# COMP 515: Advanced Compilation for Vector and Parallel Processors

Prof. Krishna Palem
Prof. Vivek Sarkar
Department of Computer Science
Rice University
{palem,vsarkar}@rice.edu

https://wiki.rice.edu/confluence/display/PARPROG/COMP515

# Acknowledgments

- Slides from previous offerings of COMP 515 by Prof. Ken Kennedy
  - http://www.cs.rice.edu/~ken/comp515/

# Dependence Testing

**Allen and Kennedy, Chapter 3 (up to Section 3.3.2)**

# The General Problem

```
DO i1 = L1, U1
  DO i2 = L2, U2
            …
    DO in = Ln, Un
 S1                   A(f1(i1,…,in),…,fm(i1,…,in)) = …
 S2                   … = A(g1(i1,…,in),…,gm(i1,…,in))
    ENDDO
    ...
  ENDDO
ENDDO
```

*Under what conditions is the following true for iterations $\alpha$ and $\beta$ ?*

$$f_i(\alpha) = g_i(\beta) \text{ for all } i, 1 \le i \le m$$

***Note that the number of equations equals the rank of the array,***

***and the number of variables is twice the number of loops that enclose both array references (two iteration vectors)***

# Basics: Complexity

A subscript equation is said to be

- ZIV if it contains no index (zero index variable)
- SIV if it contains only one index (single index variable)
- MIV if it contains more than one index (multiple index variables)

For Example:

```
A(5,I+1,j) = A(1,I,k) + C

        First subscript equation is ZIV

        Second subscript equation is SIV

        Third subscript equation is MIV
```

# Terminology: Indices and Subscripts

Index: **Index variable for some loop surrounding a pair of references**

Subscript: ***A*** **PAIR of subscript positions in a pair of array references (corresponds to dependence equation for that dimension)**

**For Example:**

```
A(I,j) = A(I,k) + C

        ‹I,I› is the first subscript

        ‹j,k› is the second subscript
```

# Basics: Separability

- A subscript is separable if its indices do not occur in other subscripts
- If two different subscripts contain the same index they are coupled

For Example:

```
A(I+1,j) = A(k,j) + C

        Both subscripts are separable

A(I,j,j) = A(I,j,k) + C

        Second and third subscripts are coupled
```

# Basics: Coupled Subscript Groups

- Why are they important?

  Ignoring coupled subscripts may lead to imprecision in dependence testing

e.g., is there a loop-carried dependence on *A* in the following loop?

```
  DO I = 1, 100
S1      A(I+1,I) = B(I) + C
S2      D(I) = A(I,I) * E
  ENDDO
```

# Basics: Conservative Testing

- Consider only linear subscript expressions
- Finding integer solutions to system of linear Diophantine Equations is NP-Complete
- Most common approximation is Conservative Testing

  See if you can assert

  "No dependence exists between two subscripted references of the same array"
- Never incorrect, may be less than optimal

# Dependence Testing: Overview

- Partition subscripts of a pair of array references into separable and coupled groups
- Classify each subscript as ZIV, SIV or MIV
- For each separable subscript apply single subscript test. If not done goto next step
- For each coupled group apply multiple subscript test
- If still not done, merge all direction vectors computed in the previous steps into a single set of direction vectors

# Step 1: Subscript Partitioning

- **Partitions the subscripts into separable and minimal coupled groups**
- **Notations**

// $S$ is a set of $m$ subscript pairs $S_1, S_2, \ldots S_m$ each enclosed in

// $n$ loops with indexes $I_1, I_2, \ldots I_n$, which is to be

// partitioned into separable or minimal coupled groups.

// $P$ is an output variable, containing the set of partitions

// $n_p$ is the number of partitions

# Step 2: Classify as ZIV/SIV/MIV

- Easy step
- Just count the number of different indices in a subscript equation

# Step 3: Applying Single Subscript Tests

- ZIV Test
- SIV Test
  - Strong SIV Test
  - Weak SIV Test
    - Weak-zero SIV
    - Weak Crossing SIV
- SIV Tests in Complex Iteration Spaces

# ZIV Test

```
  DO j = 1, 100
S         A(e1) = A(e2) + B(j)
  ENDDO
```

e1,e2 are constants or loop invariant symbols

If (e1-e2)!=0 No Dependence exists

Program analyses that can improve the accuracy of this test include constant propagation, value numbering, and symbolic “definitely different” analysis (inferring that e1 = e2 + nonzero-constant)

# Strong SIV Test

- Strong SIV subscripts are of the form

$$\langle ai + c_1, ai + c_2 \rangle$$

where a ≠ 0

- For example the following are strong SIV subscripts

$$\langle i + 1, i \rangle$$

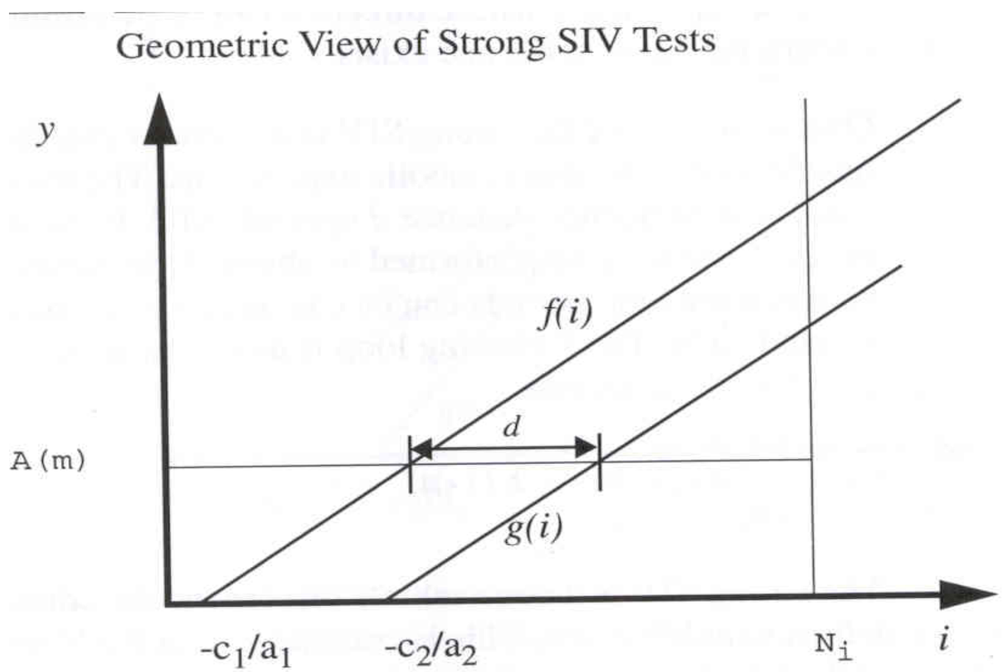
$$\langle 4i + 2, 4i + 4 \rangle$$

- Strong subscripts are also referred to as "uniformly generated"

# Strong SIV Test Example

```
DO k = 1, 100
   DO j = 1, 100
S1       A(j+1,k) = ...
S2              ... = A(j,k) + 32
   ENDDO
ENDDO
```

# Strong SIV Test




$$d = i' - i = \frac{c_1 - c_2}{a}$$

Dependence exists if there is an integer value of d within loop bounds,

$$|d| \le U - L$$

# Weak SIV Tests

- Weak SIV subscripts are of the form

$$\langle a_1 i + c_1, a_2 i + c_2 \rangle$$

where $a_1 \neq 0$ (without loss of generality)

- For example the following are weak SIV subscripts

$$\langle i + 1, 5 \rangle$$

$$\langle 2i + 1, i + 5 \rangle$$

$$\langle 2i + 1, -2i \rangle$$

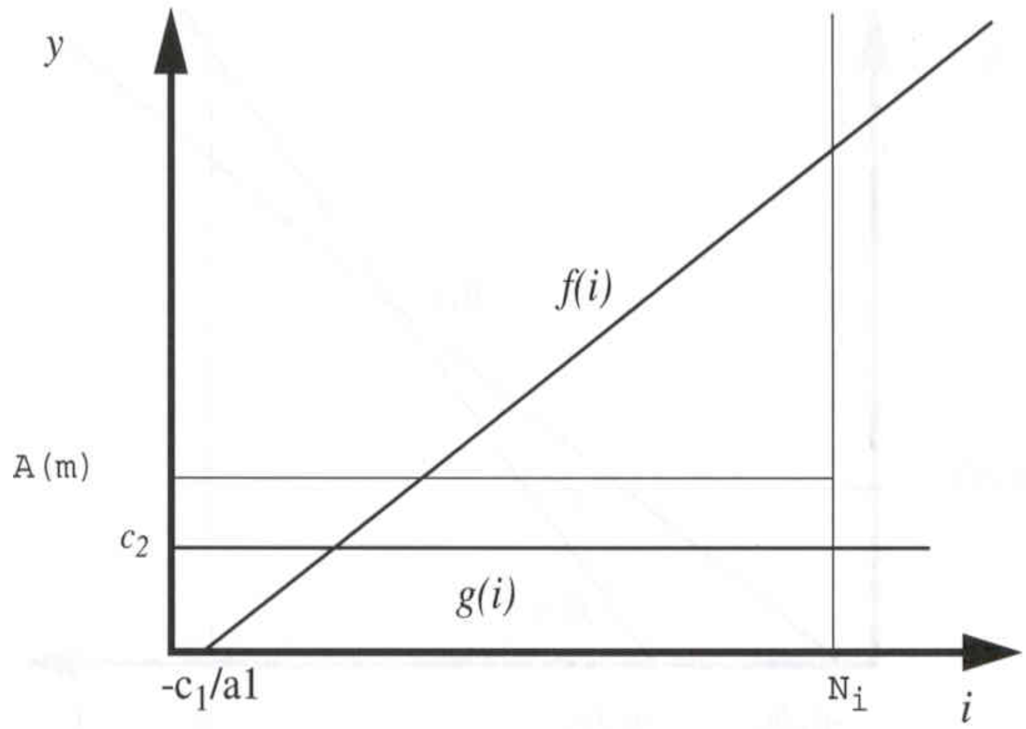
# Weak-zero SIV Test

- Special case of Weak SIV where one of the coefficients ($a_2$) of the index is zero
- The test consists merely of checking whether the solution is an integer and is within loop bounds

$$i = \frac{c_2 - c_1}{a_1}$$

# Weak-zero SIV Test

Geometric View of Weak-zero SIV Subscripts



# Weak-zero SIV & Loop Peeling

```
      DO i = 1, N
S1        Y(i, N) = Y(1, N) + Y(N, N)
      ENDDO
```

*Can be loop peeled to...*

```
      Y(1, N) = Y(1, N) + Y(N, N)
      DO i = 2, N-1
S1        Y(i, N) = Y(1, N) + Y(N, N)
      ENDDO
      Y(N, N) = Y(1, N) + Y(N, N)
```

# Weak-crossing SIV Test

- Special case of Weak SIV where the coefficients of the index are equal in magnitude but opposite in sign i.e., $a_2 = -a_1$
- The test consists merely of checking whether the solution index

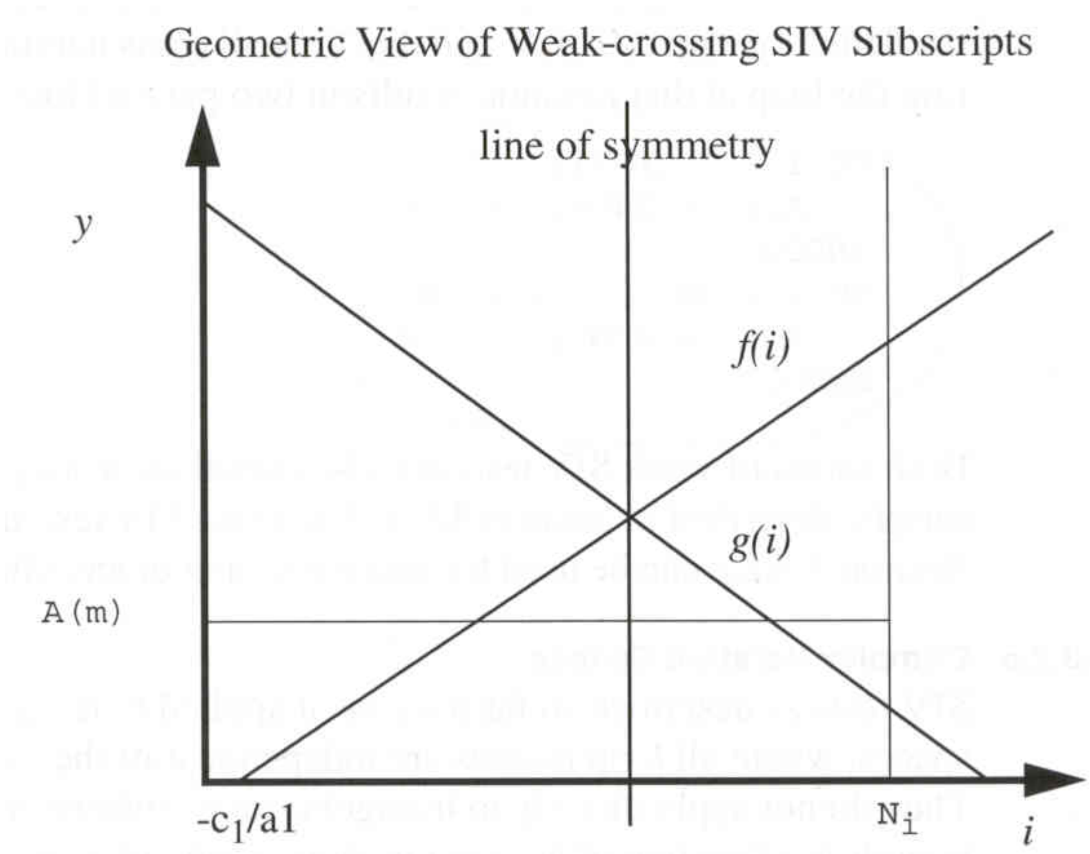
  is 1. within loop bounds and is

  2. either an integer or has a non-integer

  part equal to 1/2

$$i = \frac{c_2 - c_1}{2a_1}$$

# Weak-crossing SIV Test



# Weak-crossing SIV & Loop Splitting

```
        DO i = 1, N
S1      A(i) = A(N-i+1) + C
        ENDDO
```

*This loop can be split into...*

```
        DO i = 1,(N+1)/2
              A(i) = A(N-i+1) + C
        ENDDO
        DO i = (N+1)/2 + 1, N
              A(i) = A(N-i+1) + C
        ENDDO
```

# Complex Iteration Spaces

- **Till now we have applied the tests only to rectangular iteration spaces**
- **These tests can also be extended to apply to triangular or trapezoidal loops**
  - **Triangular: One of the loop bounds is a function of at least one outer loop index**
  - **Trapezoidal: Both the loop bounds are functions of at least one outer loop index**

# Complex Iteration Spaces

- For example consider this special case of a strong SIV subscript

```
  DO I = 1,N
          DO J = L0 + L1*I, U0 + U1*I
S1                 A(J + d) =
S2                 = A(J) + B
          ENDDO
  ENDDO
```

# Complex Iteration Spaces

- **Strong SIV test gives dependence if**

$$|d| \le U_0 - L_0 + (U_1 - L_1)I$$

$$I \ge \frac{|d| - (U_0 - L_0)}{U_1 - L_1}$$

- **Unless this inequality is violated for all values of `I` in its iteration range, we must assume a dependence in the loop**

# Index Set Splitting

```
DO I = 1,100
   DO J = 1, I
S1       A(J+20) = A(J) + B
   ENDDO
ENDDO
```

For values of

$$I < \frac{|d| - (U_0 - L_0)}{U_1 - L_1} = \frac{20 - (-1)}{1} = 21$$

there is no dependence

# Index Set Splitting

- This condition can be used to partially vectorize S1 by Index set splitting as shown

```
    DO I = 1,20
            DO J = 1, I
S1a                 A(J+20) = A(J) + B
            ENDDO
    ENDDO

DO I = 21,100
            DO J = 1, Ix
S1b                 A(J+20) = A(J) + B
            ENDDO
  ENDDO
```

Now the inner loop for the first nest can be vectorized

# Coupling makes these tests imprecise

```
  DO I = 1,100
          DO J = 1, I
S1                A(J+20,I) = A(J,19) + B
          ENDDO
  ENDDO
```

- We will report dependence even if there isn't any
- But such cases are very rare

# Breaking Conditions

- Consider the following example

```
      DO I = 1, L
S1          A(I + N) = A(I) + B
      ENDDO
```

- If `L<=N`, then there is no dependence from $S_1$ to itself
- `L<=N` is called the Breaking Condition

# Using Breaking Conditions

- Using breaking conditions the compiler can generate alternative code

```
      IF (L<=N) THEN
              A(N+1:N+L) = A(1:L) + B
      ELSE
              DO I = 1, L
S1                      A(I + N) = A(I) + B
              ENDDO
      ENDIF
```