COMP 322: Parallel and Concurrent Programming

Lecture 30: Parallel Graph Algorithms

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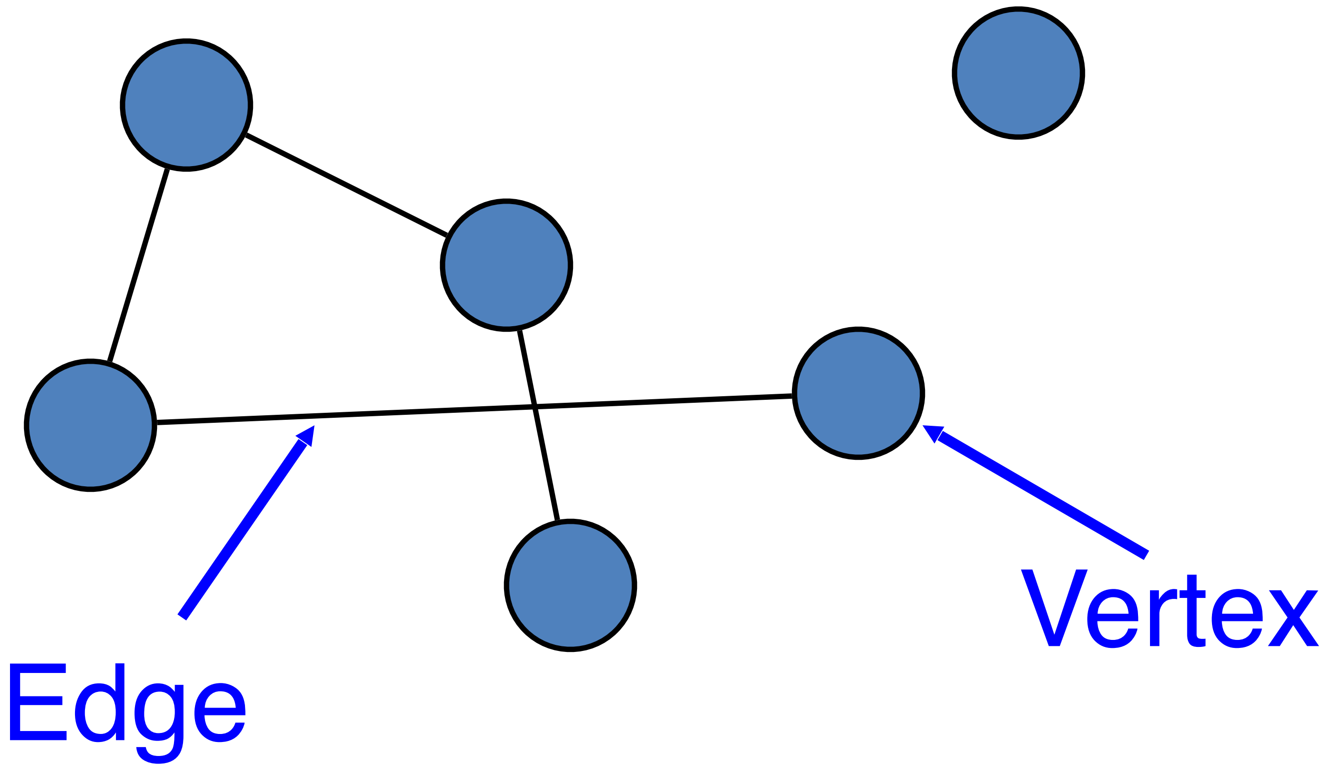
Some slides in this presentation are adopted from Aydin Buluç: “Parallel Graph Algorithms”, LBNL, CS267, Spring 2016,
Hall Perkins, “Data Structures”, CSE 374, University of Washington

Graphs

Graph G = (V,E)

- a set of vertices and a set of edges between vertices



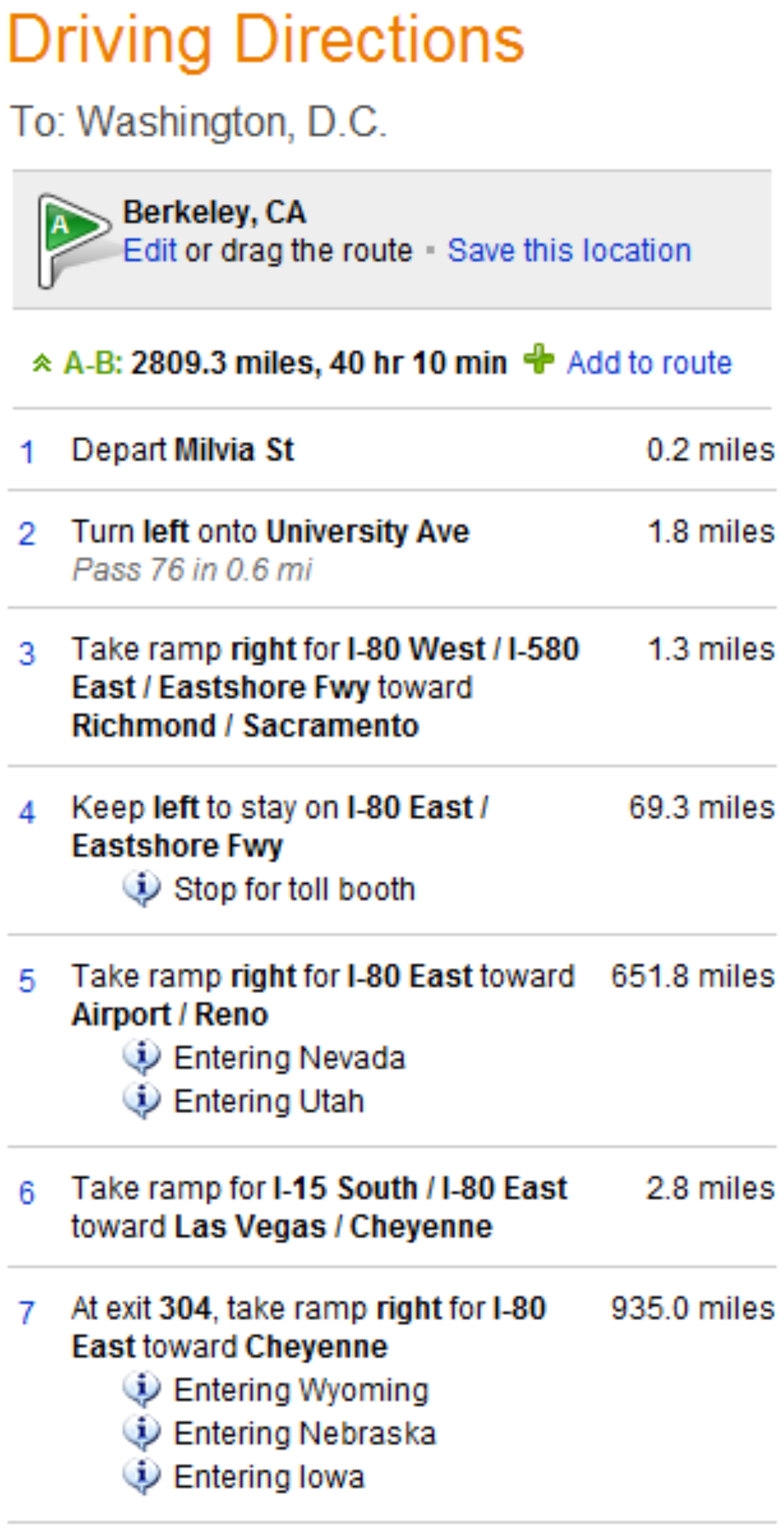
n=|V| (number of vertices)

m=|E| (number of edges)

D=diameter (max #hops between any pair of vertices)

- Edges can be directed or undirected, weighted or not.
- They can even have attributes (i.e. semantic graphs)
- Sequences of edges $<u_1,u_2>$, $<u_2,u_3>$, … ,$<u_{n-1},u_n>$ is a **path** from u_1 to u_n. Its **length** is the sum of its weights.

Routing in transportation networks



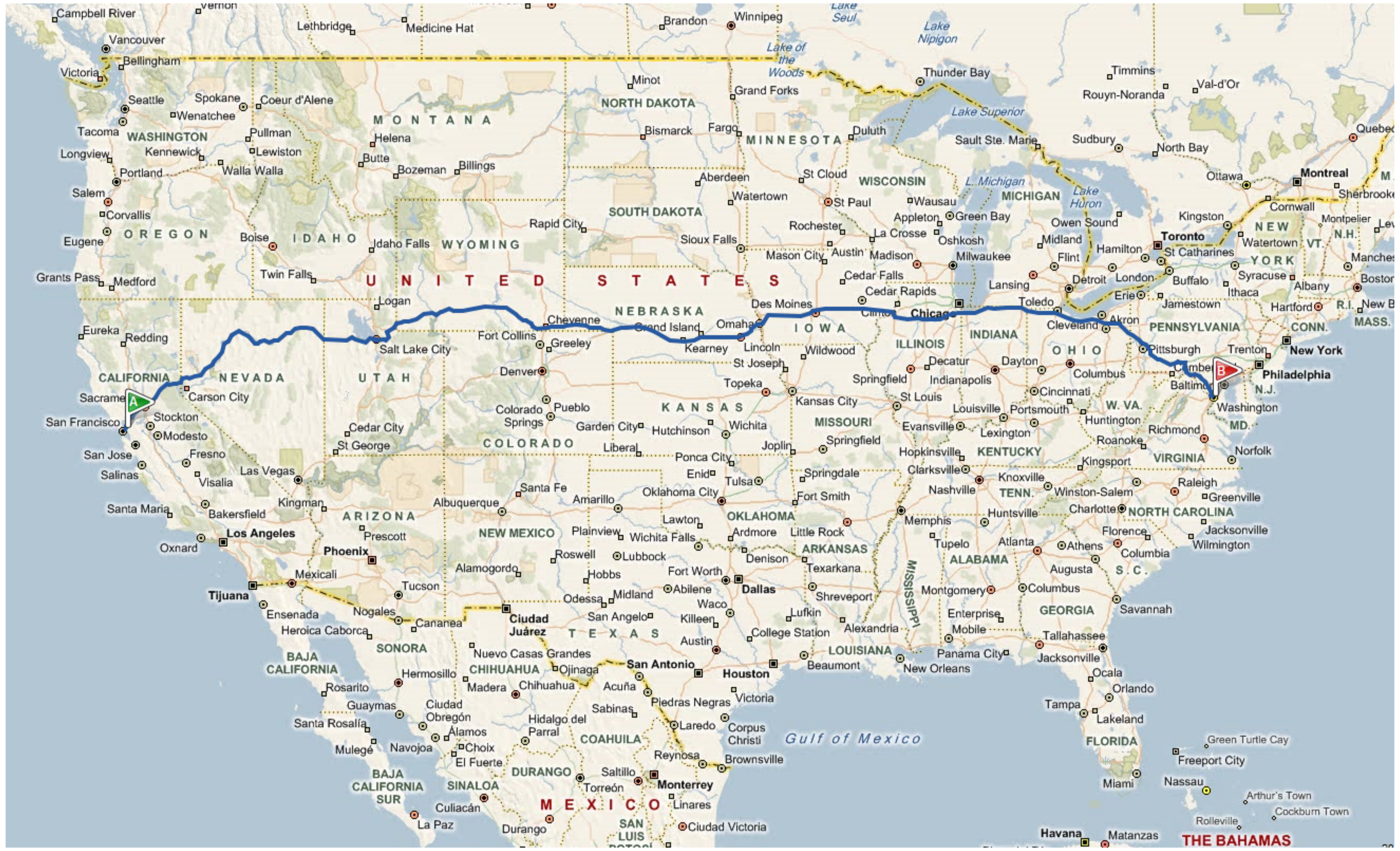
Driving Directions

To: Washington, D.C.

Berkeley, CA
Edit or drag the route · Save this location

A-B: **2809.3 miles, 40 hr 10 min** Add to route

1	Depart **Milvia St**	0.2 miles
2	Turn **left** onto **University Ave** *Pass 76 in 0.6 mi*	1.8 miles
3	Take ramp **right** for **I-80 West / I-580 East / Eastshore Fwy** toward **Richmond / Sacramento**	1.3 miles
4	Keep **left** to stay on **I-80 East / Eastshore Fwy** Stop for toll booth	69.3 miles
5	Take ramp **right** for **I-80 East** toward **Airport / Reno** Entering Nevada Entering Utah	651.8 miles
6	Take ramp for **I-15 South / I-80 East** toward **Las Vegas / Cheyenne**	2.8 miles
7	At exit **304**, take ramp **right** for **I-80 East** toward **Cheyenne** Entering Wyoming Entering Nebraska Entering Iowa	935.0 miles

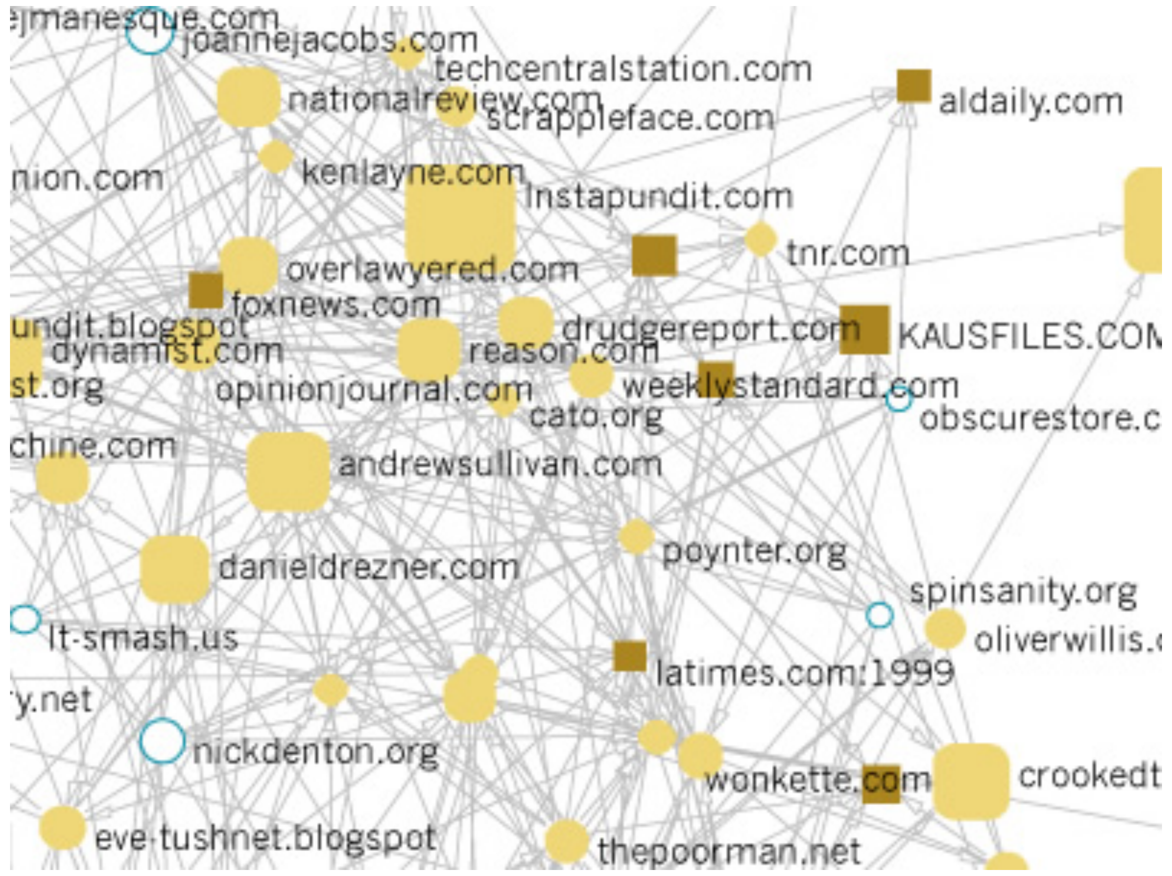


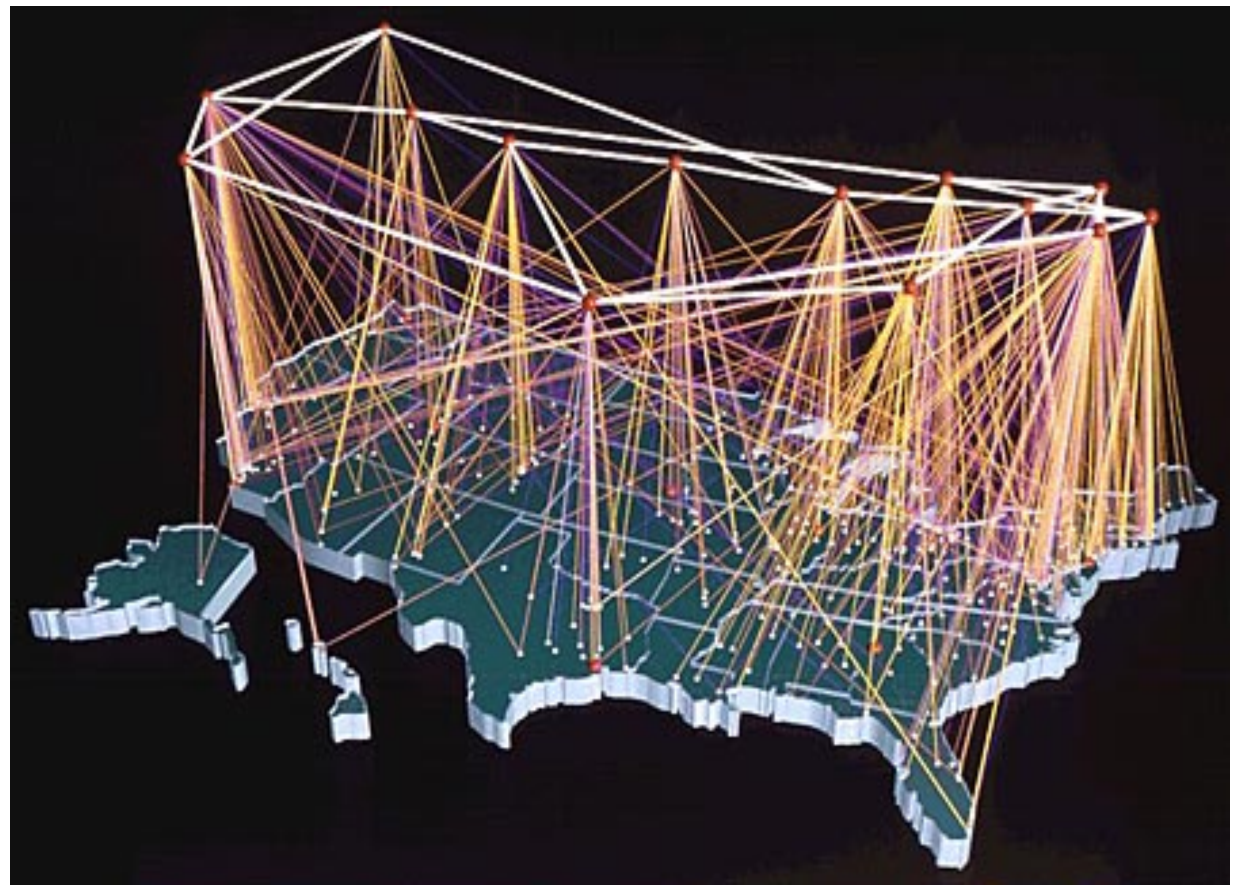
Road networks, Point-to-point shortest paths: 15 seconds (naïve) → **10 microseconds**

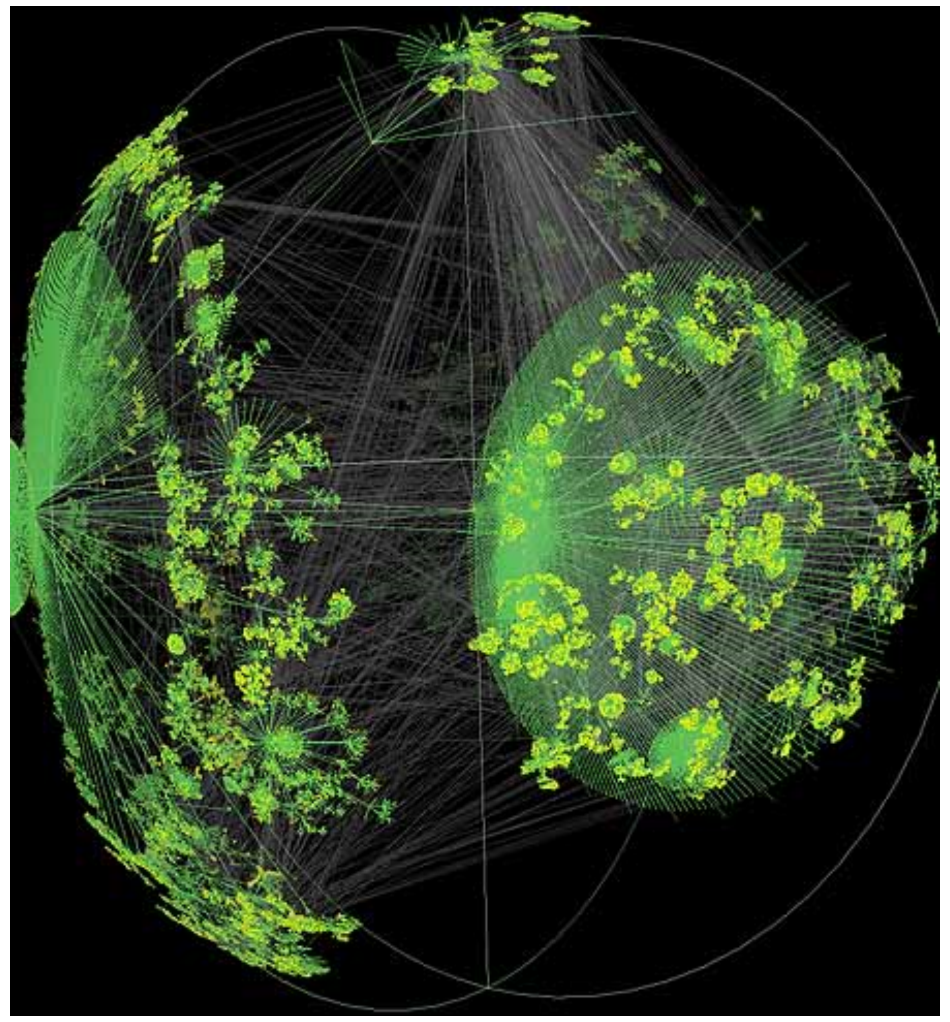
H. Bast et al., "Fast Routing in Road Networks with Transit Nodes", Science 27, 2007.

Internet and the WWW

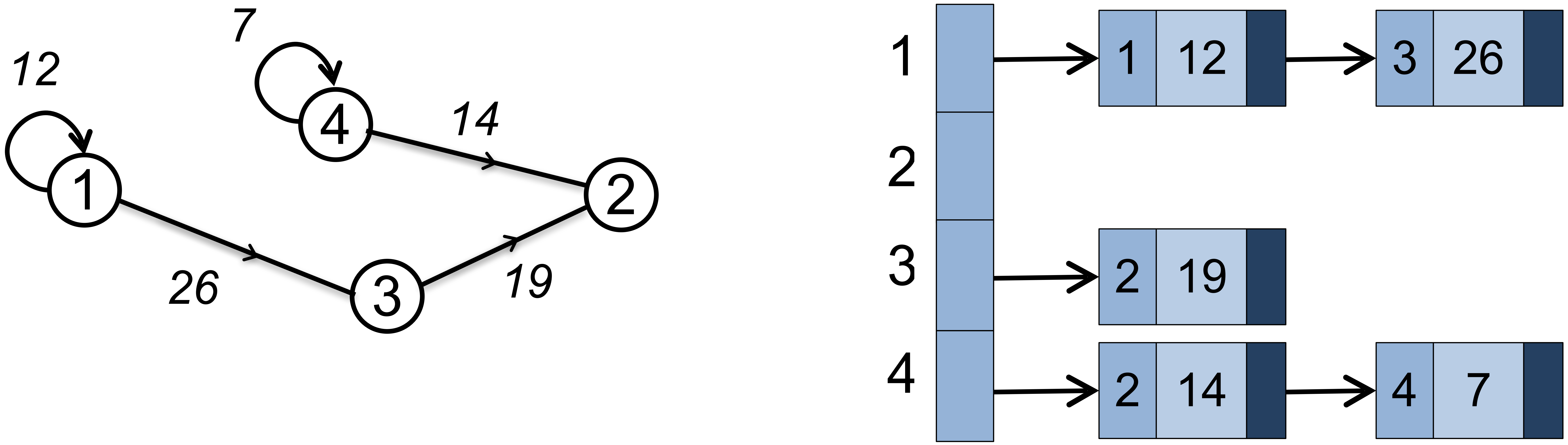
- The world-wide web can be represented as a directed graph
  - — Web search and crawl: traversal
  - — Link analysis, ranking: Page rank and HITS
  - — Document classification and clustering
- Internet topologies (router networks) are naturally modeled as graphs







Adjacency List graph representation



Graph Algorithms

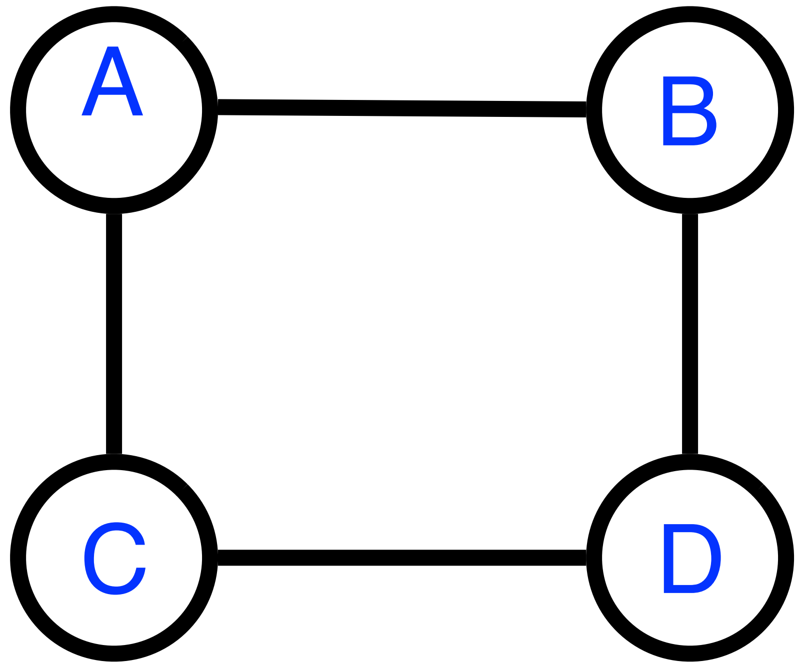
- Traversals
  - DFS, BFS
- Finding paths
  - Single-source shortest paths (Dijkstra, Bellman-Ford)
  - All-pairs shortest-paths (Floyd-Warshall)
- Maximal independent sets
- Decomposition (connected components, strongly connected components)
- Maximum cardinality matching
- Connecting
  - Minimum spanning tree

Spanning Tree Definition

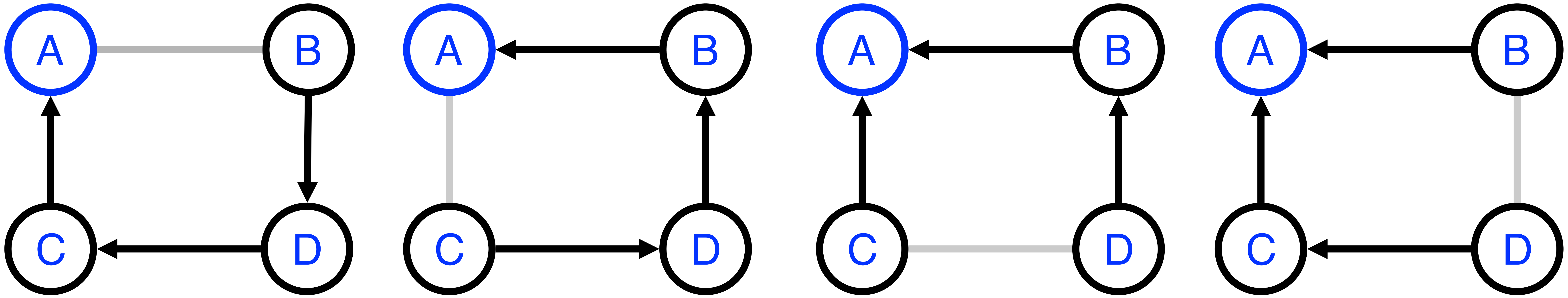
- A spanning tree, T, of a connected undirected graph G is
  - rooted at some vertex of G
  - defined by a parent map for each vertex
  - contains all the vertices of G, i.e. spans all vertices
  - contains exactly |v| - 1 edges
    - adding any other edge will create a cycle
  - contains no cycles (a tree!)
- The edges involved in T are a subset of the edges in G

An Example Graph with 4 possible spanning trees rooted at vertex A

Example Undirected Graph:



Spanning Trees (edges are directed from child to parent):



Vertex	Parent
A	null
B	D
C	A
D	C

Vertex	Parent
A	null
B	A
C	D
D	B

Vertex	Parent
A	null
B	A
C	A
D	B

Vertex	Parent
A	null
B	A
C	A
D	C

Sequential Spanning Tree Algorithm

```
class V  {
  V [] neighbors; // adjacency list for input graph
  V parent; // output value of parent in spanning tree

  boolean makeParent(V n) {
    if (parent == null) { parent = n; return true; }
    else return false; // return true if n became parent
  } // makeParent

  void compute() {
    for (int i=0; i<neighbors.length; i++) {
      final V child = neighbors[i];
      if (child.makeParent(this))
        child.compute(); // recursive call
     }
  } // compute
} // class V
. . . // main program
root.parent = root; // Use self-cycle to identify root
root.compute();
. . .
```

Exercise: Parallel Spanning Tree Algorithm using object-based isolated construct

```
class V  {
  V [] neighbors; // adjacency list for input graph
  V parent; // output value of parent in spanning tree

  boolean makeParent(V n) {
    if (parent == null) { parent = n; return true; }
    else return false; // return true if n became parent
  } // makeParent

  void compute() {
    for (int i=0; i<neighbors.length; i++) {
      final V child = neighbors[i];
      if (child.makeParent(this))
        child.compute(); // recursive call
    }
  } // compute
} // class V
. . . // main program
root.parent = root; // Use self-cycle to identify root
root.compute();
. . .
```

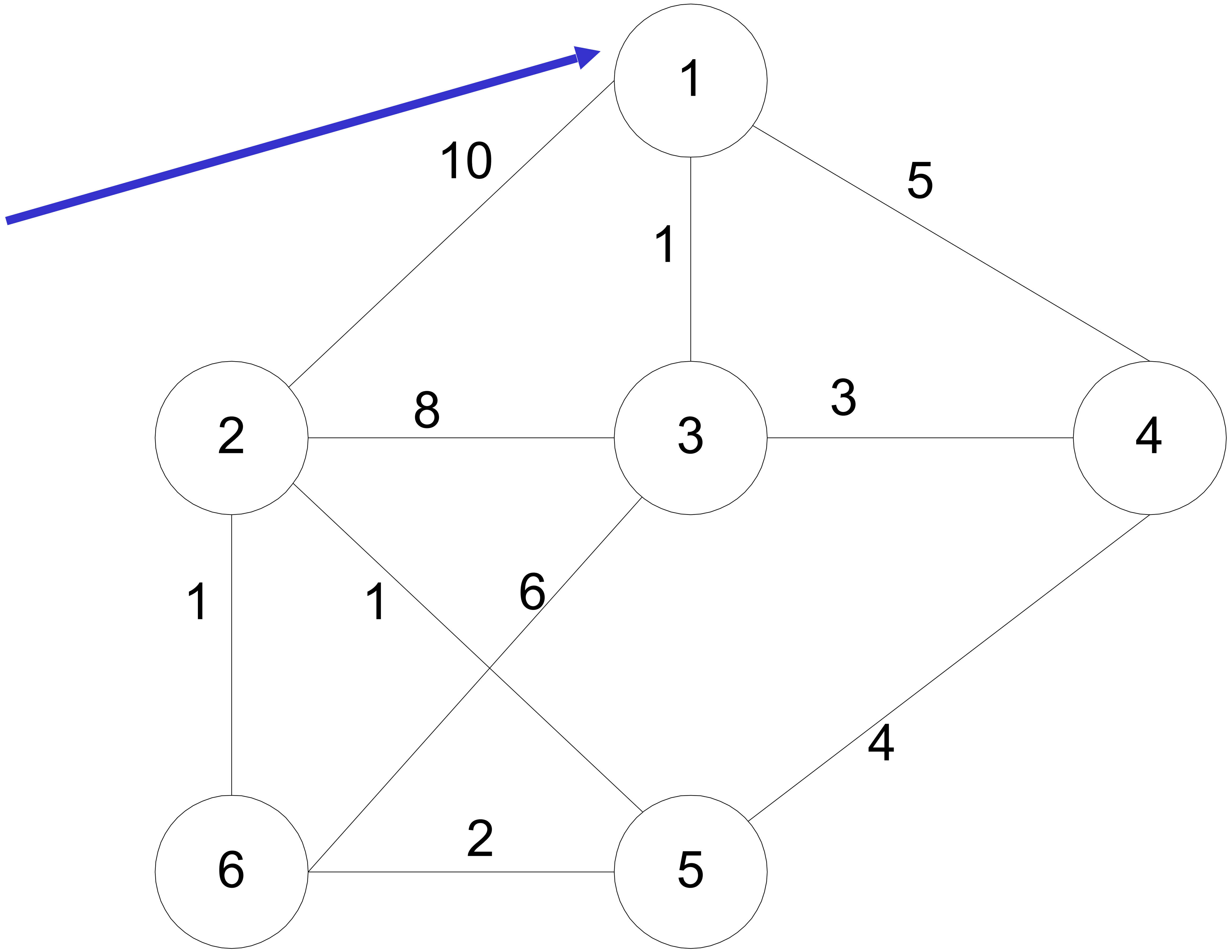
Minimum Spanning Tree

- For graphs that have edge weights
- Spanning tree with a minimum weight
- Sequential algorithms:
  - Prim’s algorithm: greedy, grow a single tree by adding nodes closest to it
  - Kruskal’s algorithm: greedy, add lightest edges that don’t create a cycle
  - Boruvka’s algorithm: combination of Prim’s and Kruskal’s
    - Can be parallelized

Prim’s Algorithm

Starting from empty T, choose a vertex at random and initialize

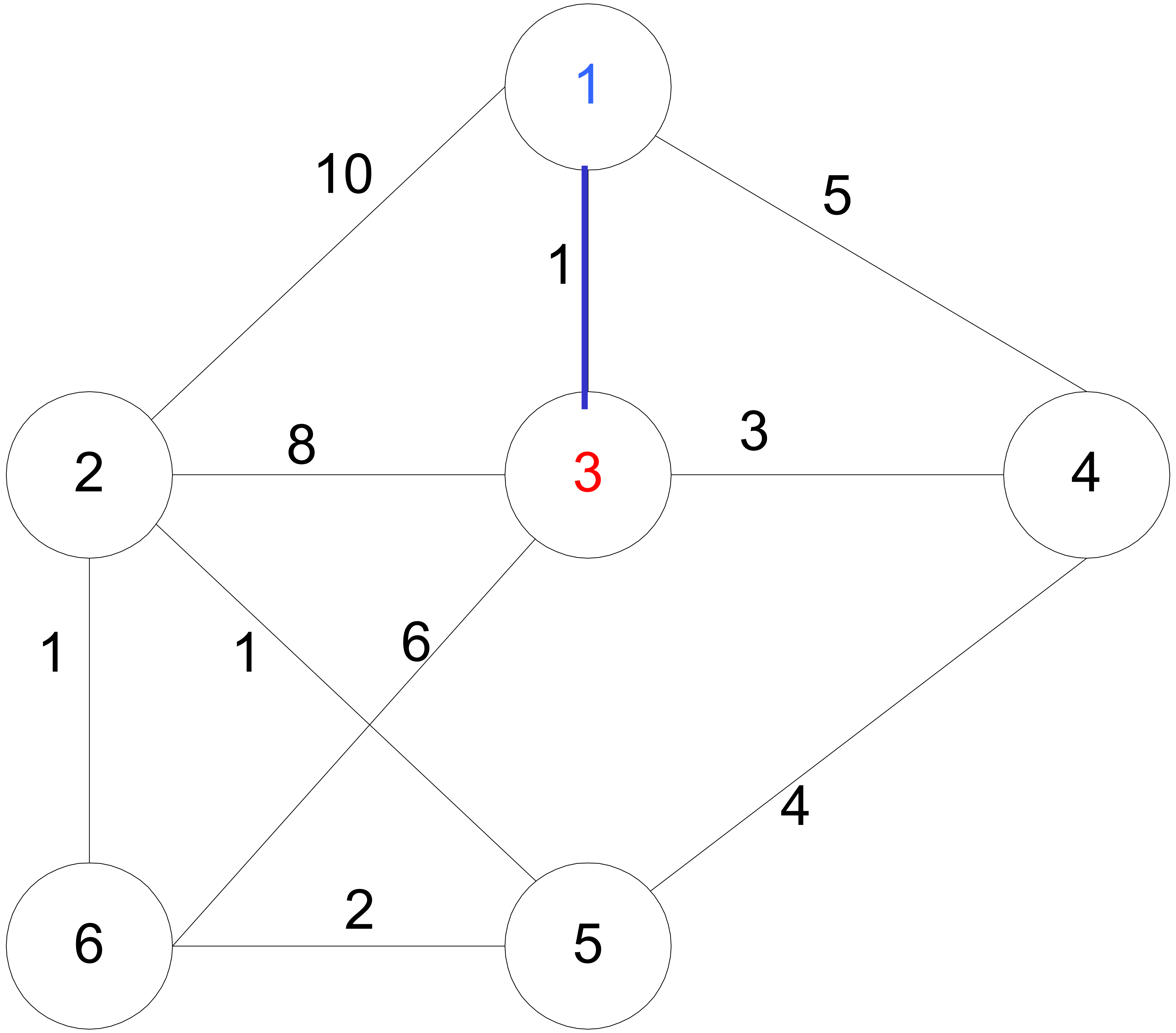
V = {1), E’ ={}



Prim’s Algorithm

Choose the vertex u not in V such that edge weight from u to a vertex in V is minimal (greedy!)

V={1,3} E’= {(1,3) }



Prim’s Algorithm

Repeat until all vertices have been chosen

Choose the vertex u not in V such that edge weight from v to a vertex in V is minimal (greedy!)

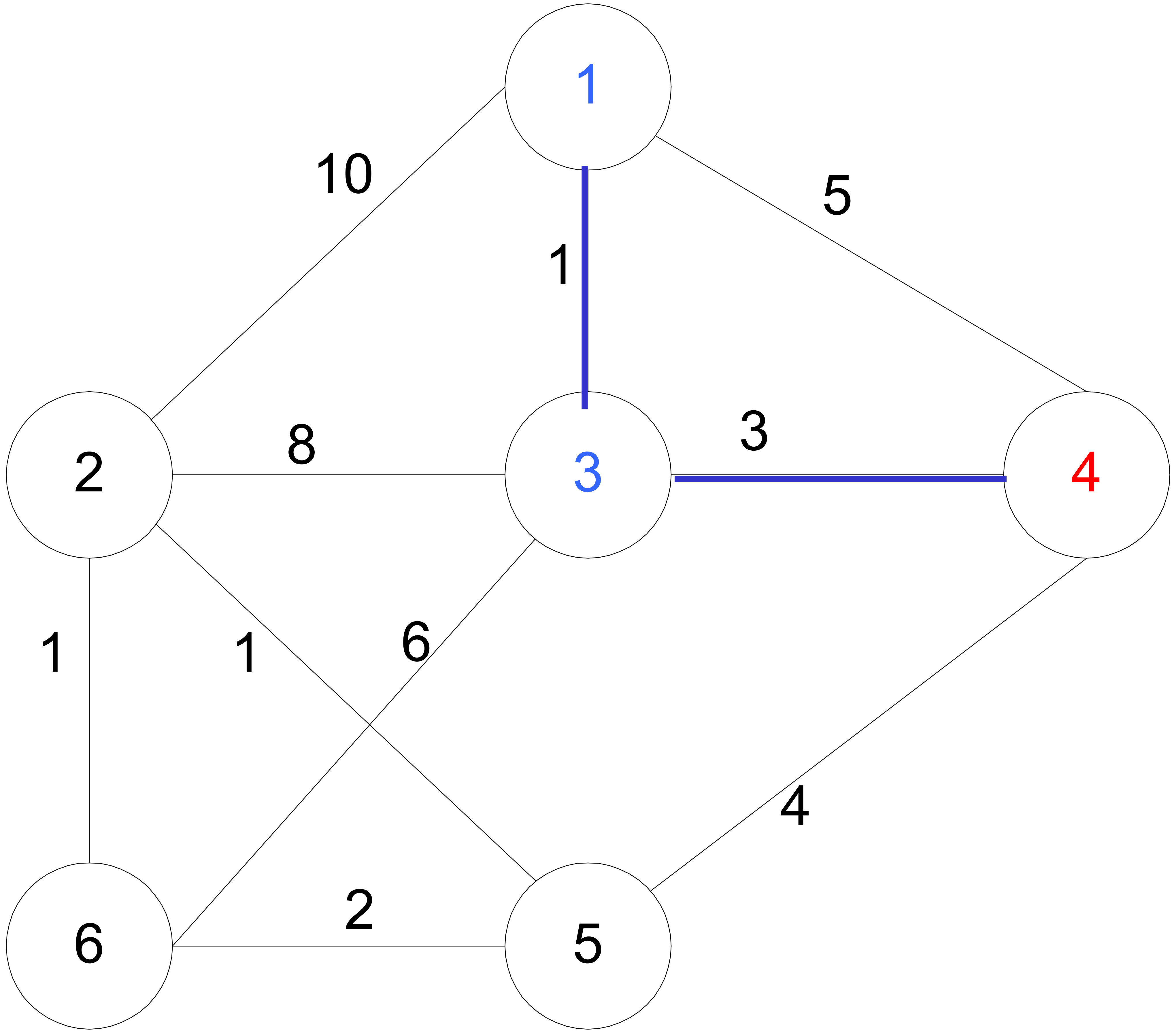
V= {1,3,4} E’= {(1,3),(3,4)}

V={1,3,4,5} E’={(1,3),(3,4),(4,5)}

….

V={1,3,4,5,2,6}

E’={(1,3),(3,4),(4,5),(5,2),(2,6)}



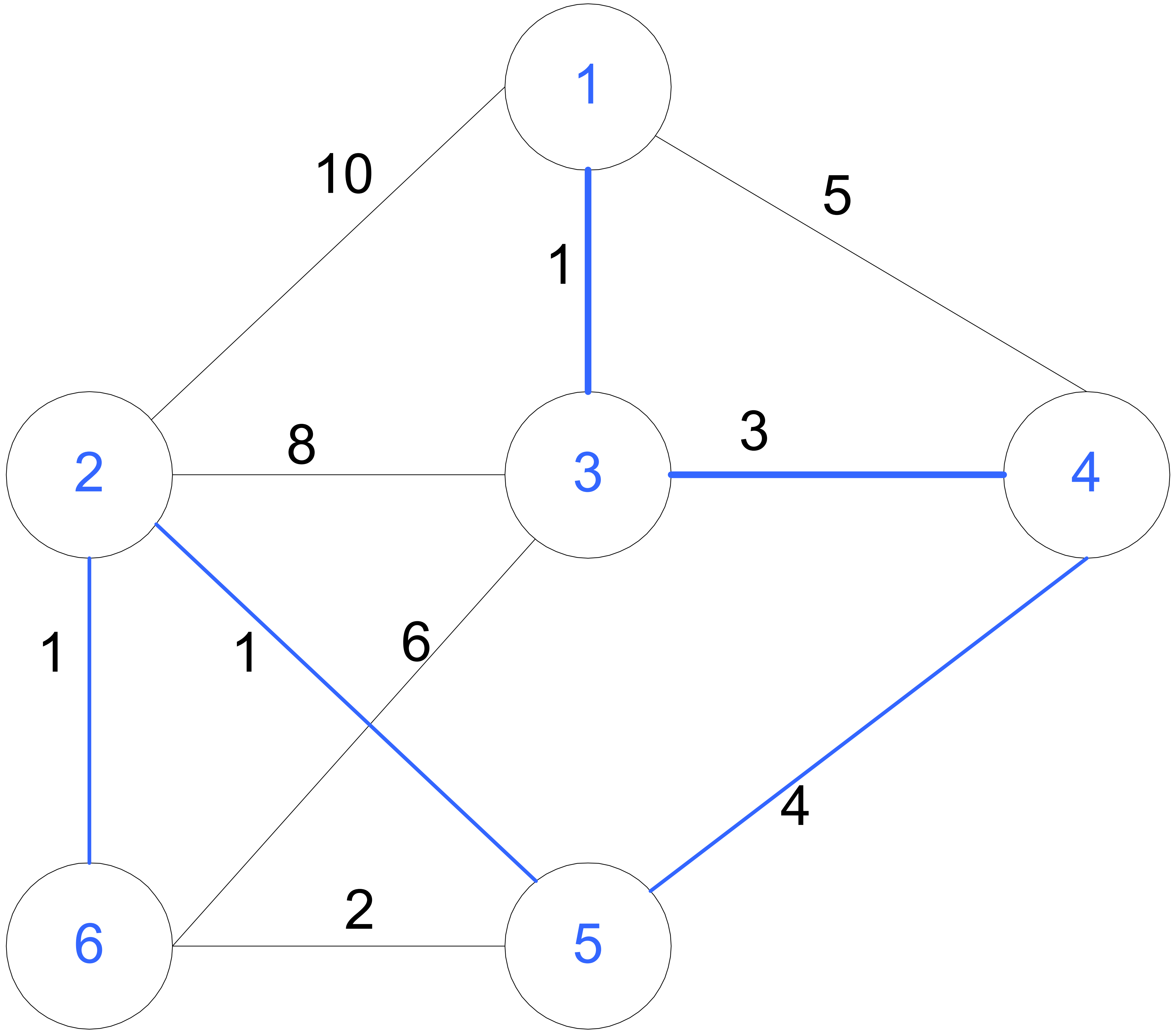
Prim’s Algorithm

Repeat until all vertices have been chosen

V={1,3,4,5,2,6}

E’={(1,3),(3,4),(4,5),(5,2),(2,6)}

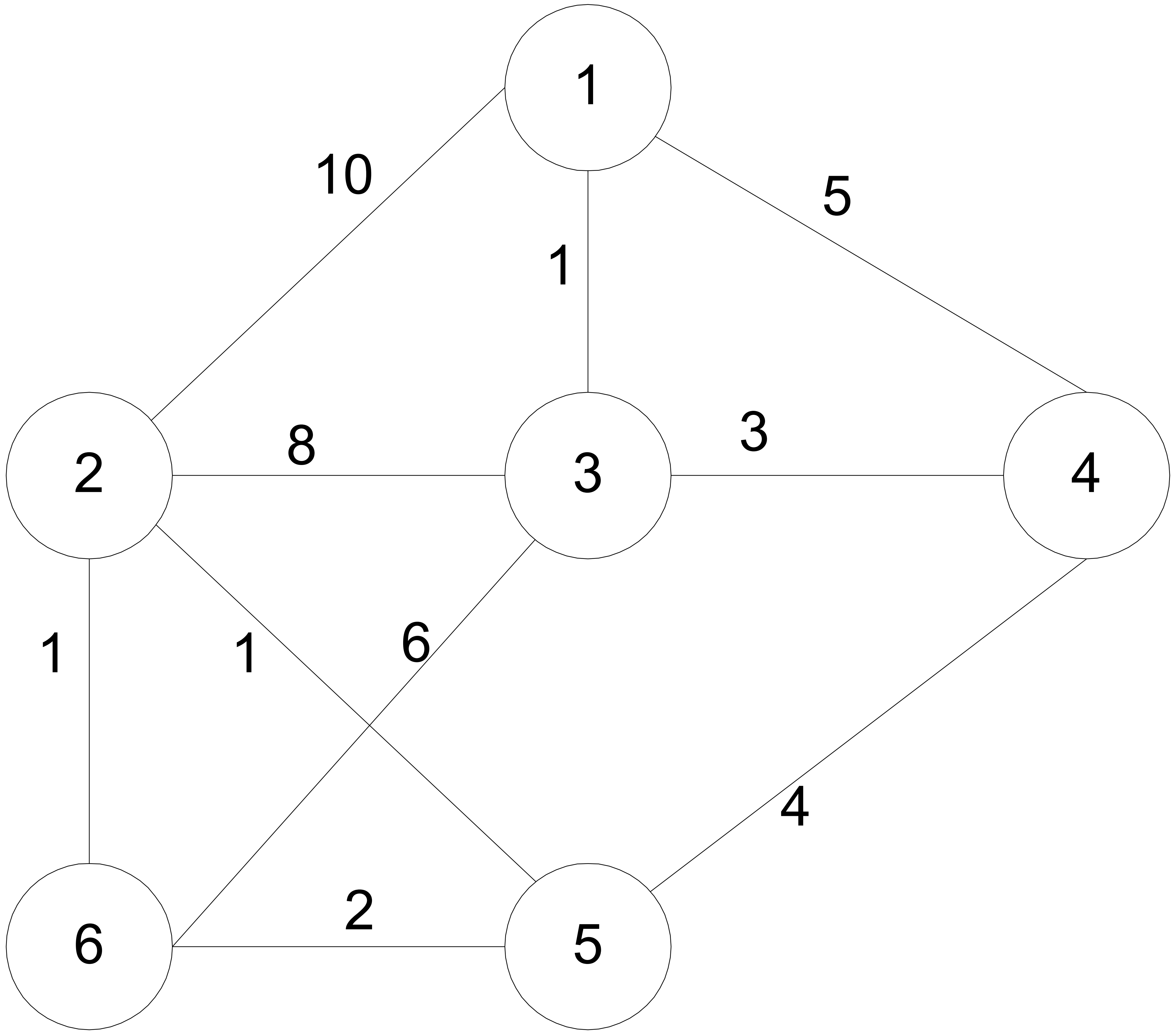
Final Cost: 1 + 3 + 4 + 1 + 1 = 10



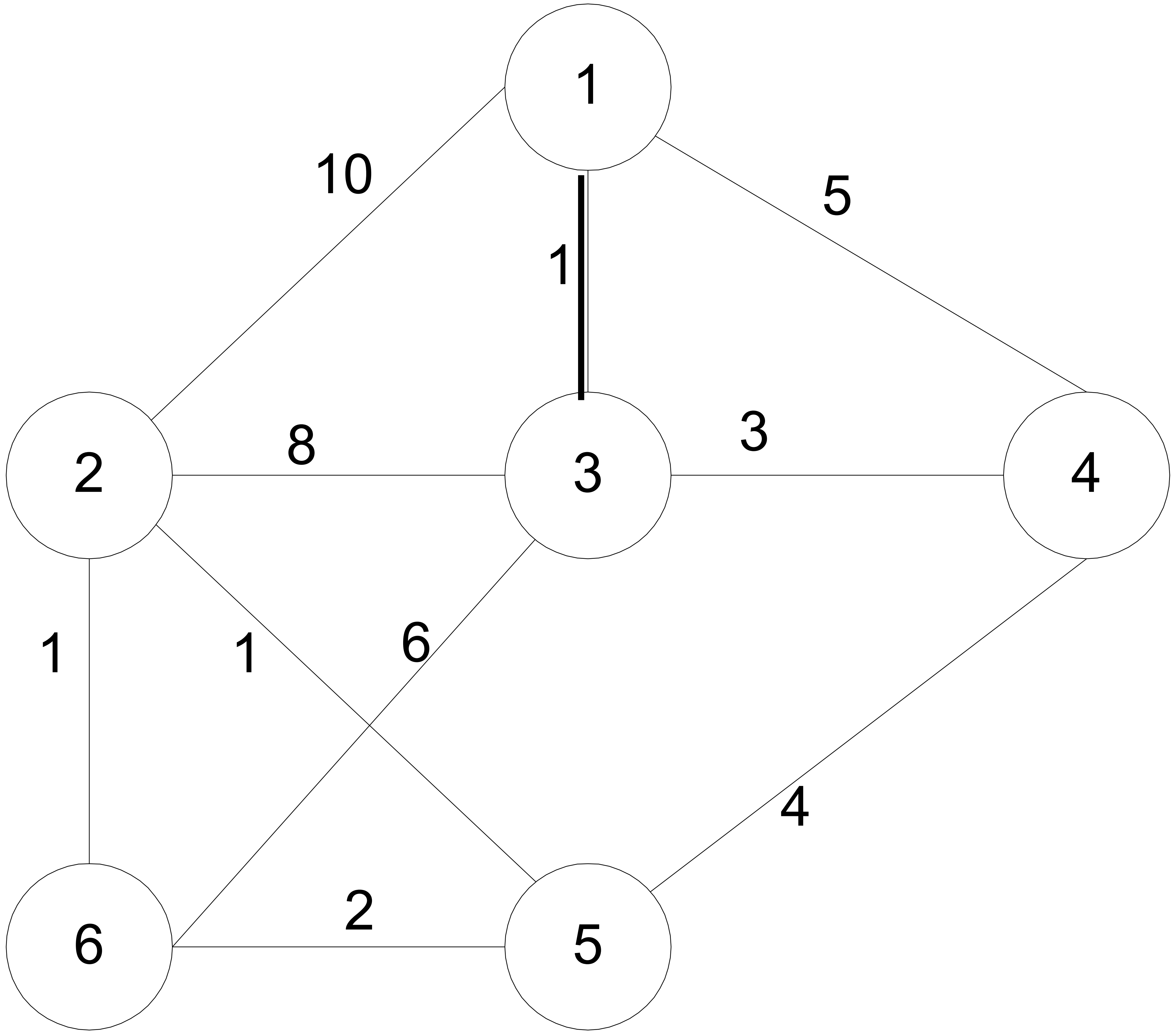
Kruskal’s Algorithm

- Select edges in order of increasing cost
- Accept an edge to expand tree or forest only if it does not cause a cycle
- Implementation using adjacency list, priority queues and disjoint sets

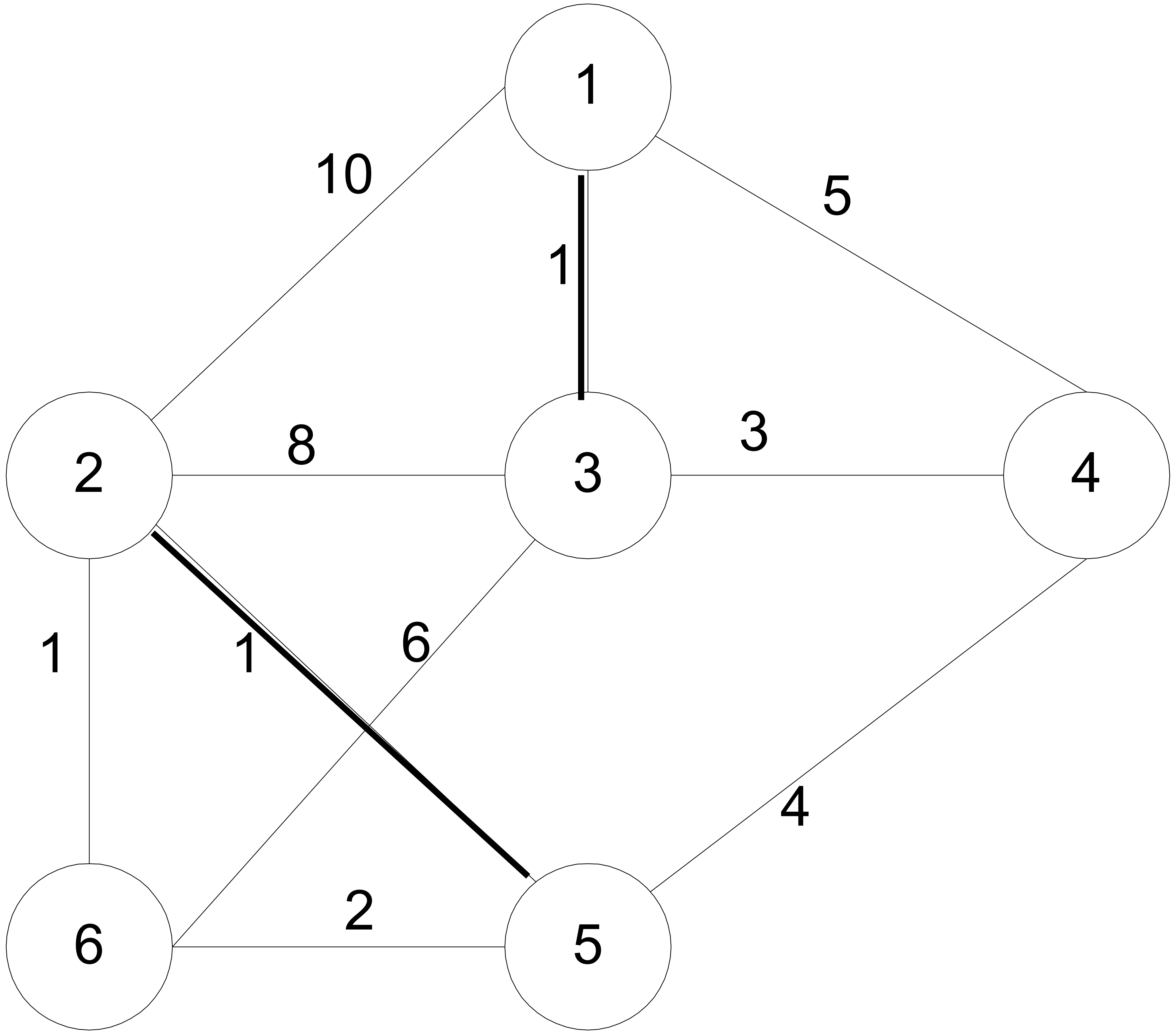
Kruskal’s Algorithm



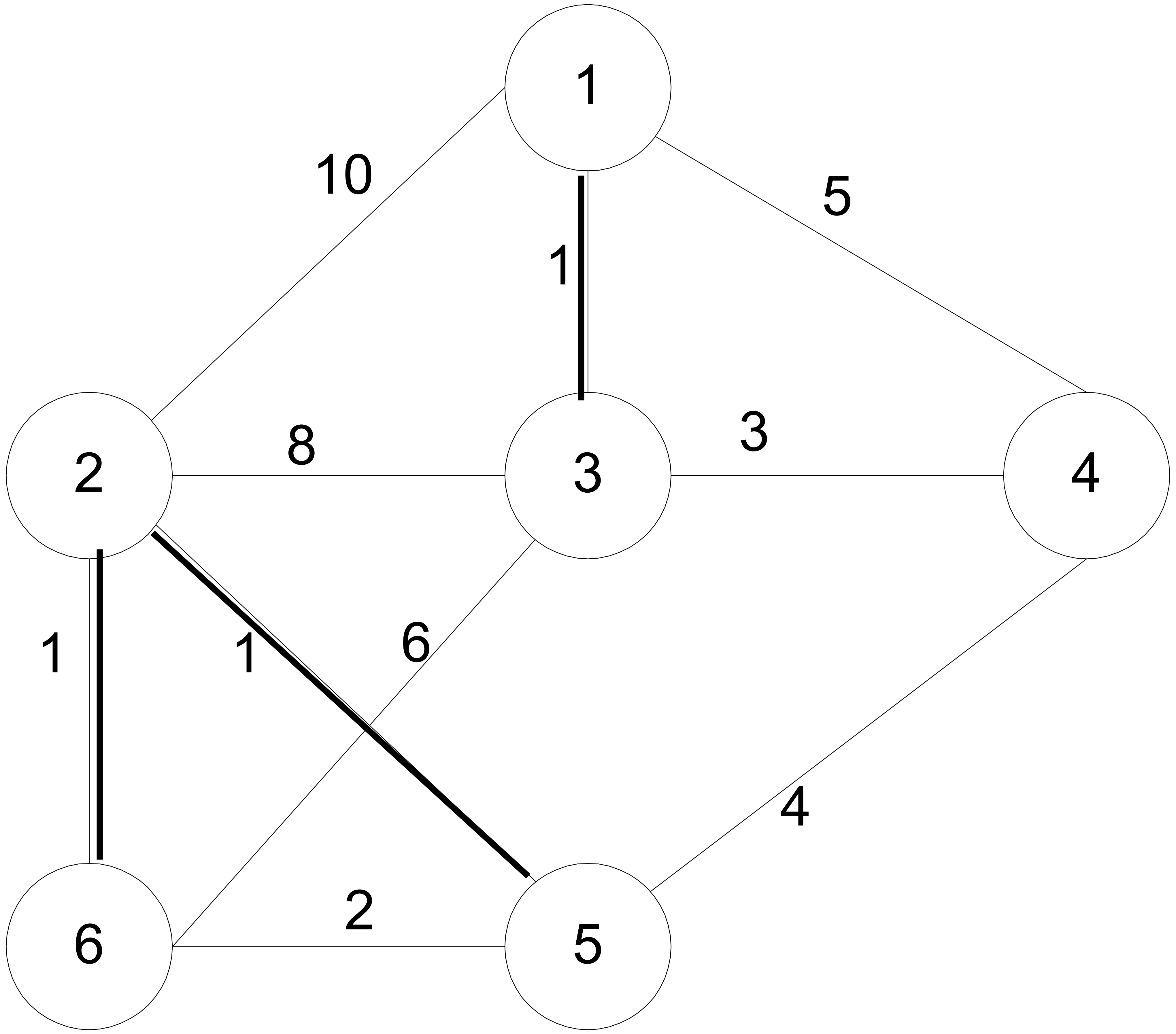
Kruskal’s Algorithm



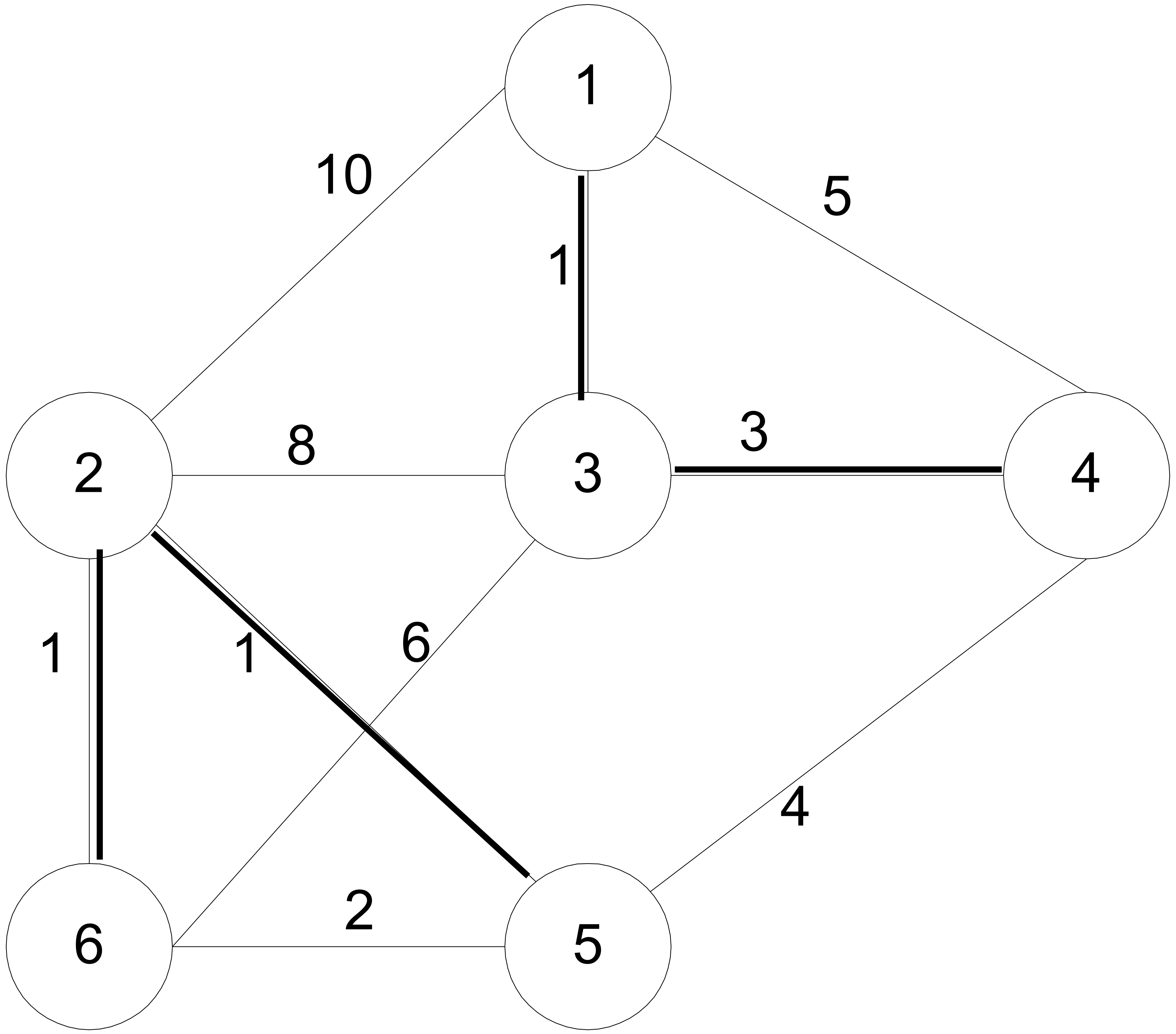
Kruskal's Algorithm



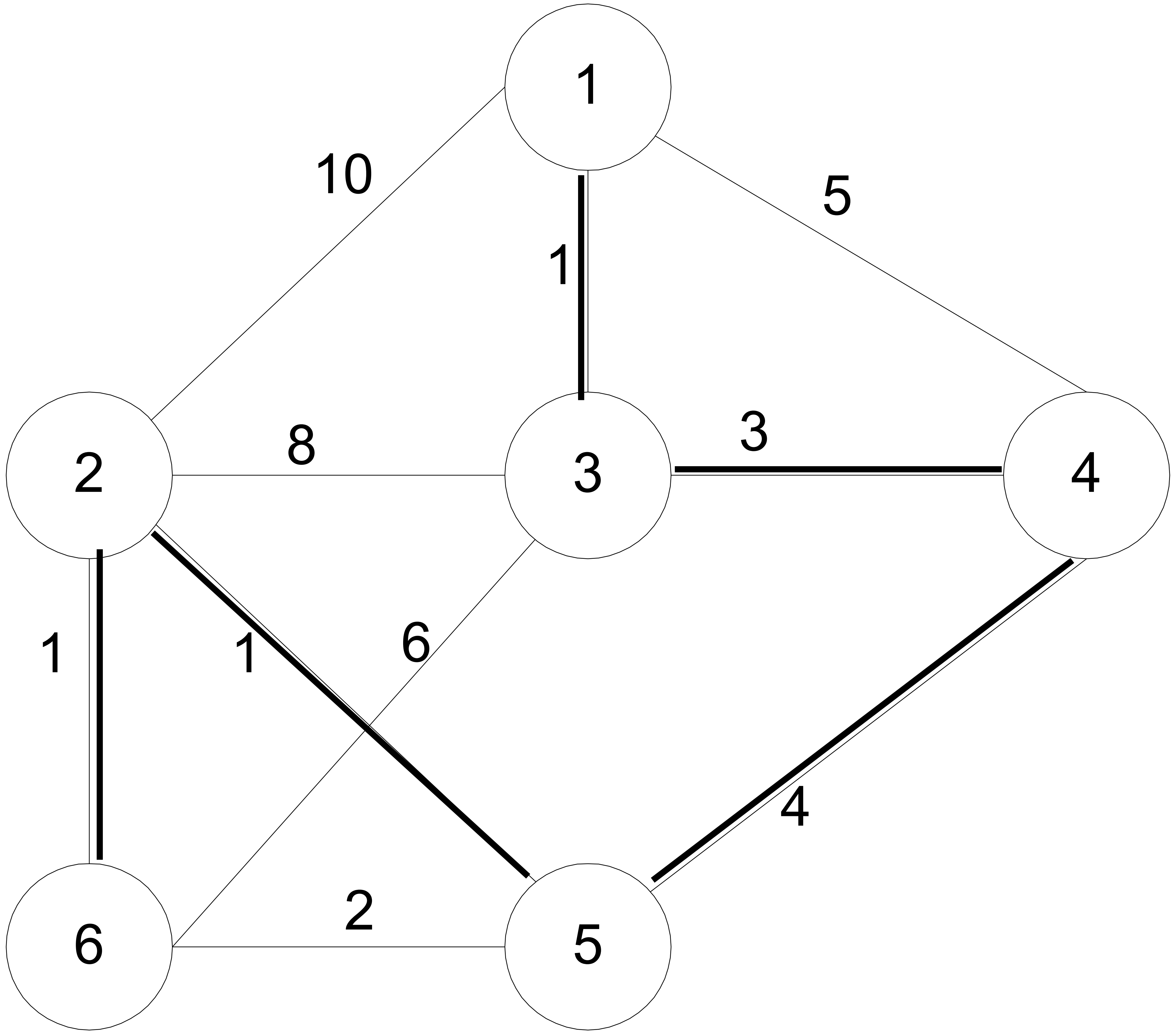
Kruskal's Algorithm



Kruskal’s Algorithm



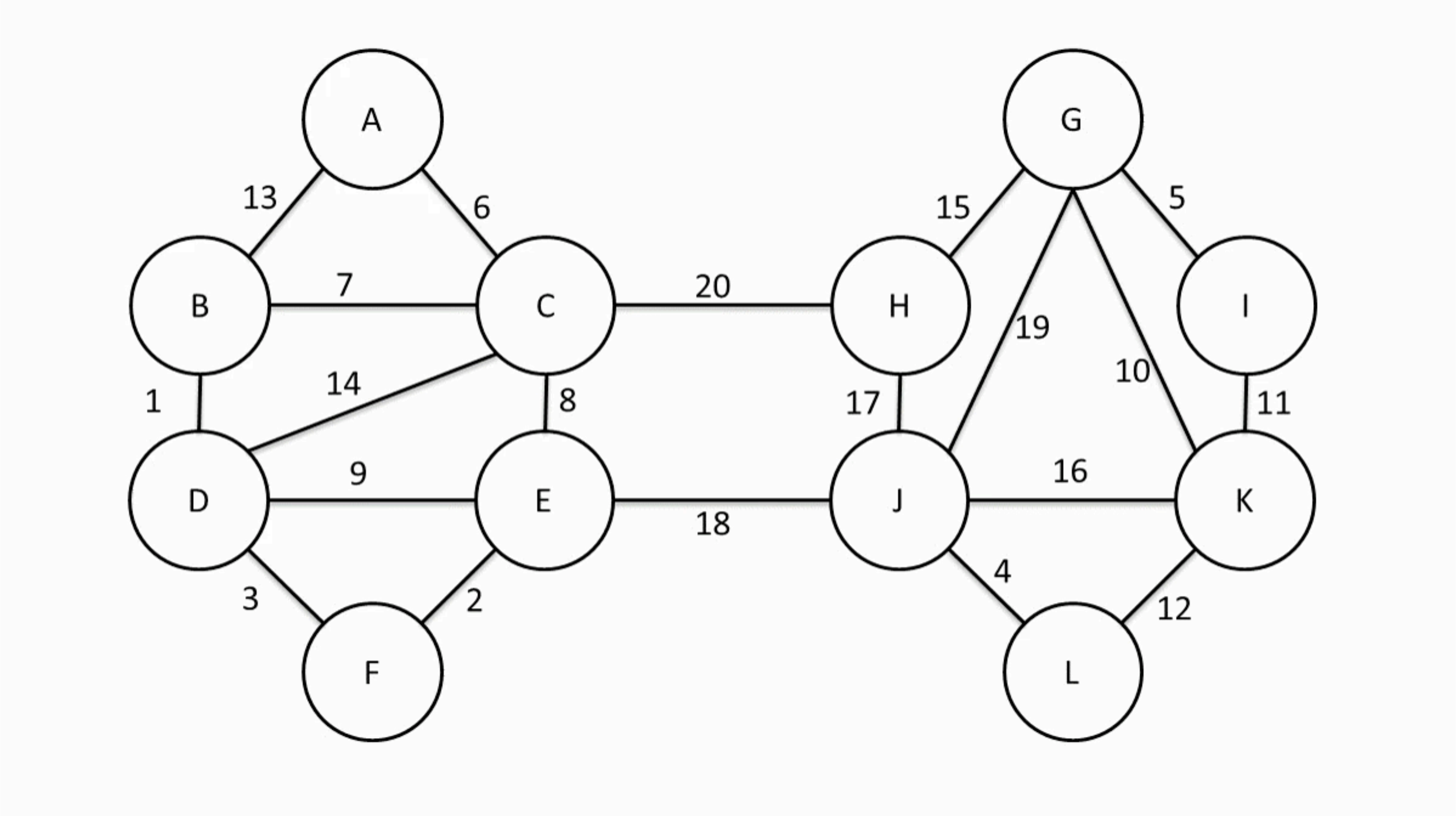
Kruskal's Algorithm



Boruvka’s Algorithm

- Combination of Prim’s and Kruskal’s
- Grow a tree (component) by picking the lightest edge connected to it, just like Prim
- Connect the trees when the lightest edge is between them, just like Kruskal
- Growing of each tree can be done in parallel
- Component contraction
    - Each component represented by a single node
    - When connecting two components, contract the edge and make a single node to represent the two

Boruvka’s Algorithm



Animation: Randy Cornell, Texas State University

Parallel Boruvka's Algorithm

- Java threads or async tasks picking up components off the worklist
  - You don't want too many threads of tasks, tune for the machine
  - Worklist has to allow concurrent access
- Grow components in parallel
- When inspecting the closest node to expand the component, have to synchronize
  - Other thread or task could be also accessing it
  - Careful not to introduce deadlock
- When contracting an edge, have to synchronize
- When there's only a single component left, you are done