Comp 311
Functional Programming

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Announcements

- Homework 0 is “due” next Tuesday
- Homework 1 will also be assigned next Tuesday

Defining Constants

Including Constant Definitions

- We can include constant definitions within functions by using `val`

```
val x = expr
val y: Type = expr
```

- We refer to expressions prefixed with a sequence of constant definitions as compound expressions

Place After The Requires Clause and Before the “Result” Expression

```
def cost(ticketPrice: Int) = {
  require (0 <= ticketPrice & ticketPrice <= 1000)

  val fixedCost: Int = 18000
  val perAttendeeCost: Int = 4

  fixedCost + perAttendeeCost * attendance(ticketPrice)
} ensuring (result => 0 <= result)
```

Place After The Requires Clause and Before the “Result” Expression

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def cost(ticketPrice: Int) = {
  require (0 <= ticketPrice & ticketPrice <= 1000)

  val fixedCost = 18000
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  fixedCost + perAttendeeCost * attendance(ticketPrice)
} ensuring (result => 0 <= result)
```

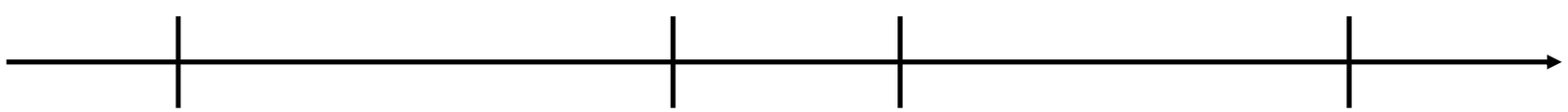
Reducing Compound Expressions

1. While there are one or more *val* bindings in the compound expression:
    i. Compute the right-hand-side value of the first *val* binding using the rules for reducing simple expressions.
    ii. Substitute the reduced value on the right-hand-side for all occurrences of the left-hand-side symbolic name throughout the remainder of this compound expression.
    iii. Drop the first *val* binding since it has now been fully reduced and substituted.
2. Reduce the resulting simple expression using the rules for reducing expressions.

Conditional Functions On Ranges

Conditional Functions On Ranges

- Often a computation falls into distinct cases depending on which of a finite set of ranges a value falls into
- In such cases, it can help to break the number line into distinct regions that we must handle separately:

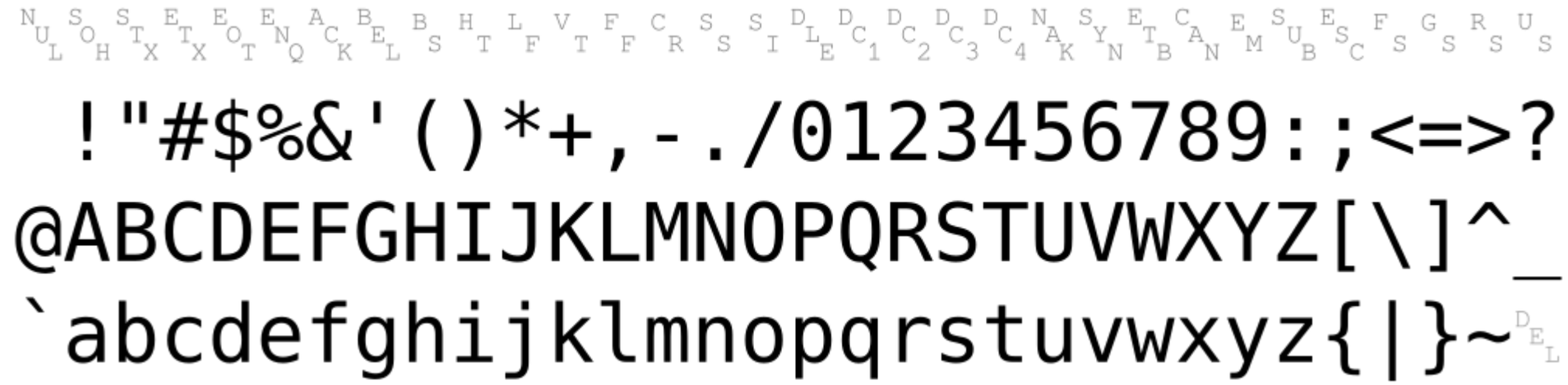


Designing Conditional Functions

- First Example – Graduated Income Tax (Single Filer):
  - Up to $9,075: 10%
  - $9,075 to $36,900: 15%
  - $36,901 to $89,350: 25%
  - $89,351 to 186,350: 28%
  - $186,351 to $405,100: 33%
  - $405,101 to $406,750: 35%
  - $405,751 or more: 39.6%

Designing Conditional Functions

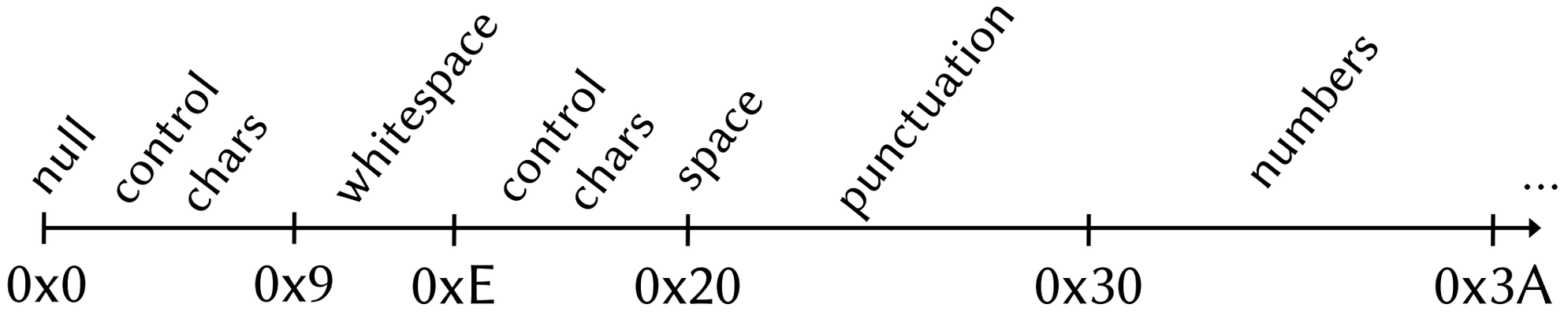
- Second Example – ASCII Character Classes



- We follow the Design Recipe

ASCII Codepoint Descriptions: Data Analysis and Definition

- We use `Int`s to ASCII character values.
- We use `String`s to describe the character class of a given ASCII codepoint.
- ASCII characters are defined in the range [0, 127].
- We break the number line into the relevant intervals:



Contract

```
/**
 * Given an ASCII character codepoint,
 * return a String describing the type of
 * character represented by that codepoint.
 */
def describeAsciiChar(char: Int): String = {
  require(0 <= char & char <= 127)
  ...
} // no ensuring clause
```

Function Application Examples

We should develop at least one example per case, as well as borderline cases

```
"Null" == describeChar(0)

"Whitespace" == describeChar(10)

"Lowercase Letter" == describeChar(97)

...
```

Our Function Template for Conditional Functions

```
/**
 * Given an ASCII character codepoint,
 * return a String describing the type of
 * character represented by that codepoint.
 */
def describeAsciiChar(char: Int): String = {
  require(0 <= char & char <= 127)
  if (char == 0x0) { "Null" }
  else if (char <= 0x08) { "Control Character" }
  else if (char <= 0x0D) { "Whitespace" }
  else if (char <= 0x19) { "Control Character" }
  else if (char == 0x20) { "Whitespace" }
  else if (char <= 0x29) { "Punctuation" }
  else if (char <= 0x39) { "Number" }
  else if (char <= 0x40) { "Punctuation" }
  else if (char <= 0x5A) { "Uppercase Letter" }
  else if (char <= 0x60) { "Punctuation" }
  else if (char <= 0x7A) { "Lowercase Letter" }
  else if (char <= 0x7E) { "Punctuation" }
  else { "Control Character" } // 0x7F
}
```

Our Function Template for Conditional Functions

```
/**
 * Given an ASCII character codepoint,
 * return a String describing the type of
 * character represented by that codepoint.
 */
def describeAsciiChar(char: Int): String = {
  require(0 <= char & char <= 127)

  if (char == 0x0) "Null"
  else if (char <= 0x08) "Control Character"
  else if (char <= 0x0D) "Whitespace"
  else if (char <= 0x19) "Control Character"
  else if (char == 0x20) "Whitespace"
  else if (char <= 0x29) "Punctuation"
  else if (char <= 0x39) "Number"
  else if (char <= 0x40) "Punctuation"
  else if (char <= 0x5A) "Uppercase Letter"
  else if (char <= 0x60) "Punctuation"
  else if (char <= 0x7A) "Lowercase Letter"
  else if (char <= 0x7E) "Punctuation"
  else "Control Character" // 0x7F
}
```

Remarks On Conditional Functions

- The clauses in a conditional function need not all have the same form

- Avoid factoring out code into a helper function until there is more than one place to call the helper

- There is more we could to do improve this example, but we need to learn more of Core Scala first.

Conditional Functions On Point Values

Conditional Functions On Point Values

- Often the cases on a conditional function must test for equality rather than whether values fall in a range
- This is especially common with String values
- What about Boolean values?
- Double values should not be tested this way (why?)

Example: Days in a Month

Given the name of a month, we want to return the number of days

Data Analysis and Definition

- We use `String`s to denote months
- We use `Int`s for the number of days
- Months have between 1 and 31 days (inclusive)

Contract

- We state the preconditions in documentation:

```
/**
 * Given a string identifying a month,
 * with the first (and only the first) letter capitalized,
 * returns the number of days in that month
 * for an ordinary year (non-leap) year.
 */
def days(month: String): Int = {
  ...
} ensuring (result => 0 < result & result <= 31)
```

- How can we improve the precondition? What data types would we want?

A Function Template for Conditional Functions on Point Values

```
/**
 * Given a string identifying a month,
 * with the first (and only the first) letter capitalized,
 * returns the number of days in that month
 * for an ordinary year (non-leap) year.
 */
def days(month: String): Int = {
  month match {
    case ... => ...
    ...
  }
} ensuring (result => 0 < result & result <= 31)
```

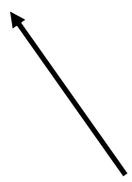
Syntax for Match

```
expr0 match {
  case Pattern1 => expr1
    ...
  case PatternN => exprN
}
```

Primitive Value Patterns

A primitive value pattern is either:

- A primitive value
- A free parameter
- The special “don’t care” pattern: _



That’s an underscore

Matching a Primitive Value With a Pattern

A primitive value ***v*** matches:

- Itself (e.g., 5 matches 5)
- A free parameter (e.g., x matches 5)
- The special “don’t care” pattern _
  - _ should only be used as the final clause of a match (why?)

Meaning of a Match Expression

- To reduce a match expression:

```
expr0 match {
  case Pattern1 => expr1
    ...
  case PatternN => exprN
}
```

  - Reduce **$expr_0$** to a value **v**
  - Find the first pattern **k** matching **v** (if it exists) and reduce to **$expr_k$** (replacing all occurrences of **k** with **v** if **k** is a free parameter)

- Failure to match a pattern results in a new form of exceptional condition (a `MatchError`)

Using Match for Point Value Matching

```
/**
 * Given a string identifying a month,
 * with the first (and only the first) letter capitalized,
 * returns the number of days in that month
 * for an ordinary year (non-leap) year.
 */
def days(month: String): Int = {
  month match {
    case "January" => 31
    case "February" => 28
    case "March" => 31
    case "April" => 30
    case "May" => 31
    case "June" => 30
    case "July" => 31
    case "August" => 31
    case "September" => 30
    case "October" => 31
    case "November" => 30
    case "December" => 31
  }
} ensuring (result => 0 < result & result <= 31)
```

Reducing Match

```
days("September")
```
↦
```
"September" match {
    case "January" => 31
    case "February" => 28
    case "March" => 31
    case "April" => 30
    case "May" => 31
    case "June" => 30
    case "July" => 31
    case "August" => 31
    case "September" => 30
    case "October" => 31
    case "November" => 30
    case "December" => 31
  }
} ensuring (result => 0 < result & result <= 31)
```
↦
```
30
```

A Match With a Free Parameter

```
def plural(word: String): String =
  word match {
    case "deer" => "deer"
    case "fish" => "fish"
    case "mouse" => "mice"
    case x => x + "s"
  }
```

Compound Datatypes

Compound Datatypes

- Although many computations can be performed on primitive data types, it is often useful to combine data into larger structures
- We call all data of this form *compound data*
- The two simplest compound datatypes in Core Scala are tuples and arrays

Tuple Values

- A tuple value contains a sequence of values

$$(v_1, \ldots, v_N)$$

- There is one empty tuple `()`

- Tuples of length one do not exist (why?)

- The value type of a tuple is simply the tuple of the corresponding value types

$$(T_1, \ldots, T_N)$$

Tuple Types

- The empty tuple has the special type `Unit`
- The static type of a tuple expression:

$$(e_1, \ldots e_N)$$

is

$$(T_1, \ldots, T_N)$$

where

$$e_1 : T_1, \ldots e_N : T_N$$

Tuple Types

- Tuple types allow us to combine data of distinct types. For example:

```
(Int, Boolean, String)
```

- However, tuple types restrict the length of any corresponding tuple value

Accessing Tuple Elements

- We can access the ***k*th** element of an expression **e** with static type **(T_1, ..., T_N)** using the syntax:

$$e._k$$

- The static type of this expression is T_k
- Note that tuples are 1-indexed
- Example:

```
(1,2,3)._2 ↦ 2
```

Accessing Tuple Elements

- We can access the elements of a tuple using match expressions
  - We add the following syntactic form to our definition of patterns

$$\texttt{(Pattern}_1\texttt{, ... , Pattern}_N\texttt{)}$$

- We call this new syntactic form a *tuple pattern*

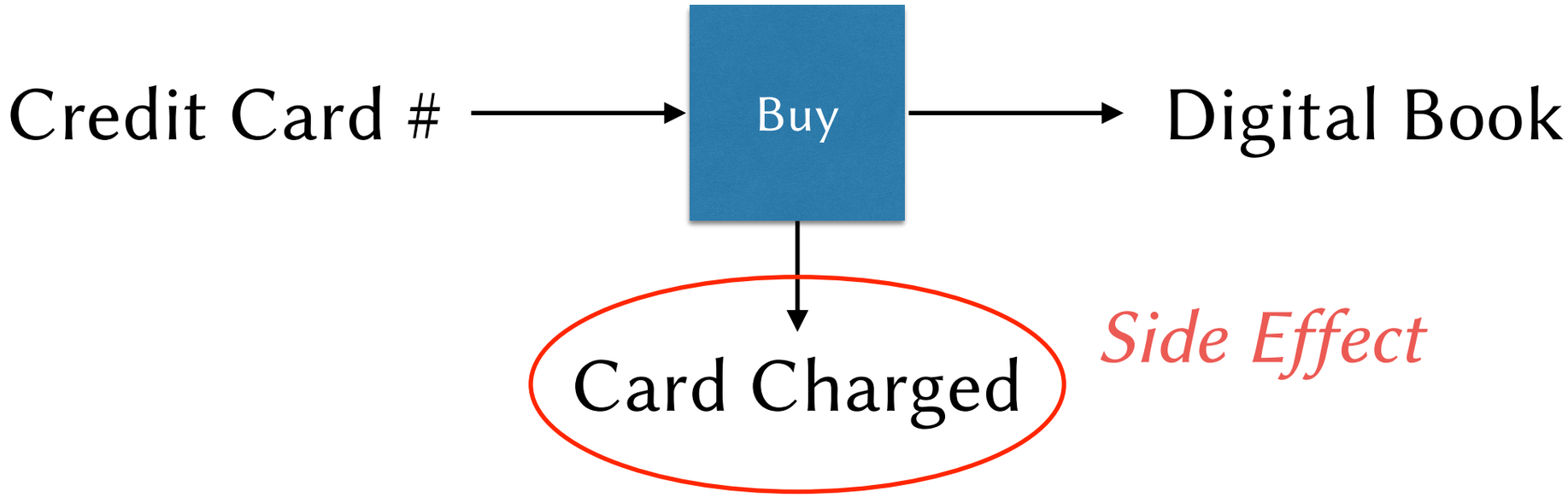
Accessing Tuple Elements

- A tuple matches a tuple pattern iff each element of the tuple matches a corresponding element of the tuple pattern, and vice versa (bijection)

- Does `(x,y,z)` match `(1,2)`?

Book Purchase (Revisited)

The *imperative* way:



Book Purchase (Revisited)

The *functional* way:

Modeled in Scala:

```
def buy(card: CreditCard): (DigitalBook, ChargeEvent) = {
  ...
}
```

Tuple Types and Arrow Types

- We can now view every arrow type as taking exactly one parameter:

- Example:

```
(Int, String, Boolean) → Int
```

Tuple Types and Arrow Types

- We can also use tuple types to denote that a function returns “multiple values”:

- Example:

```
(Int, String, Boolean) → (Int, Double)
```

Array Values

An array is a sequence of values all of the same value type

```
Array(1,2,3)
```

Array Types

- If the elements of an array value are of type T then the array is of type Array[T]

- If the expressions $e_1, \ldots, e_N$ are of static type T then the expression

$$\texttt{Array}(e_1, \ldots, e_N)$$

- has static type

$$\texttt{Array[T]}$$

Array Types

- Array types require that all elements of an array share a common type
- However, array types match array values of any length
- Contrast with tuple types

Accessing Array Values

- We can access the *k*`th` element of an expression of type `Array[T]` with the syntax:

```
expr(k)
```

- The static type of this expression is T
- Note that arrays are zero-indexed
- Example:

```
Array(1,2,3)(2) ↦ 3
```

Accessing Array Elements

- We can access the elements of an array using match expressions
  - We add the following syntactic form to our definition of patterns:

$$\texttt{Array(Pattern}_1\texttt{, ... , Pattern}_N\texttt{)}$$

- We call this new syntactic form an *array pattern*

Accessing Array Elements

An array matches an array pattern iff each element of the array matches a corresponding element of the array pattern, and vice versa

Accessing Array Elements

```
def sumOfSquares(coordinates: Array[Int]) = {
  coordinates match {
    case Array(x,y,z) => x*x + y*y + z*z
  }
}
```

Structural Data

Structural Data

- Tuples and arrays allow us to combine multiple primitive values into a single data value
- However,
  - They do not allow us to attach names to the constituent elements
  - They do not allow us to distinguish elements of conceptually distinct datatypes

Case Classes

- We can think of a case class as a tuple with its own *type* and *accessors* for its elements

Case Classes

```
case class Coordinate(x: Int, y: Int)
```

Simple Syntax for Case Classes

```
case class Name(field1: Type1, ..., fieldN: TypeN)
```

Creating Instances of a Case Class

- We construct new instances of a case class

```
case class C(field1: Type1, ..., fieldN: TypeN)
```

- with the syntax

```
C(expr1, ..., exprN)
```

- To reduce this expression, reduce each argument $expr_K$ to a value v_K, forming the value $C(v_1, ..., v_N)$
- If the types of $expr_1, ..., expr_N$ match the types of the corresponding fields, then this expression has type C

Accessing Fields of a Case Class

- Given a case class:

```
case class C(field_1: Type_1, ..., field_N: Type_N)
```

- We can access field with name $\texttt{field}_K$ of an instance $\texttt{C(v}_1\texttt{, ..., v}_N\texttt{)}$ with the expression syntax:

$$\texttt{C(v}_1\texttt{,...,v}_N\texttt{).field}_K$$

- The static type of this expression is $\texttt{Type}_K$

Accessing Fields of a Case Class

```
def magnitude(coordinate: Coordinate): Int = {
  coordinate.x * coordinate.x +
  coordinate.y * coordinate.y
}
```

Accessing Class Elements

- We can access the elements of a case class instance using match expressions
  - For each case class, we add the following syntactic form to our definition of patterns

$$\texttt{C(Pattern}_1\texttt{, ... , Pattern}_N\texttt{)}$$

- We call this new syntactic form a *class pattern*

Accessing Case Class Elements

- An instance of a case class `C(`v_1`, ..., `v_N`)` matches a class pattern `C(`P_1`, ..., `P_N`)` iff
  - The class name is identical to the class pattern name
  - Each element of the instance matches a corresponding element of the class pattern

Accessing Case Class Elements

```
def magnitude(coordinate: Coordinate): Int =
  coordinate match {
    case Coordinate(x,y) => x*x + y*y
  }
```

Key Takeaways

- Simple rules to reduce operators with value arguments
- Conditionals can reduce unevaluated expressions
- Template for conditional function on ranges (if/else)
- Template for conditional function on point values (match)
- Compound data: Tuples and Arrays
- Structural data: Case Classes

Supplemental Sides

Designing Conditional Functions

- Example – Graduated Income Tax (Single Filer):
    - Up to $9,075: 10%
    - $9,075 to $36,900: 15%
    - $36,901 to $89,350: 25%
    - $89,351 to 186,350: 28%
    - $186,351 to $405,100: 33%
    - $405,101 to $406,750: 35%
    - $405,751 or more: 39.6%
- We follow the Design Recipe

Graduated Income Tax: Data Analysis and Definition

- We use `Int`s to denote US$ values and tax percentages
- Both income and tax should be non-negative
- We break the number line into the relevant intervals

Graduated Income Tax: Data Analysis and Definition

- We use `Int`s to denote US$ values and tax percentages
- Both income and tax should be non-negative
- We break the number line into the relevant intervals

Contract

```
/**
 * Given an income in U.S. Dollars,
 * returns the dollar value of tax
 * owed for a single tax payer, using
 * 2014-2015 IRS tax brackets.
 */
def incomeTax(income: Int) = {
  require(income >= 0)
  ...
} ensuring (_ >= 0)
```

Function Application Examples

We should develop at least one example per case, as well as borderline cases

```
100 = incomeTax(1000)

907 = incomeTax(9075)

907 + 138 = incomeTax(10000)

...
```

Our Function Template for Conditional Functions

```
/**
 * Given an income in U.S. Dollars,
 * returns the dollar value of tax
 * owed for a single tax payer, using
 * 2014-2015 IRS tax brackets.
 */
def incomeTax(income: Int): Int = {
  require(income >= 0)

  if (income <= cutoff0) {
   ...
  } else if (income <= cutoff1) {
   ...
  } else if (income <= cutoff2) {
   ...
  } else if (income <= cutoff3) {
   ...
  } else if (income <= cutoff4) {
   ...
  } else if (income <= cutoff5) {
   ...
  } else if (income <= cutoff6) {
   ...
  } else { // income > cutoff6
   ...
  }
} ensuring (_ >= 0)
```

Defining Our Constant Values in One Place

```
val bracket0 = 0
val cutoff0 = 0

val bracket1 = 100
val cutoff1 = 9075

val bracket2 = 150
val cutoff2 = 36900

val bracket3 = 250
val cutoff3 = 89350

val bracket4 = 280
val cutoff4 = 186350

val bracket5 = 330
val cutoff5 = 405100

val bracket6 = 350
val cutoff6 = 406750

val bracket7 = 396
val cutoff7 = Int.MaxValue
```

As We Fill In Cases, We Find a Common Pattern

```
/**
 * Given:
 *   an income in U.S. Dollars
 *   the next lowest cutoff in U.S. Dollars
 *   a tax percentage for the bracket above the cutoff
 * Returns the income tax due for the given income
 */
def incomeTaxForBracket(income: Int, cutoff: Int, bracket: Int): Int = {
  require(income >= 0)
  (income - cutoff) * bracket / divisor + incomeTax(cutoff)
} ensuring (_ >= 0)
```

And Now We Call This New Function to Fill in the The Income Tax Function Template

```
/**
 * Given an income in U.S. Dollars, returns the dollar value of tax
 * owed for a single tax payer, using 2014-2015 IRS tax brackets.
 */
def incomeTax(income: Int): Int = {
  require(income >= 0)

  if (income <= cutoff0) {
    bracket0
  } else if (income <= cutoff1) {
    incomeTaxForBracket(income, cutoff0, bracket1)
  } else if (income <= cutoff2) {
    incomeTaxForBracket(income, cutoff1, bracket2)
  } else if (income <= cutoff3) {
    incomeTaxForBracket(income, cutoff2, bracket3)
  } else if (income <= cutoff4) {
    incomeTaxForBracket(income, cutoff3, bracket4)
  } else if (income <= cutoff5) {
    incomeTaxForBracket(income, cutoff4, bracket5)
  } else if (income <= cutoff6) {
    incomeTaxForBracket(income, cutoff5, bracket6)
  } else { // income > cutoff6
    incomeTaxForBracket(income, cutoff6, bracket7)
  }
} ensuring (_ >= 0)
```