Lecture 14: Iterative Averaging Revisited, SPMD pattern

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Worksheet #13: Parallelism in Java Streams, Parallel Prefix Sums

1. What output will the following Java Streams code print?
   - C1 or C2: The sorted() operation is a no-op
   - C2 or C1: if the printing is performed in parallel!

2. Which stream operation in this example could benefit from a parallel prefix sum implementation, and why?

   The filter operation since parallel prefix can be used to compute the indices in the output array

```java
1. Arrays
2. .asList("a1", "a2", "b1", "c2", "c1")
3. .parallelStream()
4. .filter(s -> s.startsWith("c"))
5. .sorted()
6. .map(String::toUpperCase)
7. .forEach(System.out::println);
```
Recap: Parallel Filter Operation

Given an array **input**, produce an array **output** containing only elements such that \( f(elt) \) is true, i.e.,

\[
\text{output} = \text{input}.\text{parallelStream()}.\text{filter(f)}.\text{toArray()}
\]

Example: **input** [17, 4, 6, 8, 11, 5, 13, 19, 0, 24]

\[ f: \text{is elt > 10} \]

**output** [17, 11, 13, 19, 24]

Parallelizable?

- Finding elements for the output is easy
- But getting them in the right place seems hard
Recap: Parallel prefix to the rescue

1. Parallel map to compute a **bit-vector** for true elements (can use Java streams)
   
   **input**  
   
   [17, 4, 6, 8, 11, 5, 13, 19, 0, 24]  
   
   **bits**  
   
   [1, 0, 0, 0, 1, 0, 1, 1, 0, 1]  

2. Parallel-prefix sum on the bit-vector (not available in Java streams)
   
   **bitsum**  
   
   [1, 1, 1, 1, 2, 2, 3, 4, 4, 5]  

3. Parallel map to produce the output (can use Java streams)
   
   **output**  
   
   [17, 11, 13, 19, 24]  

   **output** = new array of size **bitsum[n-1]**  

   **FORALL** (i=0; i < input.length; i++){
      
      **if** (bits[i]==1)
      
      output[bitsum[i]-1] = input[i];  
   }
One-Dimensional Iterative Averaging Example

- Initialize a one-dimensional array of (n+2) double’s with boundary conditions, myVal[0] = 0 and myVal[n+1] = 1.
- In each iteration, each interior element myVal[i] in 1..n is replaced by the average of its left and right neighbors.
  - Two separate arrays are used in each iteration, one for old values and the other for the new values
- After a sufficient number of iterations, we expect each element of the array to converge to myVal[i] = (myVal[i-1]+myVal[i+1])/2, for all i in 1..n

Illustration of an intermediate step for n = 8 (source: Figure 6.19 in Lin-Snyder book)
Iterative Averaging structure is akin to Finite Difference solution to the One-Dimensional Heat Equation

- In the 1D heat equation, $u(j,t)$ represents the temperature at position $j$ in a 1D object at $n\text{th}$ time step (different use of “$n$” from our code)

- We can use this principle in both space and time to compute
  - $u(j,n+1) = f( u(j-1,n), u(j,n), u(j+1,n) )$

for multiple time steps $n$ until we reach a convergence with an acceptably small error between time steps
- These algorithms are also referred to as “stencil codes”
- See also: [https://en.wikipedia.org/wiki/Heat_equation](https://en.wikipedia.org/wiki/Heat_equation)
HJ code for One-Dimensional Iterative Averaging using nested forseq-forall structure (Lecture 10)

1. `double[] myVal=new double[n+2]; myVal[n+1] = 1;`
2. `double[] myNew=new double[n+2]; myNew[n+1] = myVal[n+1];`
3. `forseq(0, m-1, (iter) -> {`
4. `// Compute MyNew as function of input array MyVal`
5. `forall(1, n, (j) -> { // Create n tasks`
6. `myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;`
7. `}); // forall`
8. `// Swap myVal & myNew;`
9. `temp=myVal; myVal=myNew; myNew=temp;`
10. `// myNew becomes input array for next iteration`
11. }); // for
Converting forseq-forall version into a forall-forseq version with barriers

1. double[] gVal = new double[n+2]; gVal[n+1] = 1;
2. double[] gNew = new double[n+2];
3. forallPhased(1, n, (j) -> { // Create n tasks
4.  // Initialize myVal and myNew as local pointers
5.  double[] myVal = gVal; double[] myNew = gNew;
6.  forseq(0, m-1, (iter) -> {
7.   // Compute MyNew as function of input array MyVal
8.   myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
9.   next(); // Barrier before next iteration of iter loop
10.  // Swap local pointers, myVal and myNew
11.  double[] temp = myVal; myVal = myNew; myNew = temp;
12.  // myNew becomes input array for next iteration
13.  }); // forseq
14.}); // forall
General Approach for Iteration Grouping (Loop Chunking)

Without chunking:
for (iter : [0:iterations-1]) {
    forall (j : [1:n])
        myNew[j] = (myVal[j-1] + myVal[j+1])/2;
    Swap myNew and myVal
}

With chunking (replace “forall” by “forall-for”):
for (iter : [0:iterations-1]) {
    forall (g : [0:ng-1])
        for (j : myGroup(g,[1:n],ng)
            myNew[j] = (myVal[j-1] + myVal[j+1])/2;
        Swap myNew and myVal
}
Example: HJ for One-Dimensional Iterative Averaging with Slide 7 code with chunking (Lecture 11)

1. double[] myVal = new double[n+2]; myVal[n+1] = 1;
2. double[] myNew = new double[n+2]; myNew[n+1] = myVal[n+1];
3. int nc = numWorkerThreads();
4. forseq(0, m-1, (iter) -> {
5.     // Compute MyNew as function of input array MyVal
6.     forallChunked(1, n, n/nc, (j) -> {
7.         myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
8.     }); // forallChunked
9.     temp = myVal; myVal = myNew; myNew = temp; // Swap myVal & myNew;
10.    // myNew becomes input array for next iteration
11. }); // for
Chunking forall loop in Slide 8 - inner chunked loop goes inside forseq-iter loop

1. double[] gVal = new double[n+2]; gVal[n+1] = 1;
2. double[] gNew = new double[n+2];
3. HjRegion1D iterSpace = newRectangularRegion1D(1,n);
4. int nc = numWorkerThreads();
5. forallPhased(0, nc-1, (jj) -> {
464 // Create nc tasks
6. // Initialize myVal and myNew as local pointers
7. double[] myVal = gVal; double[] myNew = gNew;
8. forseq(0, m-1, (iter) -> {
9. forseq(myGroup(jj, iterSpace, nc), (j) -> {
10. // Compute MyNew as function of input array MyVal
11. myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
12. }); // forseq
13. next(); // Barrier before executing next iteration of iter loop
14. // Swap local pointers, myVal and myNew
15. double[] temp = myVal; myVal = myNew; myNew = temp;
16. // myNew becomes input array for next iter
17. }); // forseq
18.}); // forall
Single Program Multiple Data (SPMD)

Basic idea

• Run the same code (program) on P workers

• Use the “rank” --- an ID ranging from 0 to (P-1) --- to determine what data is processed by which worker
  — Hence, “single-program” and “multiple-data”
  — Rank is equivalent to index in a top-level “forall (point[i] : [0:P-1])” loop

• Lower-level programming model than dynamic async/finish parallelism
  — Programmer’s code is essentially at the worker level (each forall iteration is like a worker),
  — Work distribution is managed by programmer by using barriers and other synchronization constructs
  — Harder to program but can be more efficient for restricted classes of applications (e.g. for OneDimAveraging, but not for NQueens)

• Convenient for hardware platforms that are not amenable to efficient dynamic task parallelism
  — General-Purpose Graphics Processing Unit (GPGPU) accelerators
  — Distributed-memory parallel machines
Slide 11 viewed as exemplar of SPMD pattern

1. double[] gVal=new double[n+2]; gVal[n+1] = 1;
2. double[] gNew=new double[n+2];
3. HjRegion1D iterSpace = newRectangularRegion1D(1,m);
4. int nc = numWorkerThreads();
5. forAllPhased(1, nc, (jj) -> {

6. // Initialize myVal and myNew as local pointers
7. double[] myVal = gVal; double[] myNew = gNew;
8. forseq(0, m-1, (iter) -> {
9.   forseq(myGroup(jj, iterSpace, nc), (j) -> {
10.  // Compute MyNew as function of input array MyVal
11. myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
12. }); // forseq
13. next(); // Barrier before executing next iteration of iter loop
14. // Swap local pointers, myVal and myNew
15. double[] temp=myVal; myVal=myNew; myNew=temp;
16. // myNew becomes input array for next iter
17. }); // forseq
18. }); // forAllPhased

Instead of async-finish, this SPMD version creates one task per worker, uses myGroup() to distribute work, and use barriers to synchronize workers.
Announcements & Reminders

- Midterm Exam on Thursday, Mar. 11th from 7pm - 9pm in Canvas
- Quiz for Unit 3 (topics 3.1 - 3.7) due Monday, Mar. 15th by 11:59pm
- HW3 available today, due Monday, April 5th by 11:59pm
  - Checkpoint 1 due Wednesday, March 24th by 11:59pm
Worksheet #14: Iterative Averaging Revisited

Answer the questions in the table below for the versions of the Iterative Averaging code shown in slides 7, 8, 10, 11.

Write in your answers as functions of m, n, and nc.

<table>
<thead>
<tr>
<th>Slide 7</th>
<th>Slide 8</th>
<th>Slide 10</th>
<th>Slide 11</th>
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<tbody>
<tr>
<td>How many tasks are created (excluding the main program task)?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many barrier operations (calls to next() per task) are performed?</td>
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<td></td>
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</tbody>
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