
def fnName(arg0: type0, …, argk: typek) =
  expr
```

The Substitution Rule Works as Before

```
def square(x: Double) = x * x
```

```
square(2.0 * 3.0) ↦
   square(6.0) ↦
    6.0 * 6.0 ↦
       36.0
```