

COMP 322: Fundamentals of Parallel Programming

Lecture 14: Iterative Averaging Revisited, SPMD pattern

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Worksheet #13 solution:

Parallelism in Java Streams, Parallel Prefix Sums

1. What output will the following Java Streams code print?

C1		C2	The sorted() operation is a no-op if the printing is performed in parallel!
C2	or	C1	

2. Which stream operation in this example could benefit from a parallel prefix sum implementation, and why?

The filter operation since parallel prefix can be used to compute the indices in the output array

```
1. Arrays
2.   .asList("a1", "a2", "b1", "c2", "c1")
3.   .parallelStream()
4.   .filter(s -> s.startsWith("c"))
5.   .sorted()
6.   .map(String::toUpperCase)
7.   .forEach(System.out::println);
```



Recap: Parallel Filter Operation

[Credits: David Walker and Andrew W. Appel (Princeton), Dan Grossman (U. Washington)]

Given an array `input`, produce an array `output` containing only elements such that `f (elt)` is `true`, i.e., `output = input.parallelStream.filter(f).toArray`

```
Example: input [17, 4, 6, 8, 11, 5, 13, 19, 0, 24]
         f: is elt > 10
         output [17, 11, 13, 19, 24]
```

Parallelizable?

- Finding elements for the output is easy
- But getting them in the right place seems hard



Recap: Parallel prefix to the rescue

1. Parallel map to compute a **bit-vector** for true elements (can use Java streams)

input [17, 4, 6, 8, 11, 5, 13, 19, 0, 24]

bits [1, 0, 0, 0, 1, 0, 1, 1, 0, 1]

2. Parallel-prefix sum on the bit-vector (not available in Java streams)

bitsum [1, 1, 1, 1, 2, 2, 3, 4, 4, 5]

3. Parallel map to produce the output (can use Java streams)

output [17, 11, 13, 19, 24]

```
output = new array of size bitsum[n-1]
FORALL(i=0; i < input.length; i++){
    if(bits[i]==1)
        output[bitsum[i]-1] = input[i];
}
```



One-Dimensional Iterative Averaging Revisited

- Initialize a one-dimensional array of $(n+2)$ double's with boundary conditions, $\text{myVal}[0] = 0$ and $\text{myVal}[n+1] = 1$.
- In each iteration, each interior element $\text{myVal}[i]$ in $1..n$ is replaced by the average of its left and right neighbors.
 - Two separate arrays are used in each iteration, one for old values and the other for the new values
- After a sufficient number of iterations, we expect each element of the array to converge to $\text{myVal}[i] = (\text{myVal}[i-1] + \text{myVal}[i+1])/2$, for all i in $1..n$

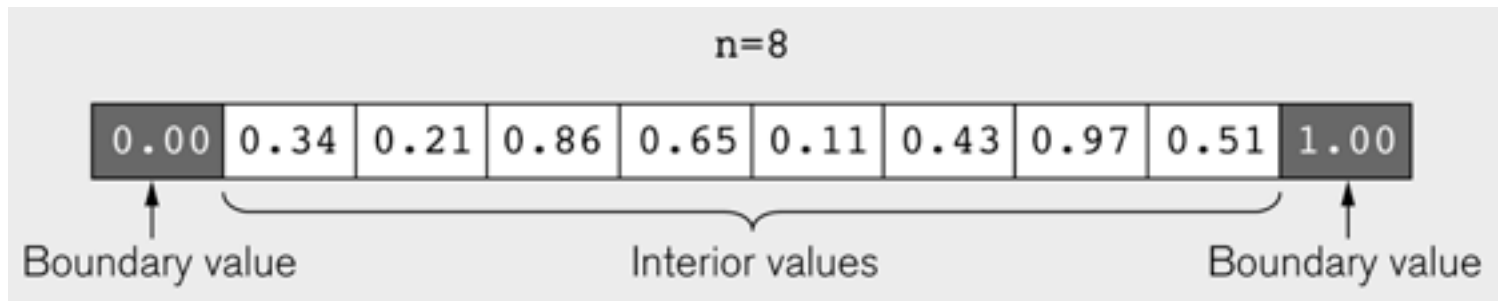


Illustration of an intermediate step for $n = 8$ (source: Figure 6.19 in Lin-Snyder book)



Iterative Averaging structure is akin to that of a Finite Difference solution to the One-Dimensional Heat Equation

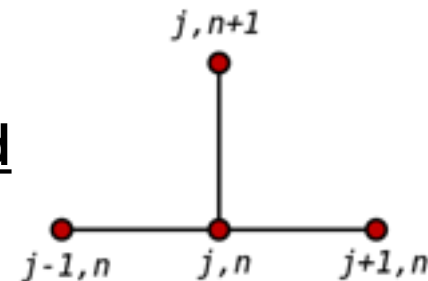
- In the 1D heat equation, $u(j,t)$ represents the temperature at position j in a 1D object at n^{th} time step (different use of “ n ” from our code)
- Recall that for a “properly-behaved” function f , and a sufficiently small h , we can approximate the first derivative of f as follows:

$$f'(a) \approx \frac{f(a+h) - f(a)}{h}$$

- We can use this principle in both space and time to compute
 - $u(j,n+1) = f(u(j-1,n), u(j,n), u(j+1,n))$

for multiple time steps n until we reach a convergence with an acceptably small error between time steps

- These algorithms are also referred to as “stencil codes”
- Source: http://en.wikipedia.org/wiki/Finite_difference_method
- See also: https://en.wikipedia.org/wiki/Heat_equation



HJ code for One-Dimensional Iterative Averaging using nested `foreachforall` structure (Lecture 11)

```
1. double[] myVal=new double[n+2]; myVal[n+1] = 1;
2. double[] myNew=new double[n+2]; myNew[n+1] = myVal[n+1];
3. foreach(0, m-1, (iter) -> {
4.     // Compute MyNew as function of input array MyVal
5.     forall(1, n, (j) -> { // Create n tasks
6.         myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
7.     }); // forall
8.     // Swap myVal & myNew;
9.     temp=myVal; myVal=myNew; myNew=temp;
10.    // myNew becomes input array for next iteration
11. }); // for
```



Converting forseq-forall version (Slide 7) into a forall-forseq version with barriers

```
1. double[] gVal=new double[n+2]; gVal[n+1] = 1;
2. double[] gNew=new double[n+2];
3. forallPhased(1, n, (j) -> { // Create n tasks
4.     // Initialize myVal and myNew as local pointers
5.     double[] myVal = gVal; double[] myNew = gNew;
6.     forseq(0, m-1, (iter) -> {
7.         // Compute MyNew as function of input array MyVal
8.         myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
9.         next(); // Barrier before next iteration of iter loop
10.        // Swap local pointers, myVal and myNew
11.        double[] temp=myVal; myVal=myNew; myNew=temp;
12.        // myNew becomes input array for next iteration
13.    }); // forseq
14. }); // forall
```



General Approach for Iteration Grouping (Loop Chunking)

Without chunking:

```
for (iter : [0:iterations-1]) {  
  forall (j : [1:n])  
    myNew[j] = (myVal[j-1] + myVal[j+1])/2;  
  Swap myNew and myVal  
}
```

With chunking (replace “forall” by “forall-for”):

```
for (iter : [0:iterations-1]) {  
  forall (g : [0:ng-1])  
    for(j : myGroup(g,[1:n],ng)  
      myNew[j] = (myVal[j-1] + myVal[j+1])/2;  
    Swap myNew and myVal
```



Example: HJ code for One-Dimensional Iterative Averaging with Slide 7 code w/ chunking (Lecture 11)

```
1. double[] myVal=new double[n+2]; myVal[n+1] = 1;
2. double[] myNew=new double[n+2]; myNew[n+1] = myVal[n+1];
3. int nc = numWorkerThreads();
4. forseq(0, m-1, (iter) -> {
5.     // Compute MyNew as function of input array MyVal
6.     forallChunked(1, n, n/nc, (j) -> { // Create nc tasks
7.         myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
8.     }); // forallChunked
9.     temp=myVal; myVal=myNew; myNew=temp;// Swap myVal & myNew;
10.    // myNew becomes input array for next iteration
11. }); // for
```



General Approach for Iteration Grouping with Barriers

Barrier-based solution:

// Note that iter-loop is inserted between forall-g and for-j loops

```
forall (g : [0:ng-1])  
  for (iter : [0:iterations-1]) {  
    for(j : myGroup(g,[1:n],ng)  
      myNew[j] = (myVal[j-1] + myVal[j+1])/2;  
    next; // Barrier  
    Swap myNew and myVal  
  } // for iter
```

Also referred to as a “single program multiple data” (SPMD)



Chunking the forall loop in Slide 8 — inner chunked loop goes inside forseq-iter loop

```
1.  double[] gVal=new double[n+2]; gVal[n+1] = 1;
2.  double[] gNew=new double[n+2];
3.  HjRegion1D iterSpace = newRectangularRegion1D(1,n);
4.  int nc = numWorkerThreads();
5.  forallPhased(0, nc-1, (jj) -> { // Create nc tasks
6.      // Initialize myVal and myNew as local pointers
7.      double[] myVal = gVal; double[] myNew = gNew;
8.      forseq(0, m-1, (iter) -> {
9.          forseq(myGroup(jj,iterSpace,nc), (j) -> {
10.             // Compute MyNew as function of input array MyVal
11.             myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
12.          }); // forseq
13.         next(); // Barrier before executing next iteration of iter loop
14.         // Swap local pointers, myVal and myNew
15.         double[] temp=myVal; myVal=myNew; myNew=temp;
16.         // myNew becomes input array for next iter
17.     }); // forseq
18. }); // forall
```



Single Program Multiple Data (SPMD)

Basic idea

- Run the same code (program) on P workers
- Use the “rank” --- an ID ranging from 0 to $(P-1)$ --- to determine what data is processed by which worker
 - Hence, “single-program” and “multiple-data”
 - Rank is equivalent to index in a top-level “forall (point[i] : [0:P-1])” loop
- Lower-level programming model than dynamic async/finish parallelism
 - Programmer’s code is essentially at the worker level (each forall iteration is like a worker), and work distribution is managed by programmer by using barriers and other synchronization constructs
 - Harder to program but can be more efficient for restricted classes of applications (e.g. for OneDimAveraging, but not for NQueens)
- Convenient for hardware platforms that are not amenable to efficient dynamic task parallelism
 - General-Purpose Graphics Processing Unit (GPGPU) accelerators
 - Distributed-memory parallel machines



Slide 12 viewed as exemplar of SPMD pattern

```
1. double[] gVal=new double[n+2]; gVal[n+1] = 1;
2. double[] gNew=new double[n+2];
3. HjRegion1D iterSpace = newRectangularRegion1D(1,m);
4. int nc = numWorkerThreads();
5. forallPhased(1, nc, (jj) -> { // Create nc tasks
6.     // Initialize myVal and myNew as local pointers
7.     double[] myVal = gVal; double[] myNew = gNew;
8.     forseq(0, m-1, (iter) -> {
9.         forseq(myGroup(jj,iterSpace,nc), (j) -> {
10.            // Compute MyNew as function of input array MyVal
11.            myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
12.        }); // forseq
13.        next(); // Barrier before executing next iteration of iter loop
14.        // Swap local pointers, myVal and myNew
15.        double[] temp=myVal; myVal=myNew; myNew=temp;
16.        // myNew becomes input array for next iter
17.    }); // forseq
```

Instead of async-finish, this SPMD version creates one task per worker, uses myGroup() to distribute work, and use barriers to synchronize workers.