Center for Domain Specific Computing

Oil and Water Can Mix: An Integration of Polyhedral and AST-based Transformations

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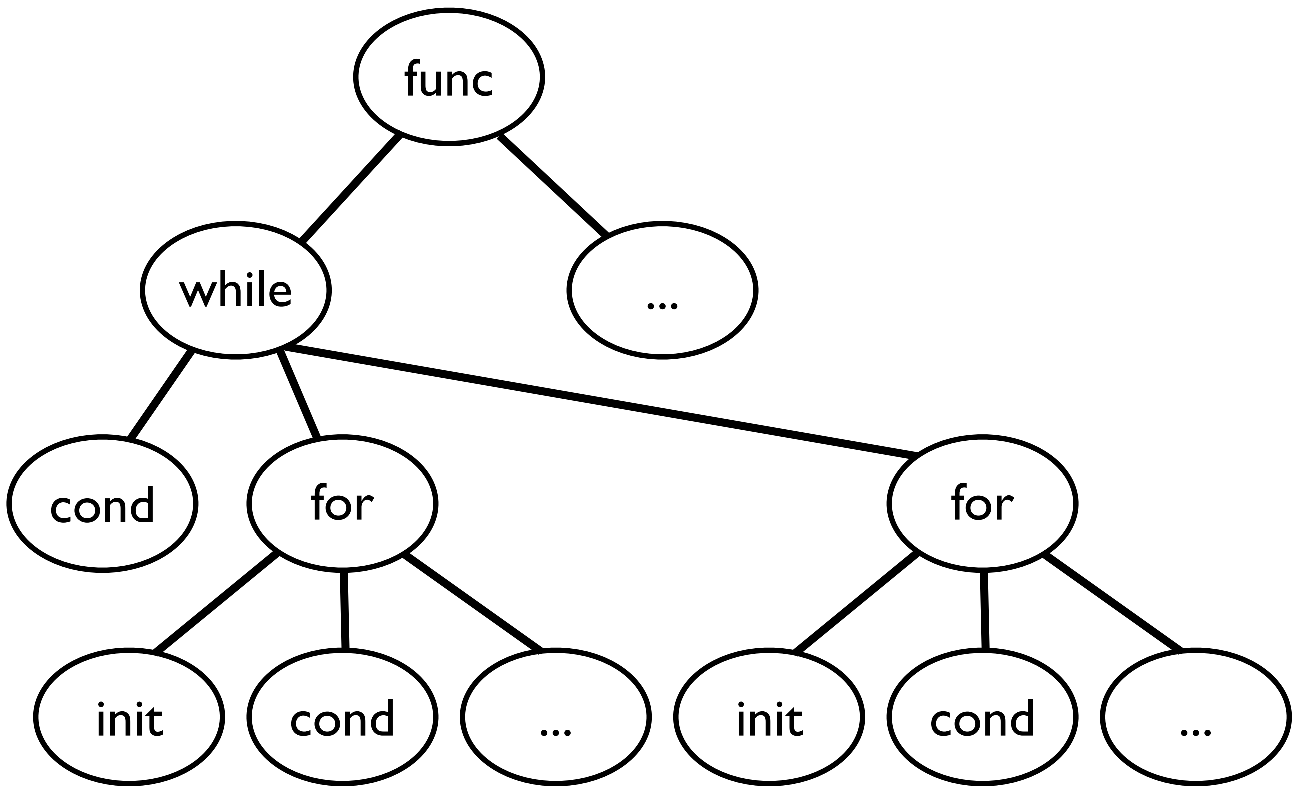
Jun Shirako, Louis-Noel Pouchet, Vivek Sarkar

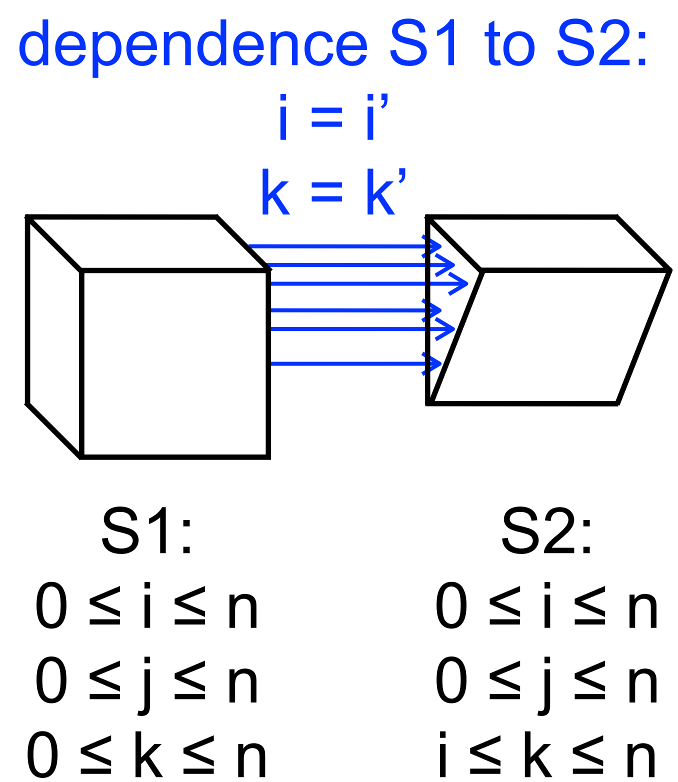
Two Views of Program Representations

AST (Abstract Syntax Tree) view



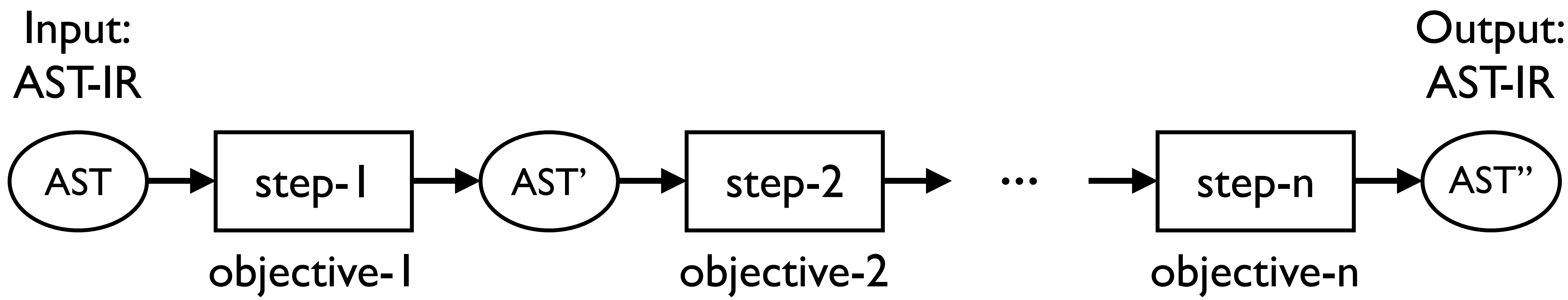
- AST captures all input programs
- Multiple steps modify AST while keeping the semantics

Polyhedral view



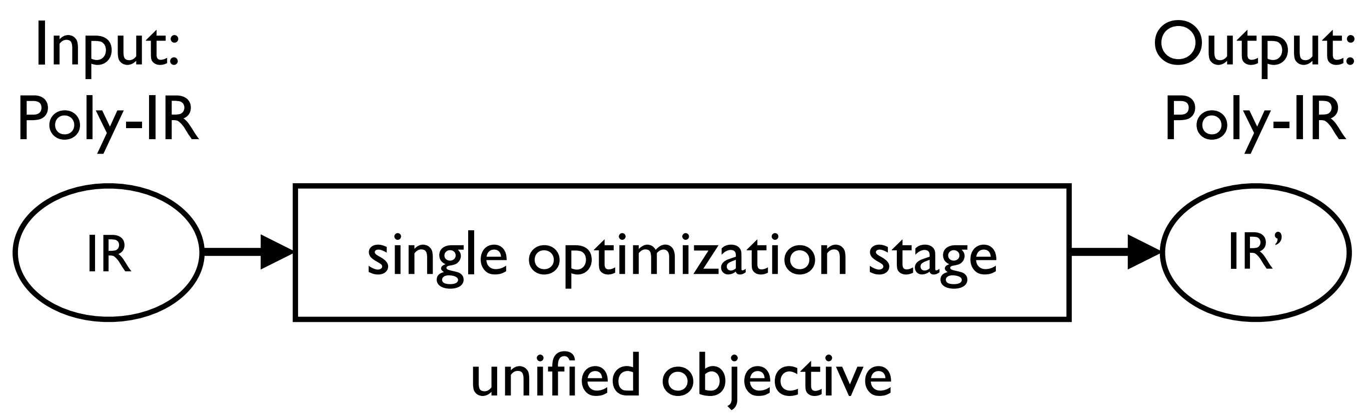
- Limited to loops whose bounds and accesses are affine expressions
- Single mathematical operation computes optimal solution

AST-based Loop Transformation Framework



- Sequence of individual loop transformations on Abstract Syntax Tree
  - Including : fusion, distribution, permutation, skewing, tiling, unroll-and-jam
  - Each step focuses on specific optimization objective:
    - Parallelism (doall, reduction, pipeline)
    - Temporal and spatial data locality
    - Vectorization efficiency
  - Analysis and cost model customized for each transformation
  - Phase-ordering problem (which comes before/after which)
    - Numerous transformations are complementary to each other

Mathematical Approach to Unified Transformation



- Polyhedral model
  - Algebraic framework for affine program representation and transformation
  - Ability to handle everything in single stage
    - Unified view that captures arbitrary loop structures
    - Generalizes loop transformations as form of affine transform
  - Complexity due to unification/generalization
    - Hard to model cost functions for unified transformations
      - Multiple objectives to be combined in a single cost model

Cost Model Example in Polyhedral Approaches

```
    // Input: sequence of two matmults
    for (i = 0; i < N; i++)
      for (j = 0; j < N; j++)
        for (k = 0; k < N; k++)
S1:       tmp[i][j] += A[i][k] * B[k][j];

    for (i = 0; i < N; i++)
      for (j = 0; j < N; j++)
        for (k = 0; k < N; k++)
S2:       D[i][j] += C[i][k] * tmp[k][j];
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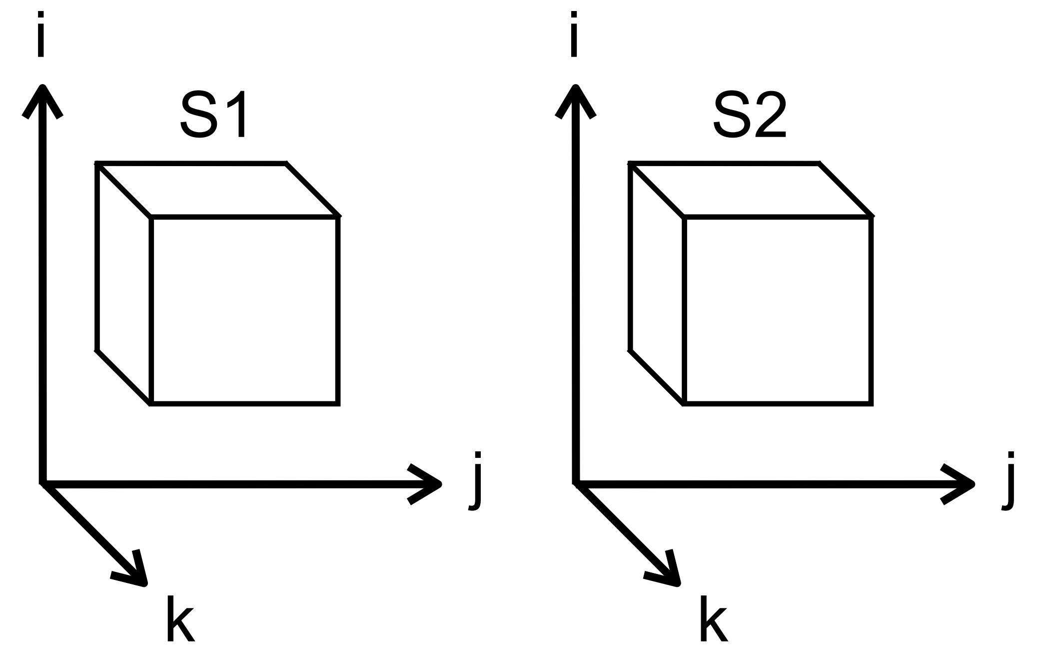

- Objective : Minimization of reuse distance
  - Better temporal data locality
  - Outer parallelism by pushing dependences inside

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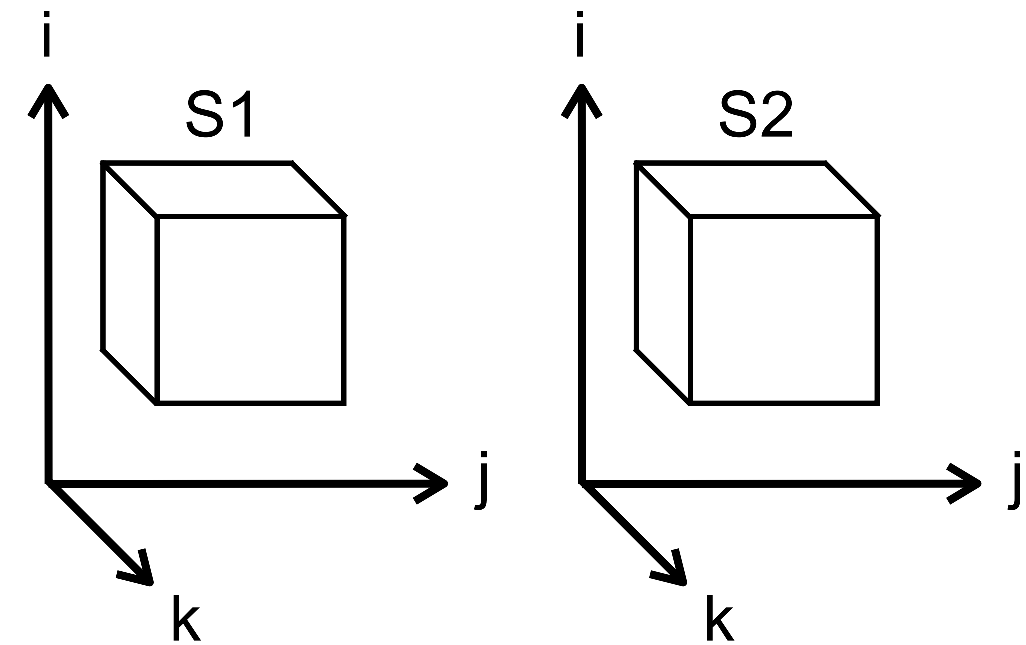
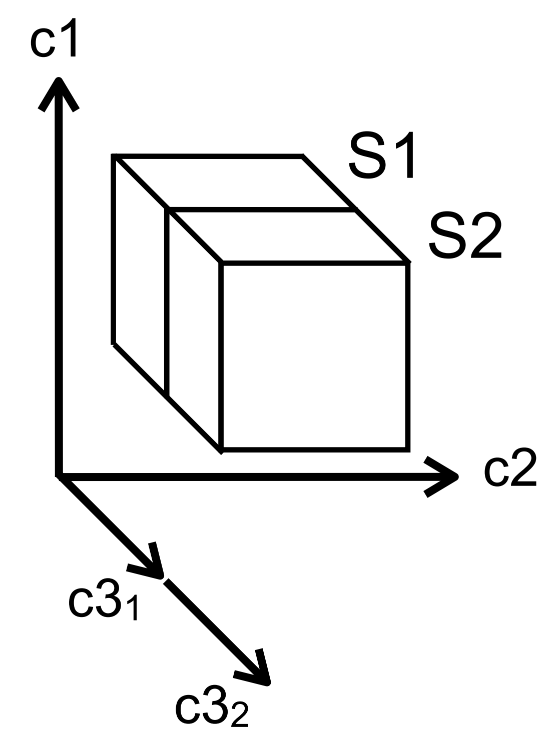



```
    // Output: Minimum reuse distance
    #pragma omp parallel for private(c2, c3)
    for (c1 = 0; c1 < N; c1++) {
      for (c2 = 0; c2 < N; c2++) {
        for (c3 = 0; c3 < N; c3++)
S1:       tmp[c2][c1] += A[c2][c3] * B[c3][c1];
        for (c3 = 0; c3 < N; c3++)
S2:       D[c3][c1] += C[c3][c2] * tmp[c2][c1];
    } }
```



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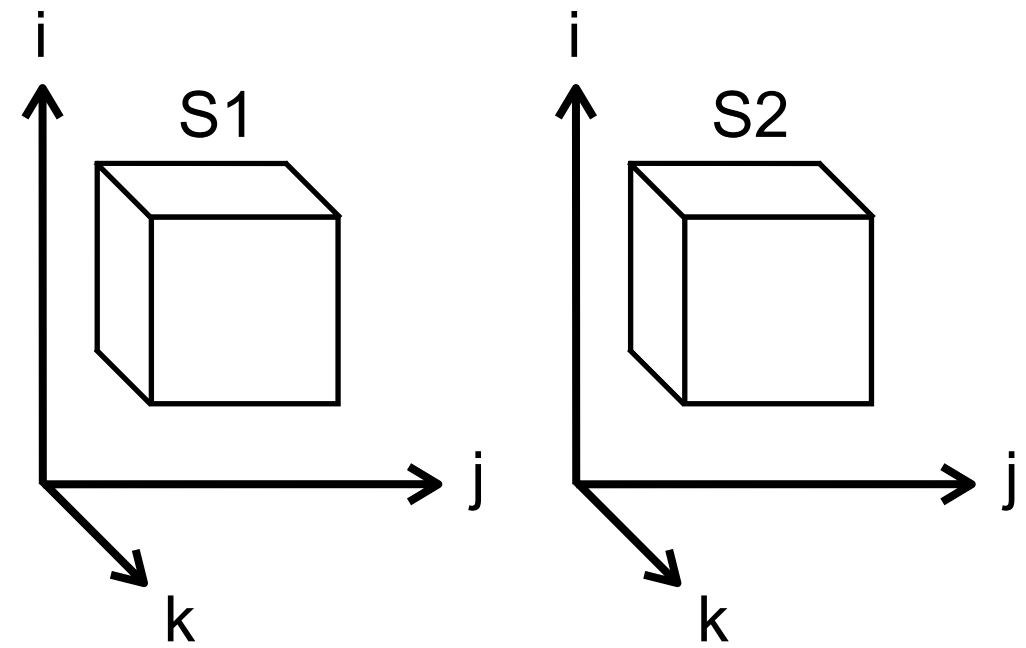


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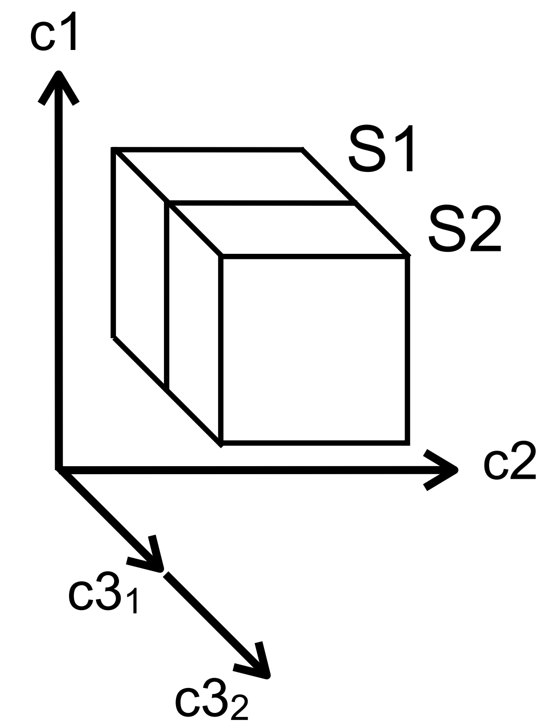
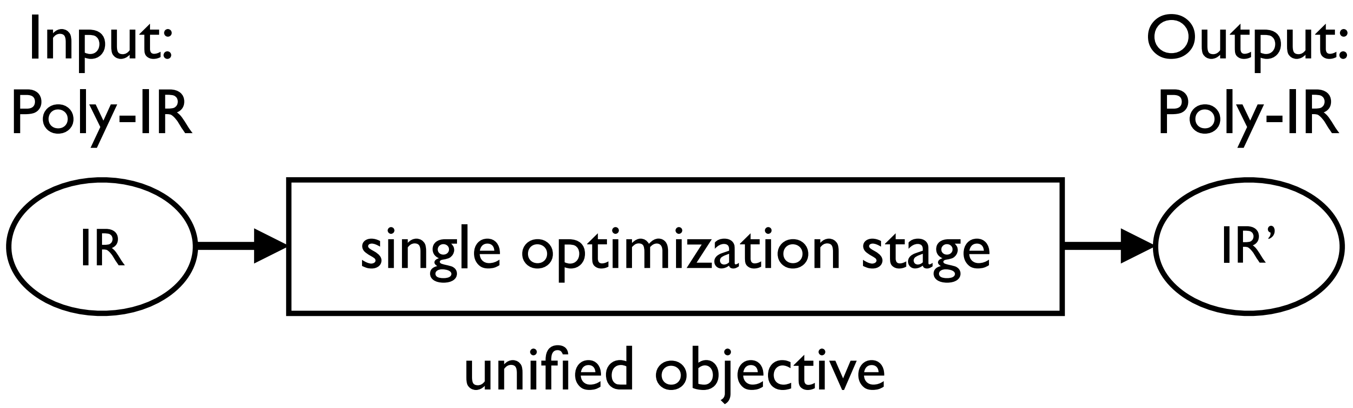


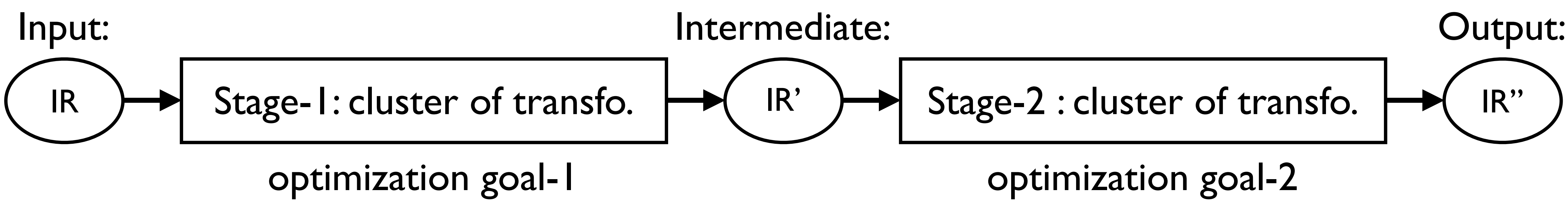
- Objective : Minimization of reuse distance
  - Better temporal data locality
  - Outer parallelism by pushing dependences inside
  - Poor spatial data locality : not modeled in this objective

Mathematical Approach to Unified Transformation



- Challenge : Combining multiple objectives for unified transformations
  - Objectives can conflict, e.g., temporal locality (fuse loop) vs. vectorization (distribute)

Mathematical Approach to Unified Transformation



- Challenge : Combining multiple objectives for unified transformations
  - Objectives can conflict, e.g., temporal locality (fuse loop) vs. vectorization (distribute)

- *Our approach --- decouple the optimization problem into two stages with different cost functions:*
  - Global - i.e., inter-loop-nest
    - Good candidate for polyhedral approach
      - Unified view that captures arbitrary loop structures (perfect & imperfect nests)
  - Local - i.e., per-loop-nest
    - Good candidate for AST-based approach
      - Well-defined sequence of transformations on perfect loop nest

Integrating Polyhedral and AST-based Transformations

- Poly+AST : two-stage approach to integration
  - Stage-1 : Polyhedral transformations
    - Finds optimal loop structures to provide sufficient data locality
      - Restricted form of affine transform
      - Extension of memory cost model for polyhedral model
    - Output : locality-optimized loop nests
  - Stage-2 : AST-based transformations
    - Input : loop nests and dependences from stage-1
    - Sequence of individual transformations per loop nest (w/ different objectives)
      - Loop skewing (increase tilability)
      - Parallelization (outermost doall / reduction / doacross)
      - Loop tiling (enhance locality and granularity of parallelism)
      - Intra-tile optimization (e.g., register-tiling, if-optimization, ...)

Outline

- Introduction
- Stage-1 : Cache-aware polyhedral transformations
- Stage-2 : AST-based transformations
- Experimental results vs. stage-of-the-art polyhedral compiler
- Conclusions

Polyhedral Representation of Program

```
    for (i = 0; i < N; i++)
     for (j = 0; j < N; j++)
      for (k = 0; k < N; k++)
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    for (i = 0; i < N; i++)
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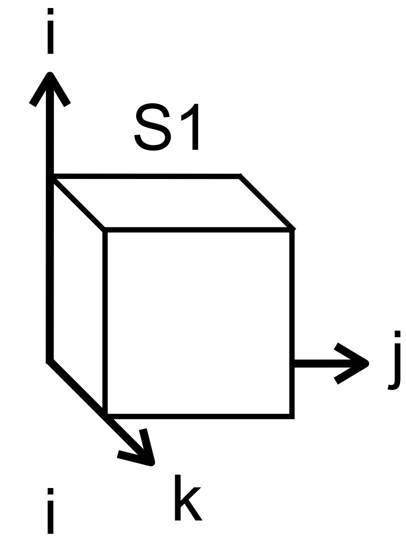
S2:    D[i][j] += C[i][k] * tmp[k][j];
```

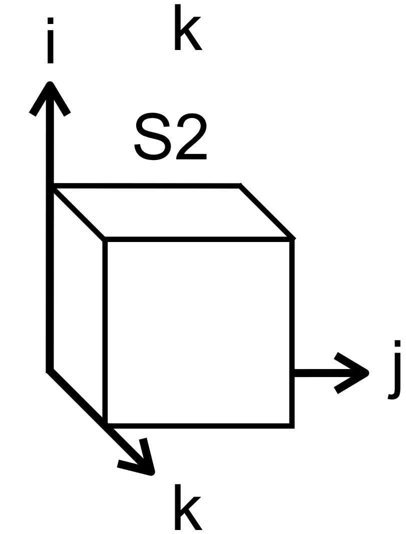
$(i, j, k) \in \mathcal{D}^{S1}$:

$0 \le i \le N\text{-}1$
$0 \le j \le N\text{-}1$
$0 \le k \le N\text{-}1$



$(i, j, k) \in \mathcal{D}^{S2}$:

$0 \le i \le N\text{-}1$
$0 \le j \le N\text{-}1$
$0 \le k \le N\text{-}1$



- Iteration domain
  - $\mathcal{D}^{S_i}$: Set of iteration instances $\boldsymbol{i} = (i_1, i_2, \ldots, i_n)$ of S_i
    - Statement S_i is enclosed in n loops

Polyhedral Representation of Program

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$0 \le i \le N\text{-}1$
$0 \le j \le N\text{-}1$
$0 \le k \le N\text{-}1$

$\langle (i, j, k), (i', j', k') \rangle \in \mathcal{D}^{S1 \rightarrow S2}$:

$0 \le i \le N\text{-}1$
$0 \le j \le N\text{-}1$
$0 \le k \le N\text{-}1$
$0 \le i' \le N\text{-}1$
$0 \le j' \le N\text{-}1$
$0 \le k' \le N\text{-}1$
$i = k'$
$j = j'$

- Iteration domain
  - $\mathcal{D}^{Si}$: Set of iteration instances $\boldsymbol{i} = (i_1, i_2, \ldots, i_n)$ of S_i
    - Statement S_i is enclosed in n loops
- Dependence polyhedron
  - $\mathcal{D}^{Si \rightarrow Sj}$: Captures dependence from S_i to S_j
    - $\langle \boldsymbol{s}, \boldsymbol{t} \rangle \in \mathcal{D}^{Si \rightarrow Sj} \Leftrightarrow \boldsymbol{t} \in \mathcal{D}^{Sj}$ depends on $\boldsymbol{s} \in \mathcal{D}^{Si}$

General Affine Program Transformation

$$\Theta^{Si}(\boldsymbol{i}) = \begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} & \dots & \alpha_{1,d} & c_1 \\ \alpha_{2,1} & \alpha_{2,2} & \dots & \alpha_{2,d} & c_2 \\ \vdots & \vdots & & \vdots & \vdots \\ \alpha_{n,1} & \alpha_{n,2} & \dots & \alpha_{n,d} & c_n \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_d \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_{1,1} i_1 + \alpha_{1,2} i_2 + \dots + \alpha_{1,d} i_d + c_1 \\ \alpha_{2,1} i_1 + \alpha_{2,2} i_2 + \dots + \alpha_{2,d} i_d + c_2 \\ \vdots \\ \alpha_{n,1} i_1 + \alpha_{n,2} i_2 + \dots + \alpha_{n,d} i_d + c_n \end{pmatrix}$$

$\boldsymbol{i} = (i_1, i_2, \dots, i_d)^T$: iteration instances of statement S_i

- Multi-dimensional affine transform
  - Θ^{Si} associates $\boldsymbol{i}$ with a *timestamp* - i.e., logical execution date (yy/mm/dd)
  - Can model any composition of loop transformations including:

    Loop fusion, distribution, permutation, skewing, tiling
- Legality requirements
  - For all dependence polyhedra : $\Theta^{Sj}(\boldsymbol{t}) \succ \Theta^{Si}(\boldsymbol{s})$, $\langle \boldsymbol{s}, \boldsymbol{t} \rangle \in \mathcal{D}^{Si \rightarrow Sj}$

Stage-1 : Cache-aware Polyhedral Transformations

- Restricted form of affine transformations
  - To focus on optimal loop structure to provide sufficient locality
  - Weaker constraints can generate simple (i.e., easy-to-optimize) codes
- Subsumes the following:
  - Loop fusion, distribution and code motion
    - Group statements with locality into a loop
  - Loop permutation
    - Optimal loop order to optimize locality
  - Loop reversal and index-set shifting
    - Increase the opportunities of fusion/permutation
  - No loop skewing (but supported in AST stage)
    - Changes array access pattern, e.g., a[i][j] to a[i+j][j]
    - Can miss spatial locality / affect memory cost analysis

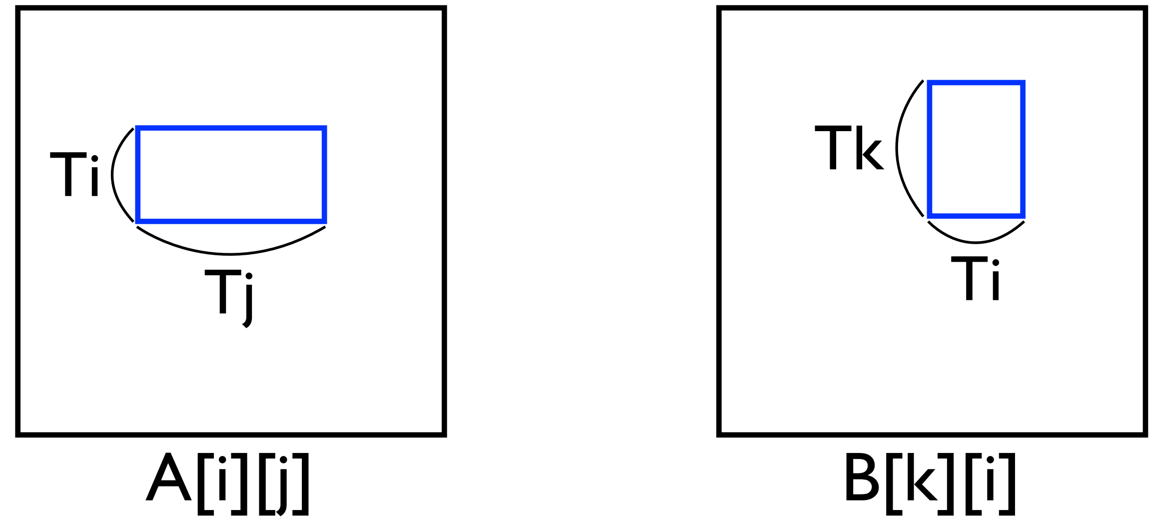
Proposed Restricted Affine Transformation

$$\Theta^{Si}(\boldsymbol{i}) = \begin{pmatrix} 0 & 0 & \dots & 0 & \beta_1 \\ \alpha_{1,1} & \alpha_{1,2} & \dots & \alpha_{1,d} & c_1 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \beta_k \\ \alpha_{k,1} & \alpha_{k,2} & \dots & \alpha_{k,d} & c_k \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \beta_d \\ \alpha_{d,1} & \alpha_{d,2} & \dots & \alpha_{d,d} & c_d \\ 0 & 0 & \dots & 0 & \beta_{d+1} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_d \\ 1 \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \alpha_{1,x}\, i_x + c_1 \\ \vdots \\ \beta_k \\ \alpha_{k,y}\, i_y + c_k \\ \vdots \\ \beta_d \\ \alpha_{1,z}\, i_z + c_d \\ \beta_{d+1} \end{pmatrix} \quad \forall k,\ \sum_{j=1}^{d} |\, \alpha_{k,j} \,| = 1$$

- Restricted forms
  - Odd row : constant offset β_k
  - Even row : linear expression of index where coefficient $\alpha_{k,x} = \pm 1$
- Symbols ⇔ transformations
  - offset β_k ⇔ fusion / distribution / code motion
  - index i_x ⇔ permutation
  - coefficient $\alpha_{k,x}$ ⇔ reversal (apply loop reversal when $\alpha_{k,x} = -1$)
  - offset c_k ⇔ index-set shifting

Cost Model to Guide Polyhedral Transfo.

```
for ti = 0, N-1, Ti
  for tj = 0, M-1, Tj
    for tk = 0, K-1, Tk
      for i = ti, ti+Ti-1
        for j = tj, tj+Tj-1
          for k = tk, tk+Tk-1
            A[i][j] += B[k][i];
```



$$DL(Ti,Tj,Tk) = DL_A(Ti,Tj,Tk) + DL_B(Ti,Tj,Tk) = Ti \times \lceil Tj / L \rceil + Tk \times \lceil Ti / L \rceil$$

$$mem\_cost(T_1, T_2, ..., T_d) = COST_{LINE} * DL(T_1, T_2, ..., T_d) / (T_1 * T_2 * ... * T_d)$$

- DL (Distinct Line) model
  - Assumes loop tiling to fit data within cache/TLB
  - Number of Distinct cache Lines accessed within a tile
    - Total cache miss counts per tile
- Average (per-iteration) memory cost
  - Defined as [total cache miss penalty per tile] / [tile size]

Profitability Analysis via DL Memory Cost

- Most profitable loop permutation order
  - Partial derivative of memory cost w.r.t. T_k :

$$\frac{\partial \text{mem\_cost}(T_1, T_2, \ldots, T_d)}{\partial T_k}$$

    - Reduction rate of memory cost when increasing T_k → Priority of permutation
      - Loop_k with most negative value → to be innermost position
    - Best loop order = descending order of $\partial \text{mem\_cost}(T_1, T_2, \ldots, T_d) / \partial T_k$
- Profitability of loop fusion
  - Comparing mem_cost(T_1, T_2, ..., T_d) before/after fusion
    - Memory cost decreased → fusion is profitable

      * tentative tile size used; final tile size selected later phase
  - Other criteria, e.g., parallelism, are also considered

Affine Transformation Algorithm

Input : S : set of statements S_i,
$PoDG$: polyhedral dependence graph,
k : current nest level, or dimension,
$niter^{Si}$: # iterators not yet scheduled in Θ^{Si}

begin

$PoDG'$:= subset of $PoDG$ w/o satisfied dependence;

$SccSet$:= compute SCCs of $PoDG'$;

/* Intra-SCC transformation (permutation) */

for each $SCC_a \in SccSet$ **do**

compute permutation at level k and get constraints on reversal ($\alpha_{k,*}$) and shifting (c_k);

/* Inter-SCC transformation (fusion / distribution) */

$FuseSet$:= compute β_k and get constraints on reversal and shifting;

for each $Fuse_a \in FuseSet$ **do**

solve constraints on reversal and shifting and compute $\alpha_{k,*}$ and c_k;

if $\exists S_i \in Fuse_a : niter^{Si} \geq 1$ **then**

recursively process the next level - i.e., k+1;

end

Output : Dimensions k ... m of schedule Θ^{Si}

Running Example : 2mm

```
// Input: sequence of two matmults
for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    for (k = 0; k < N; k++)
S1:   tmp[i][j] += A[i][k] * B[k][j];

for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
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S2:   D[i][j] += C[i][k] * tmp[k][j];
```

	tmp/D[i][j]	A/C[i][k]	B/tmp[k][j]
i	N/A	N/A	temporal
j	spatial	temporal	spatial
k	temporal	spatial	N/A

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```

```
// Output: Best permutation order
for (c1 = 0; c1 < N; c1++)    // c1 = i
  for (c2 = 0; c2 < N; c2++)    // c2 = k
    for (c3 = 0; c3 < N; c3++)    // c3 = j
S1:   tmp[c1][c3] += A[c1][c2] * B[c2][c3];

for (c1 = 0; c1 < N; c1++)    // c1 = i
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```

	tmp/D[i][j]	A/C[i][k]	B/tmp[k][j]
i	N/A	N/A	temporal
j	spatial	temporal	spatial
k	temporal	spatial	N/A

- Optimization policy
  - Permute loops as close to the DL best order as possible
  - Fuse loops if legality and profitability criteria are met

Connection between Polyhedral and AST-based Stages

- Output of polyhedral stage
  - Locality-optimized loop nests
    - Permuted with legal & profitable loop order
    - Fused statements with locality into a loop
  - Dependence information
    - $\langle \boldsymbol{s}, \boldsymbol{t} \rangle \in \mathcal{P}_e^{S_i \rightarrow S_j}$: relationship between source and target instances ***s*** and ***t***
    - Extracted as dependence vector - i.e., $\boldsymbol{d} = \boldsymbol{t} - \boldsymbol{s}$
- Input of AST-based stage
  - $loop_k$: a loop that is nested at level $k \in \{1 \dots n\}$
  - $\Delta^{loop_k} = \{\boldsymbol{d}^1, \boldsymbol{d}^2, \dots, \boldsymbol{d}^n\}$:
    - Set of dependences whose source and target statements are within $loop_k$
    - Free from affine constraints in AST-based stage

Stage-2 : AST-based Transformation

- Dependence vectors : base of analysis
  - Legality : loop skewing, loop tiling, register tiling, ...
  - Detection of parallelism
- Sequence of transformations in stage-2
  - Loop skewing
    - In order to increase permutability (i.e., applicability of tiling) and parallelism
  - Coarse-grain parallelization
    - Doall / reduction / doacross parallelism
  - Loop tiling
    - Enhance computation granularity and data locality
  - Intra-tile optimizations
    - Register-tiling (i.e., multi-dimensional unrolling)

Parallelism in Poly+AST Framework

- Loop permutation order
  - To optimize spatial and temporal data locality
  - Outermost loop is not always doall
    - Also leverage other parallelism : reduction and doacross (pipeline parallelism)

- Reduction parallelism

```
for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    S[j] += alpha * X[i][j];
```

- Doacross parallelism

```
for (i = 1; i < N-1; i++) {
  for (j = 0; j < N; j++) {

    C[i][j] = 0.33 * (C[i-1][j]
          + C[i][j] + C[i+1][j]);

} }
```

OpenMP Extensions:

- *"Tile Reduction: The First Step towards Tile aware Parallelization in OpenMP", IWOMP-2009*
- *"Expressing DOACROSS Loop Dependencies in OpenMP", IWOMP-2013*

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- Doall-only approach

```
#pragma omp for
for (j = 0; j < N; j++)
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- *"Expressing DOACROSS Loop Dependencies in OpenMP", IWOMP-2013*

Parallelism in Poly+AST Framework

- Loop permutation order
  - To optimize spatial and temporal data locality
  - Outermost loop is not always doall
    - Also leverage other parallelism : reduction and doacross (pipeline parallelism)

- Reduction parallelism

```
#pragma omp for reduction(+: S[0:N-1])
for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    S[j] += alpha * X[i][j];
```

- Doall-only approach

```
#pragma omp for
for (j = 0; j < N; j++)
  for (i = 0; i < N; i++)
    S[j] += alpha * X[i][j];
```

- Doacross parallelism

```
#pragma omp for ordered(2)
for (i = 1; i < N-1; i++) {
  for (j = 0; j < N; j++) {
#pragma omp ordered depend(sink: i-1,j)
    C[i][j] = 0.33 * (C[i-1][j]
          + C[i][j] + C[i+1][j]);
#pragma omp ordered depend(src: i,j)
} }
```

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- Doall-only approach

```
#pragma omp for
for (j = 0; j < N; j++)
  for (i = 0; i < N; i++)
    S[j] += alpha * X[i][j];
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- Doacross parallelism

```
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for (i = 1; i < N-1; i++) {
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    C[i][j] = 0.33 * (C[i-1][j]
          + C[i][j] + C[i+1][j]);
#pragma omp ordered depend(src: i,j)
} }
```

- Doall-only approach

```
#pragma omp for
for (j = 0; j < N; j++)
  for (i = 1; i < N-1; i++)
    C[i][j] = 0.33 * (C[i-1][j]
          + C[i][j] + C[i+1][j]);
```

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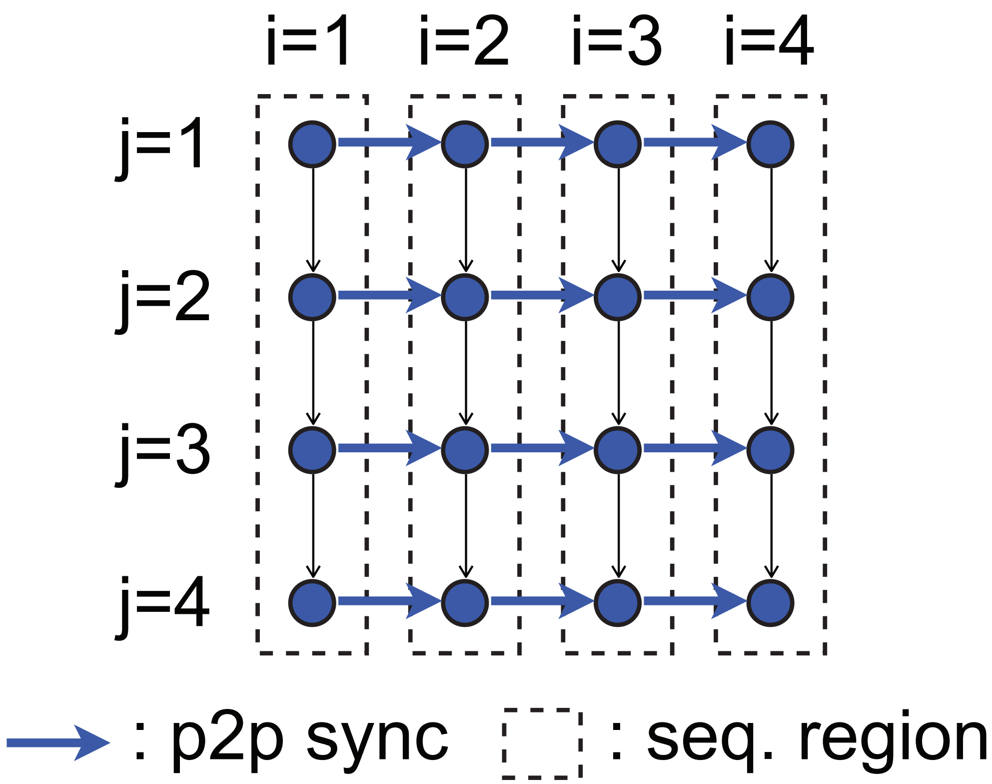
- *"Expressing DOACROSS Loop Dependencies in OpenMP", IWOMP-2013*

Pipeline Parallelism vs. Wavefront Doall

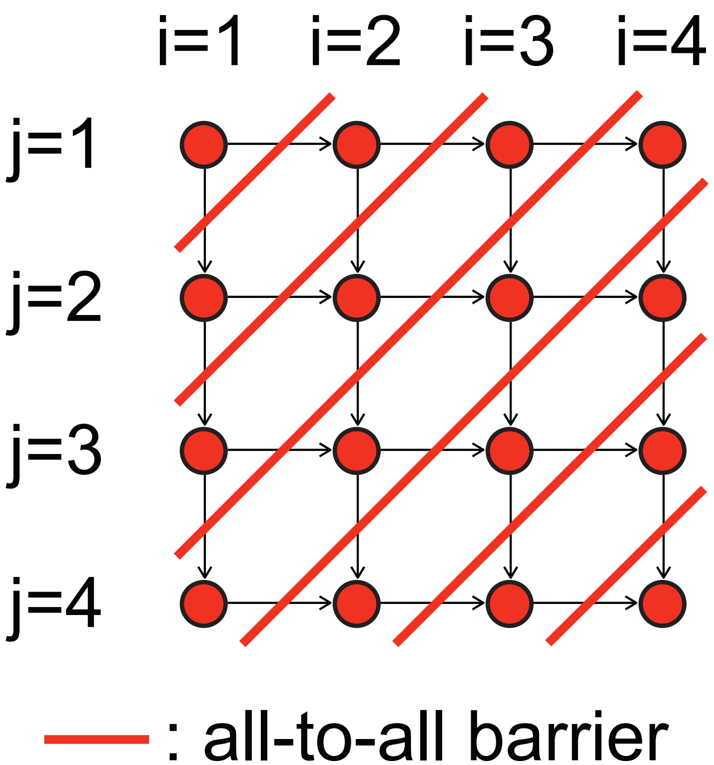
- Pipeline parallelism (OpenMP extension)

```
#pragma omp parallel for ordered(2)
for (i = 1; i < N-1; i++) {
  for (j = 1; j < N-1; j++) {
#pragma omp ordered depend(sink: i-1,j)
                    depend(sink: i,j-1)
    A[i][j] = A[i-1][j] + a[i][j-1];
#pragma omp ordered depend(src: i,j)
} }
```



- Wavefront doall with skewing

```
#pragma omp parallel
for (i = 2; i <= 2*N-4; i++) {
#pragma omp for
  for (j = max(1,i-N+2);
       j < min(N-2,i-1); j++) {
    A[i-j][j] = A[i-j-1][j] + a[i-j][j-1];
} }
```



Another Example : Jacobi-1d stencil

```
   // Input (imperfect nest)
   for (t = 0; t < time_steps; t++) {
     for (i = 1; i < n-1; i++)
S1:    b[i] = 0.33 * (a[i-1] + a[i] + a[i+1]);
     for (i = 1; i < n-1; i++)
S2:    a[i] = b[i];
   }
```

Another Example : Jacobi-1d stencil

```
   // Input (imperfect nest)
   for (t = 0; t < time_steps; t++) {
     for (i = 1; i < n-1; i++)
S1:    b[i] = 0.33 * (a[i-1] + a[i] + a[i+1]);
     for (i = 1; i < n-1; i++)
S2:    a[i] = b[i];
   }

   // Stage-1: polyhedral transformation (perfect nest)
   for (c1 = 0; c1 <= time_steps-1; c1++) {
     for (c3 = 1; c3 <= n-1; c3++) {
S1:    if (c3 <= n-2) b[c3] = 0.33 * (a[c3-1] + a[c3] + a[c3+1]);
S2:    if (c3 >= 2) a[c3-1] = b[c3-1];
   } }
```

Another Example : Jacobi-1d stencil

```
    // Input (imperfect nest)
    for (t = 0; t < time_steps; t++) {
      for (i = 1; i < n-1; i++)
S1:     b[i] = 0.33 * (a[i-1] + a[i] + a[i+1]);
      for (i = 1; i < n-1; i++)
S2:     a[i] = b[i];
    }

    // Stage-1: polyhedral transformation (perfect nest)
    for (c1 = 0; c1 <= time_steps-1; c1++) {
      for (c3 = 1; c3 <= n-1; c3++) {
S1:     if (c3 <= n-2) b[c3] = 0.33 * (a[c3-1] + a[c3] + a[c3+1]);
S2:     if (c3 >= 2) a[c3-1] = b[c3-1];
    } }

    // Stage-2: skewing & parallelization
    //   - Loop nest is fully permutable
    //   - Doacross parallelization by OpenMP extensions
    #pragma omp parallel for private(c3) ordered(2)
    for (c1 = 0; c1 < time_steps; c1++) {
      for (c3 = 2*c1+1; c3 < 2*c1+n; c3++) {
    #pragma omp ordered depend(sink: c1-1,c3) depend (sink: c1,c3-1)
S1:     if (i <= n-2) b[-2*c1+c3] = 0.33*(a[-2*c1+c3-1]+a[-2*c1+c3]+a[-2*c1+c3+1]);
S2:     if (i >= 2) a[-2*c1+c3-1] = b[-2*c1+c3-1];
    #pragma omp ordered depend(source: c1,c3)
    } }
```

Another Example : Jacobi-1d stencil

```
    // Stage-2: loop tiling
    #pragma omp parallel for private(c3,c5,i) ordered(2)
    for (c1 = ...) {
      for (c3 = ...) {
    #pragma omp ordered depend(sink: c1-1,c3) depend(sink: c1,c3-1)
        ...
        for (c5 = ...) {
          if (...) B[1] = 0.33 * (A[1-1] + A[1] + A[1+1]);
          for (c7 = ...) {
S1:         b[-2*c5+c7] = 0.33 * (a[-2*c5+c7-1] + a[-2*c5+c7] + a[-2*c5+c7+1]);
S2:         a[-2*c5+c7-1] = b[-2*c5+c7-1];
          }
          if (...) A[n-2] = B[n-2];
        }
        ...
    #pragma omp ordered depend(source: c1,c3)
    } }
```

Another Example : Jacobi-1d stencil

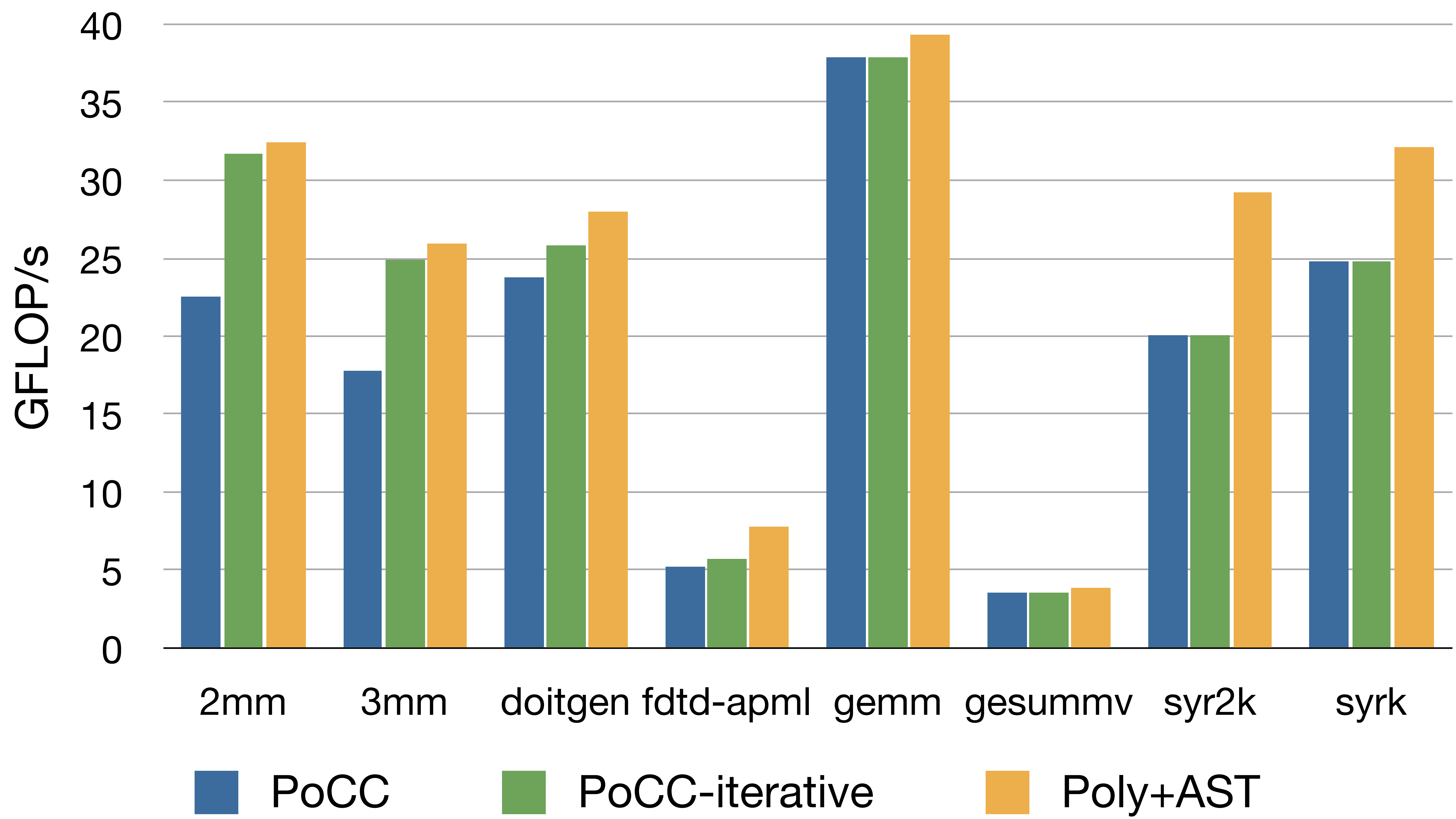
```
    // Stage-2: loop tiling
    #pragma omp parallel for private(c3,c5,i) ordered(2)
    for (c1 = ...) {
      for (c3 = ...) {
    #pragma omp ordered depend(sink: c1-1,c3) depend(sink: c1,c3-1)
        ...
        for (c5 = ...) {
          if (...) B[1] = 0.33 * (A[1-1] + A[1] + A[1+1]);
          for (c7 = ...) {
S1:         b[-2*c5+c7] = 0.33 * (a[-2*c5+c7-1] + a[-2*c5+c7] + a[-2*c5+c7+1]);
S2:         a[-2*c5+c7-1] = b[-2*c5+c7-1];
          }
          if (...) A[n-2] = B[n-2];
        }
        ...
    #pragma omp ordered depend(source: c1,c3)
    } }

    // Stage-2: register tiling (innermost by factor = 2)
          ...
          for (c7 = ...; c7 <= (...)-1; c7+=2) {
S1:         b[-2*c5+c7] = 0.33 * (a[-2*c5+c7-1]+a[-2*c5+c7]+a[-2*c5+c7+1]);
S2:         a[-2*c5+c7-1] = b[-2*c5+c7-1];
S1':        b[-2*c5+c7+1] = 0.33 * (a[-2*c5+c7+1-1]+a[-2*c5+c+1]+a[-2*c5+c7+1+1]);
S2':        a[-2*c5+c7+1-1] = b[-2*c5+c7+1-1];
          }
          ...
```

Experimental Setting

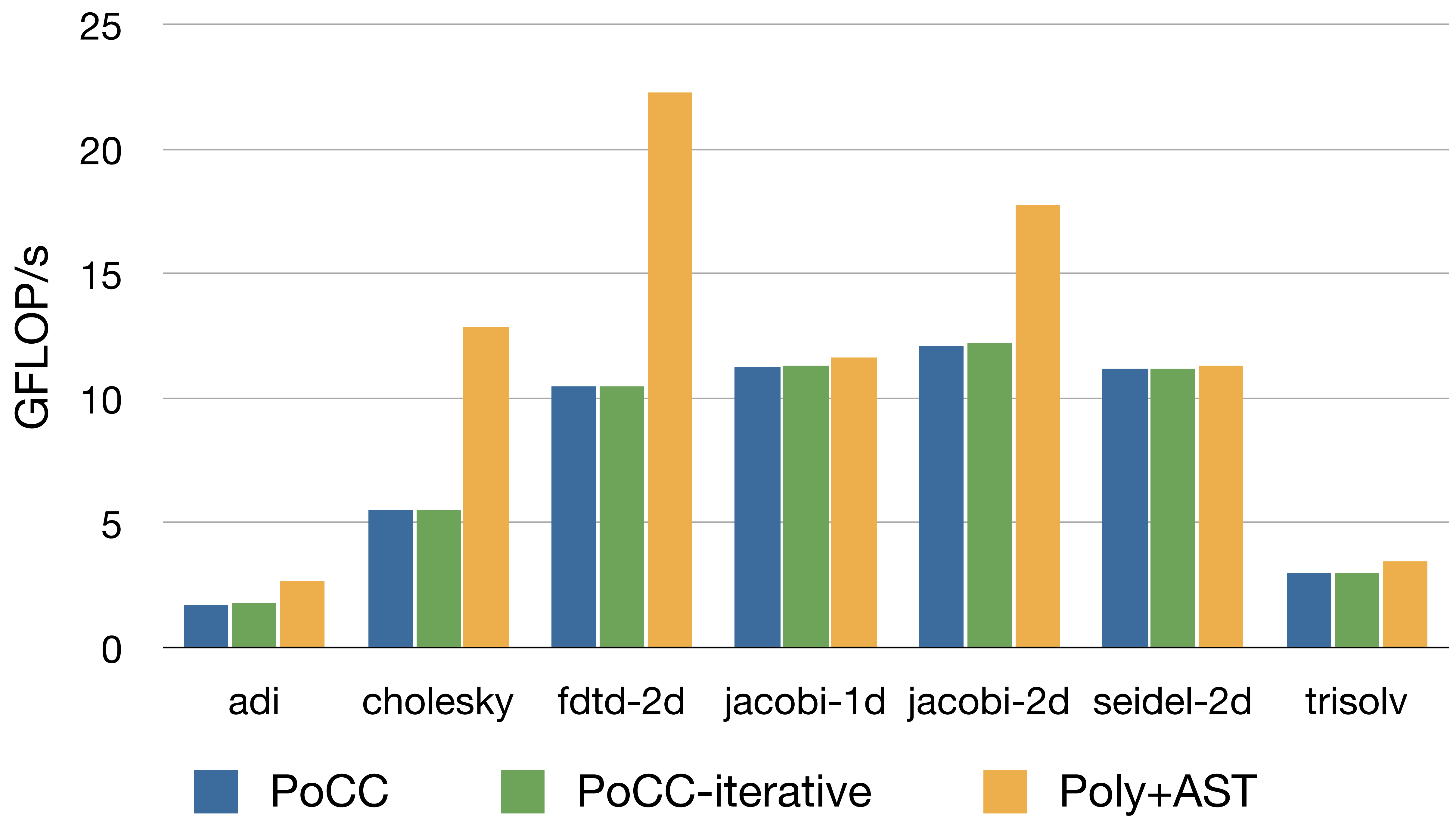
- Platforms
  - Two quad-core 2.8GHz Intel Core i7 (Nehalem) with Intel C compiler 12.0
  - Four eight-core 3.86GHz IBM Power7 with IBM XLC compiler 11.1
- Benchmarks
  - PolyBench-C 3.2 (22 benchmarks, standard/large dataset)
- Comparisons
  - PoCC : research polyhedral compiler [http://www.cs.ucla.edu/~pouchet/software/pocc]
    - PLuTo heuristic for parallelism, locality, tiling and intra-tile optimizations
    - Doall parallelism (convert doacross into wavefront doall)
  - PoCC-iterative : Iterative compilation approach [Pouchet-SC'10]
    - PoCC + empirical search for outermost fusion/distribution
  - Poly+AST : proposed integration approach
    - Doall / doacross / reduction parallelism
- Additional results in paper, e.g., ICC and XLC

GFLOP/s on Nehalem (doall dominant)



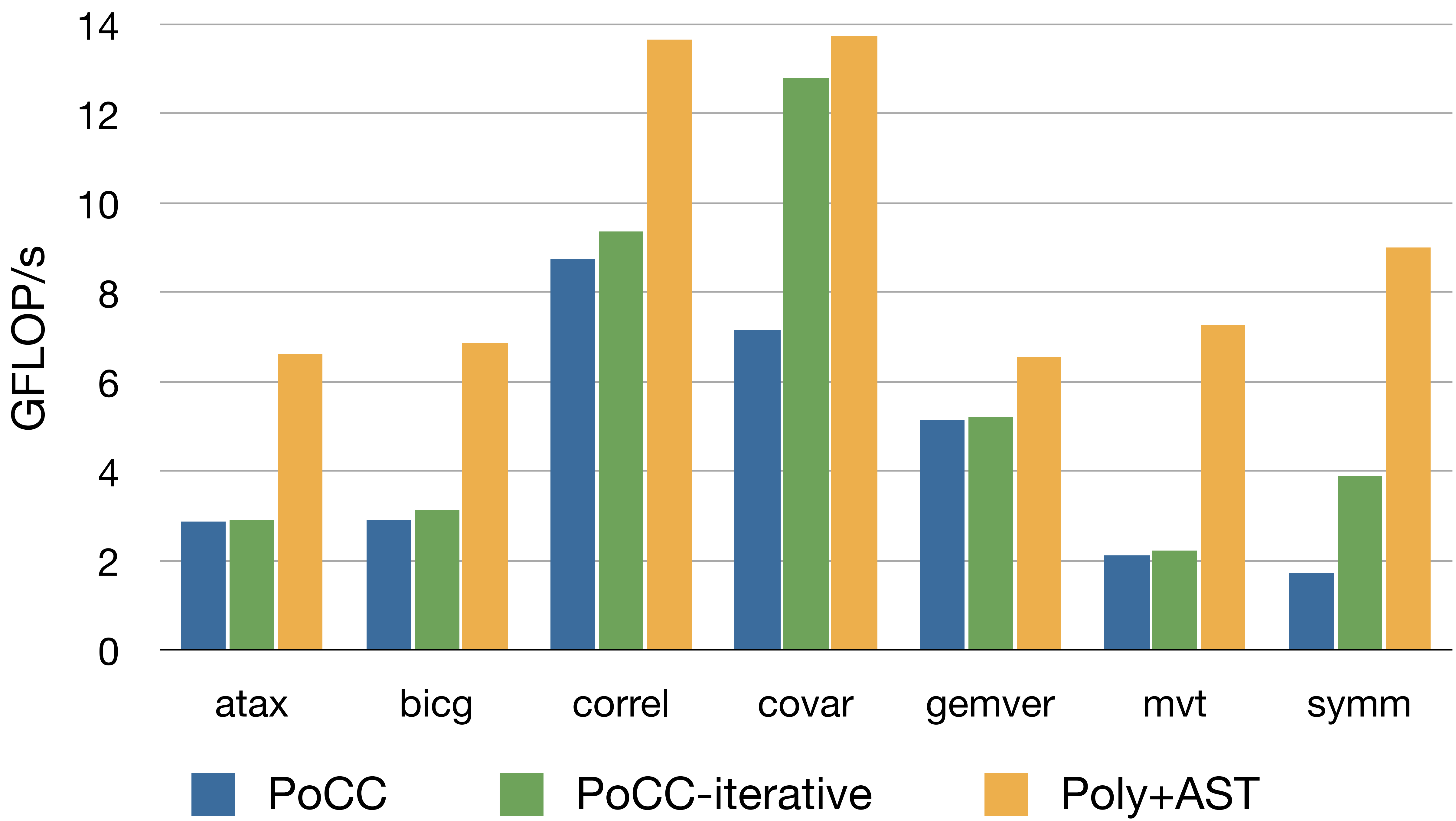
- PoCC ≤ PoCC-iterative ≤ Poly+AST
  - PoCC-iterative : empirical search for fusion/distribution
  - Poly+AST (polyhedral stage) : DL model for fusion/dist. and permutation

GFLOP/s on Nehalem (doacross-parallel dominant)



- PoCC = PoCC-iterative ≤ Poly+AST
  - adi / cholesky / fdtd-2d : loop structures (e.g., fusion, perm., index-shifting)
  - jacobi-2d : DOACROSS parallelization vs. wavefront doall by skewing

GFLOP/s on Nehalem (with reduction parallelism)



- PoCC ≤ PoCC-iterative < Poly+AST
  - Reduction support to increase flexibility of loop permutation
    - Loop order w/ better locality while keeping outermost parallelism

Transformed Codes by PoCC and Poly+AST

```
    // Input: SYMM (simplified)
    for (i = 0; i < NI; i++) {
      for (j = 0; j < NJ; j++) {
        for (k = 0; k < j - 1; k++) {
S1:       C[k][j] += alpha * A[k][i] * B[i][j];
S2:       acc[i][j] += B[k][j] * A[k][i];
        }
S3:     C[i][j] = beta * C[i][j] + alpha * A[i][i]* B[i][j] + alpha * acc[i][j];
    } }
```

Transformed Codes by PoCC and Poly+AST

```
    // PoCC optimized (omitting tiling and intra-tile optimizations)
    #pragma omp parallel for private(c2, c3)
    for (c1 = 2; c1 <= NJ-1; c1++) {
      for (c2 = 0; c2 <= NI-1; c2++) {
        for (c3 = 0; c3 <= c1+NI-1; c3++) {
S1:       if (c3 <= c1-2) acc[c2][c1] += B[c3][c1] * A[c3][c2];
S2:       if (c2 <= c1-2 && c3 >= c1) C[c2][c1] += alpha * A[c2][-c1+c3] * B[-c1+c3][c1];
S3:       if (c3 == c1+c2) C[c2][c1] = beta * C[c2][c1] + alpha * A[c2][c2] * B[c2][c1] ...
    } } }
```

doall accessing inner array dimensions; poor spatial locality

Transformed Codes by PoCC and Poly+AST

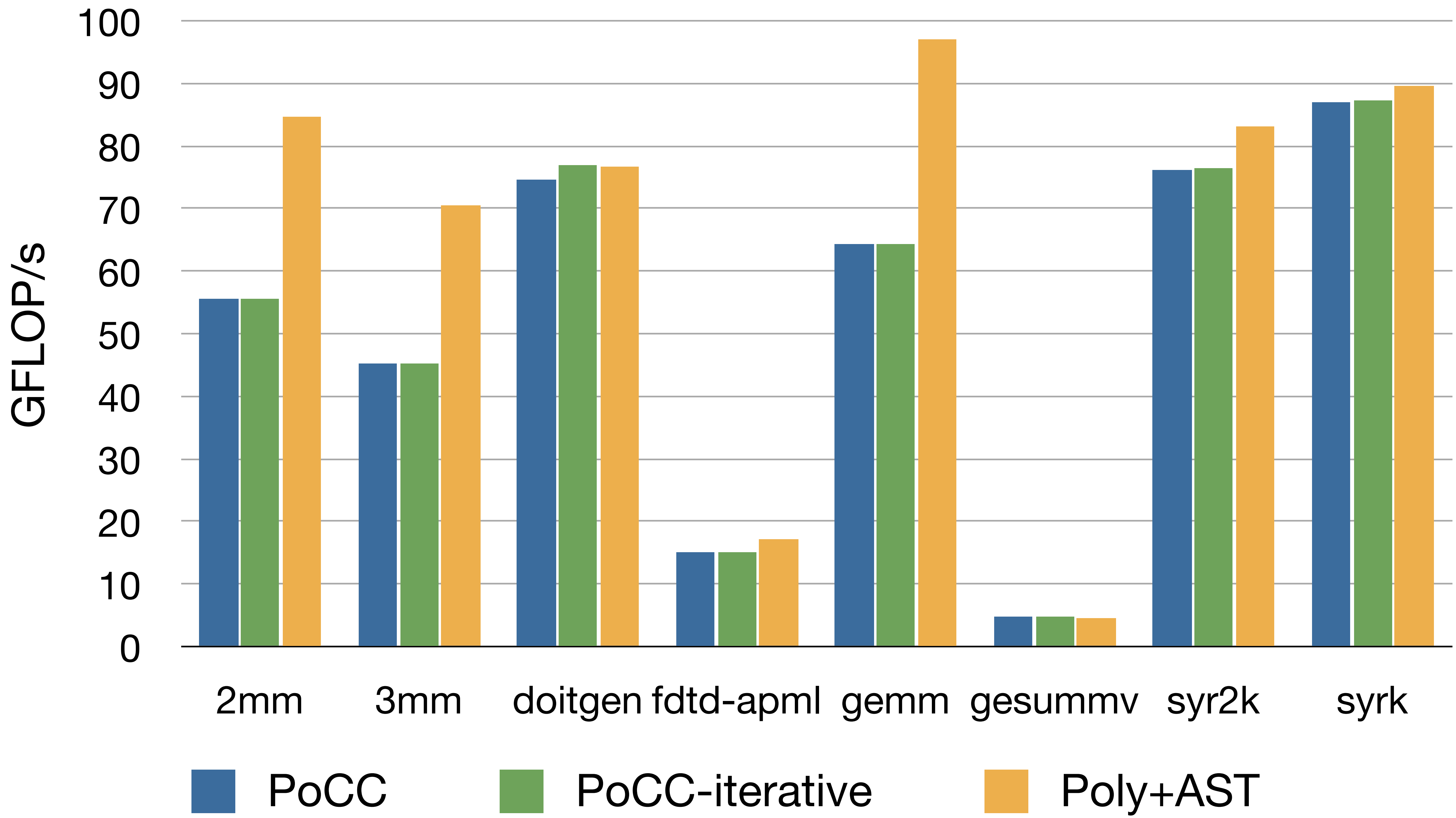
```
   // PoCC optimized (omitting tiling and intra-tile optimizations)
   #pragma omp parallel for private(c2, c3)
   for (c1 = 2; c1 <= NJ-1; c1++) {
     for (c2 = 0; c2 <= NI-1; c2++) {
       for (c3 = 0; c3 <= c1+NI-1; c3++) {
S1:      if (c3 <= c1-2) acc[c2][c1] += B[c3][c1] * A[c3][c2];
S2:      if (c2 <= c1-2 && c3 >= c1) C[c2][c1] += alpha * A[c2][-c1+c3] * B[-c1+c3][c1];
S3:      if (c3 == c1+c2) C[c2][c1] = beta * C[c2][c1] + alpha * A[c2][c2] * B[c2][c1] ...
   } } }
```

doall accessing inner array dimensions; poor spatial locality

```
   // Poly+AST optimized (omitting tiling and intra-tile optimizations)
   #pragma omp parallel for private(c3, c5) reduction(+: acc[0:NI-1][2:NJ-1])
   for (c1 = 0; c1 <= NJ-3; c1++) {
     for (c3 = 0; c3 <= NI-1; c3++) {
       for (c5 = c1 + 2; c5 <= NJ-1; c5++) {
S1:      acc[c3][c5] += B[c1][c5] * A[c1][c3];
   } } }
   #pragma omp parallel for private(c3, c5)
   for (c1 = 0; c1 <= MAX(NI-1, NJ-3); c1++) {
     for (c3 = 0; c3 <= NI-1; c3++) {
       for (c5 = 0; c5 <= NJ-1; c5++) {
S2:      if (c5 >= c1+2) C[c1][c5] += alpha * A[c1][c3] * B[c3][c5];
S3:      if (c3 == c1) C[c1][c5] = beta * C[c1][c5] + alpha * A[c1][c1] * B[c1][c5] ...
   } } }
```

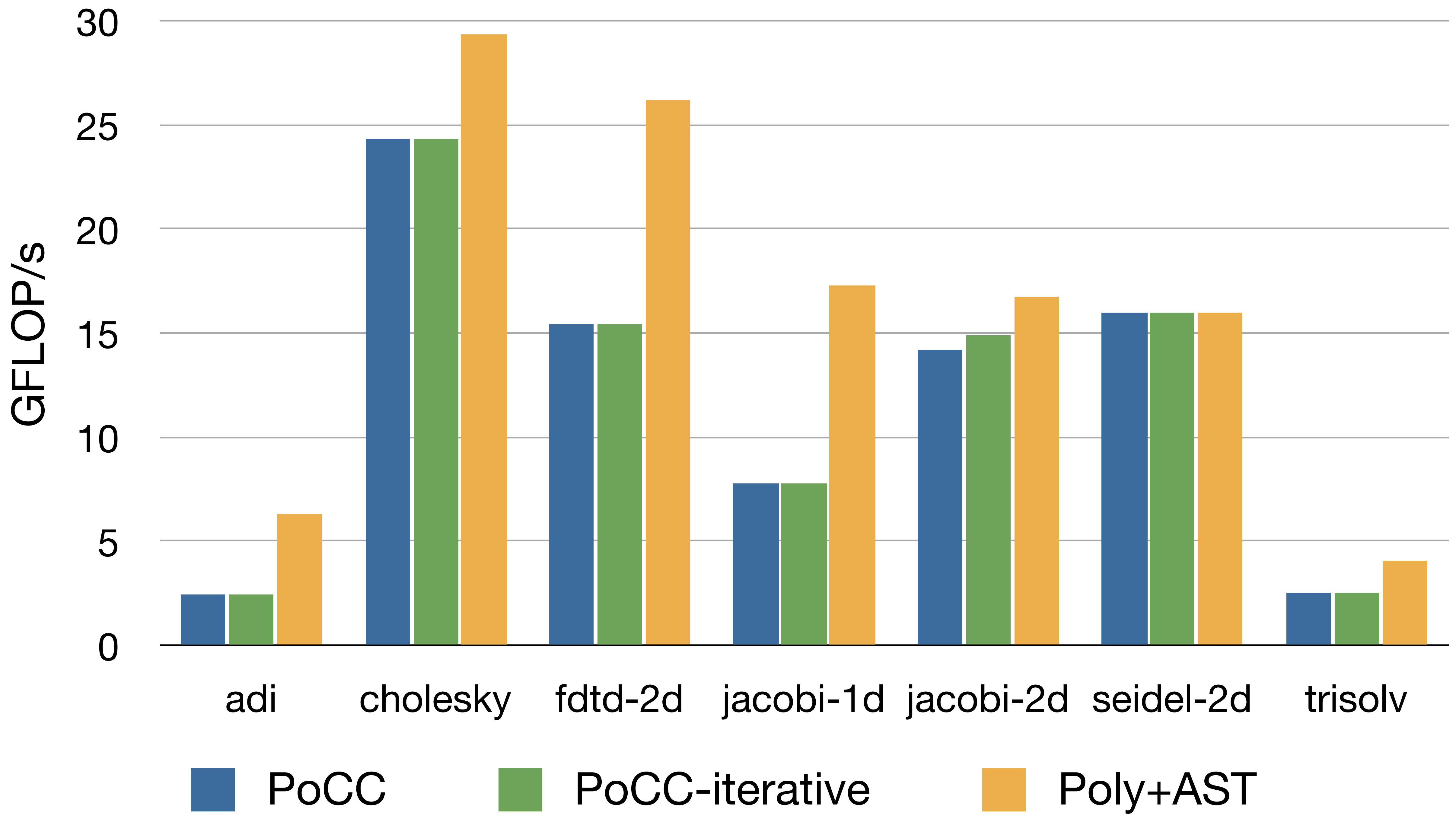
reduction / doall accessing outer array dimensions; better spatial locality

GFLOP/s on Power7 (doall dominant)



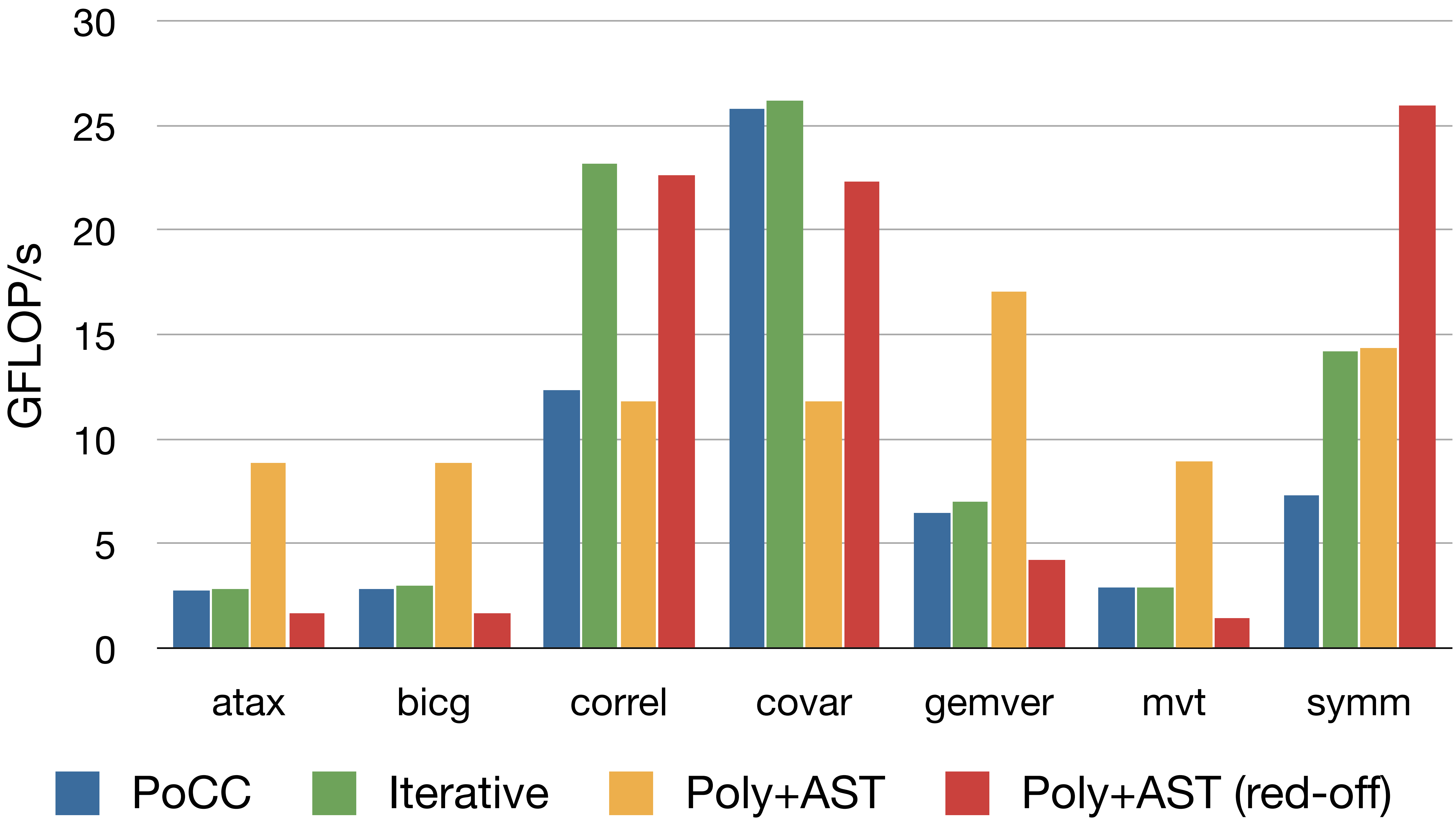
- PoCC = PoCC-iterative ≤ Poly+AST
  - Good selection of loop structures (e.g., fusion/distribution and permutation)

GFLOP/s on Power7 (doacross-parallel dominant)



- PoCC = PoCC-iterative ≤ Poly+AST
  - Efficiency of DOACROSS has more impact (32-core Power7 vs. 8-core Nehalem)

GFLOP/s on Power7 (with reduction parallelism)



- Reduction reduces performance (correl, covar and symm)
  - Sequential aggregation for final results is scalability bottleneck
  - Future work : parallel aggregation

Take-home Message

- AST-based transformations
  - Sequence of individual loop transformations
  - Difficulty in composing the optimal sequence (i.e., phase-ordering)
- Polyhedral model
  - Unification & generalization of loop transformations
  - Difficulty in modeling cost functions for whole unified transformations
- Integration of both
  - Decoupling the optimization problem into two stages
    - Polyhedral model as first stage, AST-based as second stage
  - Simpler & customized cost modeling within stage
  - Each stage leverage its strengths
  - Geometric mean speedup vs. PoCC (polyhedral optimizer)
    - 1.62x on 8-core Nehalem / 1.49x on 32-core Power7

Acknowledgements

We are grateful to P. Sadayappan at The Ohio State University and J. Ramanujam at Louisiana State University for their feedback and discussions on the polyhedral model