Lecture 12: Abstract Metrics, Parallel Speedup and Amdahl’s Law

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Abstract Performance Metrics

- Basic Idea
  - Count operations of interest, as in big-O analysis, to evaluate parallel algorithms
  - Abstraction ignores many overheads that occur on real systems

- Calls to doWork()
  - Programmer inserts calls of the form, `doWork(N)` within a task (async, future task or data-driven task) to indicate abstract execution of N application-specific abstract operation
    - e.g., in lab 3, we included one call to `doWork(1)` for each double addition, and ignore the cost of everything else
  - Abstract metrics are enabled by calling `HjSystemProperty.abstractMetrics.set(true)` at start of program execution
  - If an HJ program is executed with this option, abstract metrics can be printed at end of program execution with calls to `abstractMetrics().totalWork()`, `abstractMetrics().criticalPathLength()`, and
Abstract Performance Metrics

• Pay attention where you put doWork() calls

• What does this mean?

```java
var bottom = future(() -> . . .);
var top = future(() -> . . .)
doWork(1);
return bottom.get() + top.get();
```

• Correct:

```java
var bottom = future(() -> . . .);
var top = future(() -> . . .);

var bottomVal = bottom.get();
var topVal = top.get();
doWork(1);
return bottomVal + topVal;
```
Data Races

A data race occurs on location L in a program execution with computation graph CG if there exist steps (nodes) S1 and S2 in CG such that:

1. S1 does not depend on S2 and S2 does not depend on S1, i.e., S1 and S2 can potentially execute in parallel, and
2. Both S1 and S2 read or write L, and at least one of the accesses is a write.

• A data-race is usually considered an error. The result of a read operation in a data race is undefined. The result of a write operation is undefined if there are two or more writes to the same location.

• Note that our definition of data race includes the case that both S1 and S2 write the same value in location L, even if the data race is benign.
Parallel Speedup

• Define Speedup(P) = \( \frac{T_1}{T_P} \)
  — Factor by which \( P \) processors speeds up execution time relative to 1 processor, for fixed input size
  — For ideal executions without overhead, \( 1 \leq \text{Speedup}(P) \leq P \)
  — You see this with abstract metrics, but bounds may not hold when measuring real execution times with real overheads
  — Linear speedup
    - When Speedup(P) = \( kP \), for some constant \( k \), \( 0 < k < 1 \)

• Ideal Parallelism = WORK / CPL = \( \frac{T_1}{T_\infty} \)
  = Parallel Speedup on an unbounded (infinite) number of processors
Assume greedy schedule, input array size $S$ is a power of 2, each add takes 1 time unit

- $\text{WORK}(G) = S-1$, and $\text{CPL}(G) = \log_2(S)$
- Define $T(S,P)$ = parallel execution time for Array Sum with size $S$ on $P$ processors
- Use upper bound $T(S,P) \leq \text{WORK}(G)/P + \text{CPL}(G)$ as a worst-case estimate

$$T(S,P) \leq \text{WORK}(G)/P + \text{CPL}(G) = (S-1)/P + \log_2(S) \quad \Rightarrow \quad \text{Speedup}(S,P) = T(S,1)/T(S,P) = (S-1)/((S-1)/P + \log_2(S))$$
Define Efficiency(P) = Speedup(P)/ P = T_f/(P * T_P)

— Processor efficiency --- figure of merit that indicates how well a parallel program uses available processors

— For ideal executions without overhead, 1/P <= Efficiency(P) <= 1

— Efficiency(P) = 1 (100%) is the best we can hope for
How many processors should we use?

What should be the minimum efficiency to determine how many processors we should use?
How many processors should we use?

• Common goal: choose number P for a given input size, S, so that efficiency is at least 0.5 (50%)

• Half-performance metric
  — $S_{1/2} = \text{input size that achieves Efficiency}(P) = 0.5$ for a given P
  — Figure of merit that indicates how large an input size is needed to obtain efficient parallelism
  — A larger value of $S_{1/2}$ indicates that the problem is harder to parallelize efficiently
Array Sum: Speedup as a function of array size $S$ and number of processors $P$

- \( \text{Speedup}(S,P) = \frac{T(S,1)}{T(S,P)} = \frac{(S-1)}{(S-1)/P + \log_2(S)} \)
- Asymptotically, \( \text{Speedup}(S,P) \to \frac{(S-1)}{\log_2 S} \), as $P \to \infty$
Array Sum: Speedup as a function of array size $S$ and number of processors $P$

- $\text{Speedup}(S,P) = \frac{T(S,1)}{T(S,P)} = \frac{(S-1)}{((S-1)/P + \log_2(S))}$
- Asymptotically, $\text{Speedup}(S,P) \rightarrow \frac{(S-1)}{\log_2 S}$, as $P \rightarrow \infty$

Efficiency($P$) $\leq$ 0.5, for $P \geq 256$  
$\Rightarrow$ wasteful to use more than 256 processors for $S=2048$

Efficiency($P$) $\leq$ 0.5, for $P \geq 128$  
$\Rightarrow$ wasteful to use more than 128 processors for $S=1024$
Amdahl’s Law

If $q \leq 1$ is the fraction of WORK in a parallel program that must be executed sequentially for a given input size $S$, then the best speedup that can be obtained for that program is $\text{Speedup}(S,P) \leq 1/q$. 
Amdahl’s Law

• Upper bound on speedup simplistically assumes that work can be divided into sequential and parallel portions
  — Sequential portion of WORK = q
    - also denoted as $f_s$ (fraction of sequential work)
  — Parallel portion of WORK = 1-q
    - also denoted as $f_p$ (fraction of parallel work)

• Observation follows directly from critical path length lower bound on parallel execution time
  — CPL $\geq q \cdot T(S,1)$
  — $T(S,P) \geq q \cdot T(S,1)$
  — Speedup(S,P) = $T(S,1)/T(S,P)$ $\leq 1/q$
Illustration of Amdahl’s Law: Best Case Speedup as function of Parallel Portion