



# Data definitions and Conditionals

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# Recap of Previous Lecture

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- Primitive types and values
  - numbers, booleans, symbols
- Variable definitions (constants), function definitions
- Operators
  - Arithmetic, relational, function application
- Rules for reducing programs
  - Leftmost reduction
- Syntax Errors & Runtime Errors
- Conditional Expressions



# Challenge Problem from Previous Lecture

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Can you think of a Scheme program that exhibits different behaviors with rightmost reduction instead of leftmost?

Consider the following example:

```
(+ (/ 1 0) (+ 'A 12))
```

Error conditions can make reasoning about programs more difficult than in naive mathematics e.g., you may not preserve program behavior by replacing `(* 0 (f x))` by `0`. What if evaluating `(f x)` generates an error?

Another example: given a non-terminating function **omega**:

```
(+ (/ 1 0) (omega 0))
```



# Goals of this lecture

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- Defining compound data (Scheme *structs*)
- Template for processing *structs*
- Union (mixed) data definitions
- Conditionals
- Template for processing union data.
- Inductive (self-referential) data definitions
- Template for processing inductive data



# Simple Data Definitions

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- How do we define new forms of data in Scheme? For example, say we want to write a program for the registrar that maintains a directory of courses that can be searched ...
- Informal rose description

“A complex number is a pair with a real part and an imaginary part , which are both numbers ”
- Corresponding data definition in Scheme

```
;; Complex is a structure (make-Complex real imag)
;; where real and imag are numbers
;; NOTE: the type complex is primitive in Scheme so we
;;      capitalize the name to avoid syntax errors
(define-struct Complex (real imag))
```
- A Scheme *struct* is a tuple tagged with the struct name
- Scheme processes this definition by creating the following operations:
  - *constructor*: `make-Complex`,
  - *accessors*: `Complex-real`, `Complex-imag`
  - *recognizer*: `Complex?` (which checks the tag)



# Structs Can Represent Compound Data

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In the struct definition

```
(define-struct Complex (real imag))
```

**real** and **imag** are called **fields**.

If a struct has more than one field, it is a compound form of data because it more than one internal part. A struct with  $k$  fields can be thought of as a box with  $k$  compartments where each compartment is labeled with a distinct field name.

For example, the struct **Complex** has two fields (compartments) called **real** and **imag**.



# Operations on Structures

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Recall that the following operations are automatically generated from the **define-struct** declaration for **Complex**

- *constructor*: **make-Complex**
- *accessors*: **Complex-real**, **Complex-imag**
- *recognizer*: **Complex?**

Sample reductions for these field accessors and structure recognizers

```
(Complex-imag (make-Complex 1 2)) => 2  
(Complex? (make-Complex 3 4)) => true
```



# Structures are values

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- In a program, the structure returned by a constructor is a value and its parts are values.
- Inside a structure, the parts *must* be values. The application of a struct constructor like **make-Complex** to some argument expressions evaluates these arguments to produce *values*. At this point, the struct application becomes a *value* because all of the parts in its compartments are values.
- For example:
  - (**make-Complex** 0 (+ 2 2)) is a constructor application--not a value because the argument expression (+ 2 2) is not a value.
  - (**make-Complex** 0 (+ 2 2)) => (**make-Complex** 0 4)
  - (**make-Complex** 0 4) is a *value*. Why?
  - (**make-Complex** x y) is *not* a value. Why?





# Evaluation Rules for Structures

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Given the data definition

```
(define-struct Complex (real imag))
```

Scheme supports the following reduction rules:

```
(Complex-real (make-Complex Val1 Val2)) => Val1
```

```
(Complex-imag (make-Complex Val1 Val2)) => Val2
```

```
(Complex? (make-Complex Val1 Val2)) => true
```

```
(Complex? Val3) => false
```

where `val1 val2` are Scheme values and `val3` is not of the form `(make-Complex v1 v2)`



# The Design Recipe

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How should I go about writing programs?

1. Analyze problem and **define** any requisite **data** types
2. State the **type contract** and **purpose** for *function* that solves the problem
3. Give **examples** of function use and result
4. Select and **instantiate** a **template** for the function body
5. Write the **code** for the function.
6. **Test** the code, and confirm that tests succeeded

The order of the steps of the recipe is important. In DrScheme, steps 3 and 6 can be collapsed because the examples can be presented as calls on **check-expect**. DrScheme does not evaluate these tests until the end of the program text.



# Template for Defined Data Type

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- We start from the data definition. Example:  

```
;; A Complex is a structure (make-Complex real imag)  
;; where real and imag are numbers  
(define-struct Complex (real imag))
```
- General template for any function processing an argument of type **Complex**  

```
;; (define (f c)  
;;   ... (Complex-real c) ...  
;;   ... (Complex-imag c) ...)
```
- Type contracts for some possible functions on **Complex**  

```
;; magnitude : Complex -> number  
;; Complex-zero? : Complex -> bool  
;; conjugate: Complex -> Complex
```



# Example: write conjugate function

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Assume that we have already defined the **Complex** type  
include a template for functions that process inputs of type  
**Complex**.

```
;; Type contract
;; conjugate: Complex -> Complex

;; Examples:
(check-expect (conjugate (make-Complex 0 0)) (make-Complex 0 0))
(check-expect (conjugate (make-Complex 0 1)) (make-Complex 0 -1))
(check-expect (conjugate (make-Complex 0 -1)) (make-Complex 0 1))
(check-expect (conjugate (make-Complex 1 -1)) (make-Complex 1 1))
(check-expect (conjugate (make-Complex -1 1)) (make-Complex -1 -1))
```



# Data Type → Template → Template Instantiation → Code

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Instantiation of Complex template for magnitude?

```
;; Template instantiation
;; (define (conjugate c)
;;      ... (Complex-real c) ...
;;      ... (Complex-imag c) ...)
```

- This template instantiation is trivial but more complex examples are not.
- Templates help us write the code

```
(define (conjugate c)
  (make-Complex (Complex-real c) (- (Complex-imag c))))
```

- Sophisticated types -> sophisticated templates ...  
helping us write correct, sophisticated code



# Union (Mixed) Data Definitions

How can we define data types that include more than one kind of data?

- Use the notion of *disjoint* set union from mathematics

- Example:

```
;; A shape is either:
;;   a square (make-square s) with side s,
;;   an equilateral triangle (make-triangle s) with
;;     side s, or
;;   a circle (make-circle s) with diameter s,
;; where s is a number and square, triangle, and circle
;; are structs defined as follows.
;; (define square (size))
;; (define triangle (size))
;; (define circle (size))
```

- This data definition can be abbreviated as follows:

```
;; shape ::= (make-square s) | (make-triangle s) |
;;           (make-circle s)
;; where s is a number and square triangle, and circle...
```



# Template for Union (Mixed) Data

For the type defined on the previous slide, the general template is:

```
; (define (f ... ashape ...)
;   (cond
;     [(square? ashape) ... (square-size ashape) ...]      ;; square case
;     [(triangle? ashape) ... (triangle-size ashape) ...]  ;; triangle case
;     [(circle? ashape) ... (circle-size ashape) ...]))    ;; circle case
```

Processing mixed data requires a conditional to direct control to the appropriate case code. Note that **cond** is critical because it directs evaluation to the appropriate code, ignoring irrelevant clauses.

The template for an arbitrary union (assuming each construction is unary)

```
;; mixed-type ::= (make-S1 field) | ... | (make-SN field)
```

and struct definitions for *S1*, ..., *SN*, the general template for processing data of this type is:

```
; (define (f ... amt ...)
;   (cond
;     [(S1? amt) ... (S1-field amt) ... ]      ;; S1 case
;     ...                                       ;; cases 2, ..., N-1
;     [(SN? amt) ... (SN-field amt) ... ]))    ;; SN case
```



# Inductive Data Definitions

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How can we generate arbitrarily large data objects like lists?

- Use self-reference (induction/recursion) in typing fields

- Example:

```
:: A list-of-numbers is either
```

```
::   empty, or
```

```
::   (cons n lon)
```

```
:: where n is a number and lon is a list-of-  
numbers
```

- If we assume that **empty** is a built-in (primitive) constant (like **true**), this definition can be implemented in Scheme by the *struct*

```
(define-struct cons (first rest))
```

where **make-** is elided from the constructor



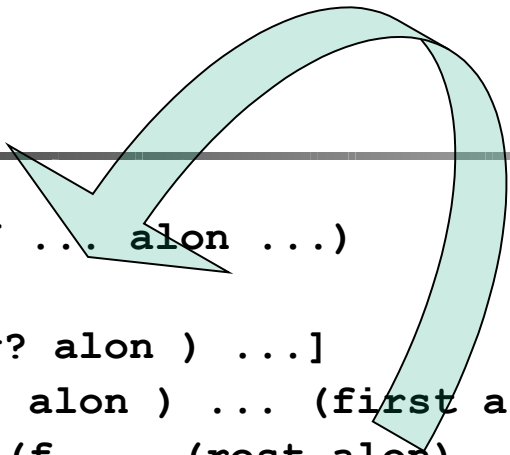
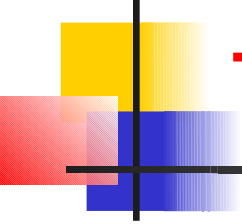


# Inductive Data Definitions

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The **cons** *struct* definition is built-in to Scheme; it is *primitive*. For the sake of brevity, the constructor is simply called **cons** rather than **make-cons** and the accessors are called **first** and **rest** rather than **cons-first** and **cons-rest**. Note that a Scheme *struct* definition does not stipulate the types of the fields of the structure. Hence, the programmer is responsible for ensuring that **cons** is used correctly. In our dialects of Scheme, **cons** ensures that its second argument is a list. (In the standard dialect, **first** is called **car** and **rest** is called **cdr** for historical reasons.)

# Template for Inductive Data Type



```
;; (define (f ... alon ...)
;;   (cond
;;     [(empty? alon ) ...]                ;; empty case
;;     [(cons? alon ) ... (first alon) ... ;; cons case
;;       ... (f ... (rest alon) ...) ...]))
```

- Processing inductive (self-referential) data requires recursion (self-reference) in the computation.
- What is `cond` ?
- This template for processing inductive data is an extension of the one on Slide 8 for processing a degenerate form of this template where there are multiple clauses (varieties) but no self-reference. The template is identical except for absence of the recursive call.



## Extended Example: Insertion Sort

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- Problem: given a list-of-numbers, sort it into ascending (non-decreasing) order.
- The solution that we will develop is the sample solution in the Scheme HW Guide.



# If Expressions

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- Simplified notation for common conditional expressions.
- Form:

`(if question result-1 result-2)`

abbreviates:

```
(cond [question  result-1]
      [else      result-2])
```

- Hence,

```
(if true result-1 result-2) => result-1
(if false result-1 result-2) => result-2
```



# Epilog

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- Reminder: work on HW01. Over the weekend, you should be able to complete problems from Section 8.3 and make substantial progress on the programs. They all process lists.
- Next class: data-directed design using other inductive types



# Example of help function

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```
; Type contract
; magnitude: Complex -> number
; Purpose: (magnitude c) computes the magnitude of the Complex number c, i.e.
; the L2 norm of (x,y) where c = (make-Complex x y)
; Examples
(check-expect (magnitude (make-Complex 0 0)) 0)
(check-expect (magnitude (make-Complex 3 4)) 5)
(check-expect (magnitude (make-Complex 4 3)) 5)
; Code
(define (magnitude c) (2norm (Complex-real c) (Complex-imag c))
; Type contract
; 2norm: number number -> number
; Purpose: (2norm x y) returns the L2 norm of the vector (x,y), i.e.
; (sqrt (+ (* x x) (* y y)))
; Examples:
(check-expect (2norm 0 0) 0)
(check-expect (2norm 3 4) 5)
(check-expect (2norm 4 3) 5)
; Template intsanction: trival
; Code:
(define (2norm x y) (sqrt (+ (* x x) (* y y))))
```