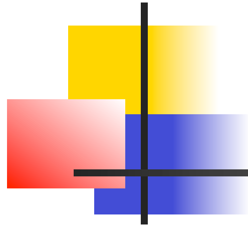
Data definitions

Prof. Robert “Corky” Cartwright

Prof. Vivek Sarkar

Department of Computer Science

Rice University



Recap of Previous Lecture

- Primitive types and values
  - numbers, booleans, symbols
- Variable definitions, function definitions
- Operators
  - Arithmetic, relational, function application
- Rules for reducing programs
  - Leftmost reduction
- Syntax Errors & Runtime Errors
- Conditional Expressions

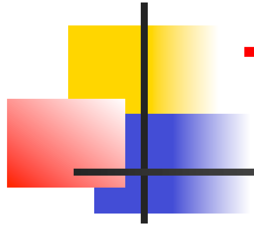
Challenge Problem from Previous Lecture

Can you think of a Scheme program that exibits different behaviors with rightmost reduction instead of leftmost?

Consider the following example:

```
(+ (/ 1 0) (+ 'A 12))
```

Error conditions can make reasoning about programs different from standard math e.g., you may not always preserve program behavior by replacing (* 0 (f x)) by 0



Today’s Goals

- Compound data definitions and templates
  - Structures
- Inductive (self-referential) compound data definitions and templates
  - Lists
- if expressions

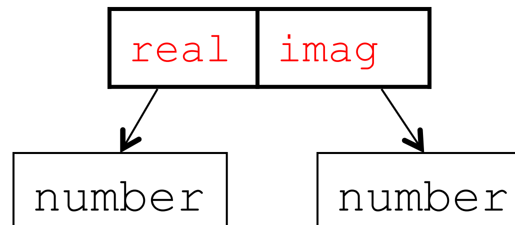
Compound Data: Structures

- Scheme structures can be used to combine a fixed number of values into a single piece of data e.g.,

- *Problem description*
  - “A complex number has a real part and an imaginary part”

- *Data definition*

```
;; cmplx is a structure (make-cmplx real imag)
;; where real and imag are real numbers
(define-struct cmplx (real imag))
```



The structure, cmplx, contains two numbers

Operations on Structures

- The following operations are automatically generated from the define-struct declaration for cmplx
  - *constructor:* `make-cmplx`
  - *accessors:* `cmplx-real`, `cmplx-imag`
  - *recognizer:* `cmplx?`

- Reductions for field accessors and structure recognizers

```
(cmplx-imag (make-cmplx 1 2)) => 2
(cmplx? (make-cmplx 3 4)) => true
```

Structures are values

- A structure returned by a constructor is a value (and hence is *not* reducible)
  - A structure is like a box with a value in each compartment
  - It may be big, but it's just like `1, true, or 'Rabbit`
  - It may be big, but it is NOT a reducible expression, like `(+ 1 2)`

- Notes:
  - `(make-cmplx 1 2)` is a value
  - `(make-cmplx x y)` is *not* a value (why not?)
  - `(make-cmplx 10 (+ 25 25))` is *not* a value (why not?)

An Aside: Converting our cmplx structure to a complex number in DrScheme

```
;; define the cmplx structure
(define-struct cmplx (real imag))

;; define eval function for converting cmplx to a Scheme
   number
(define (eval z) (+ (cmplx-real z)
                    (* (cmplx-imag z) (sqrt -1))))

;; define variables C1, C2
(define C1 (make-cmplx 1 2))
(define C2 (make-cmplx 1 -2))

;; evaluate C1 and C1*C2
> (eval C1)
1+2i
> (* (eval C1) (eval C2))
5
```

Template for Defined Data Types

- We start from the data definition. Example:

```
;; A course is a structure (make-course dept num size)
;; where dept is a symbol, and num and size are numbers
(define-struct course (dept num size))
```

- A function template must include a model of all operations that can be performed on input structure arguments e.g., here's a template for function `f` with argument `c` of type course

```
;; (define (f c)
;;      ... (course-dept c) ...
;;      ... (course-num c) ...
;;      ... (course-size c) ...)
```

Type --> Template --> Code

- Template for function processing a `course`

```
;; (define (f ... c ... )
;;       ... (course-dept c) ...
;;       ... (course-num c) ...
;;       ... (course-size c) ...)
```

- Instantiation of template for `big-class?`

```
;; (define (big-class? c)
;;       ... (course-dept c) ...
;;       ... (course-num c) ...
;;       ... (course-size c) ...)
```

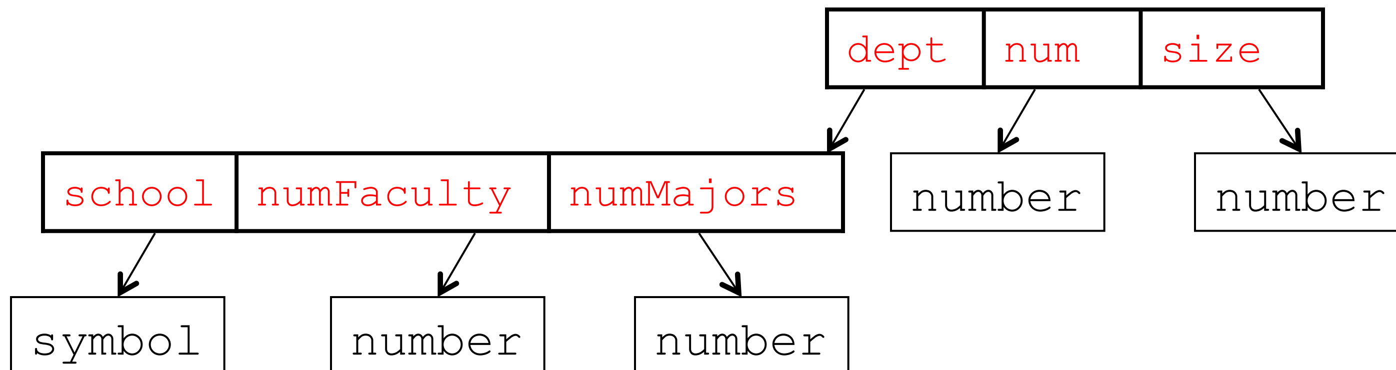
- Templates help us write the code

```
    (define (big-class? c) (>= (course-size c) 30))
```

- Sophisticated types -> sophisticated templates …
  helping us write correct, sophisticated code

Structures can be nested

```
(define-struct course (dept num size))
(define-struct department
          (school numFaculty numMajors))
```



Limitations of structures

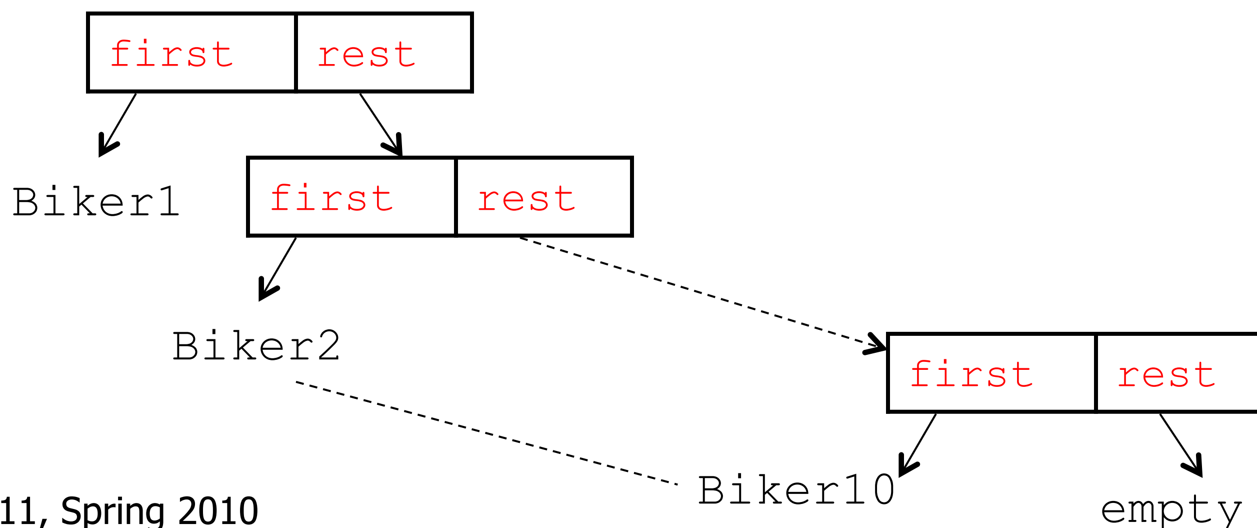
- Structures cannot contain variable numbers of elements
- Structures are impractical for large numbers of elements e.g.,

```
(define-struct BeerBikeTeam
  (rider1 rider2 … rider10
   chugger1 chugger2 … chugger10
   … )
)
```

Lists: defining Compound Data with Variable Number of Elements

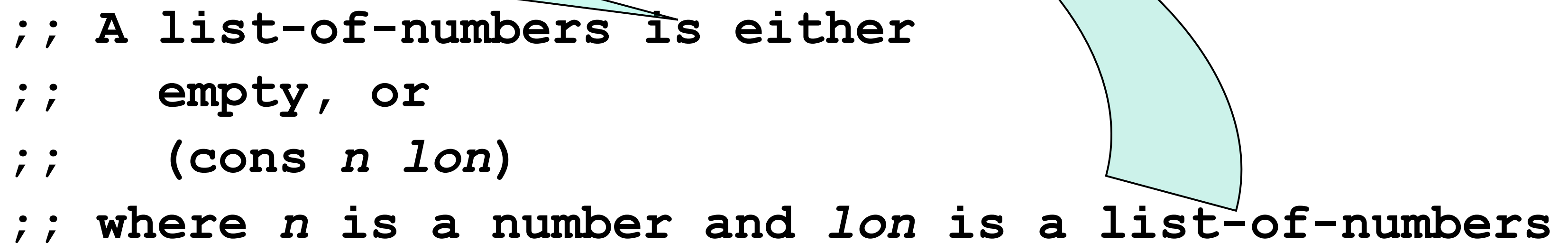
- How can we generate arbitrarily large data objects like lists?
- Use a two-element **`struct`** as a building block to chain together multiple elements e.g.,

```
(define-struct cons (first rest))
```



Inductive Data Definitions for Lists

- Use self-reference (induction/recursion)
- Example:



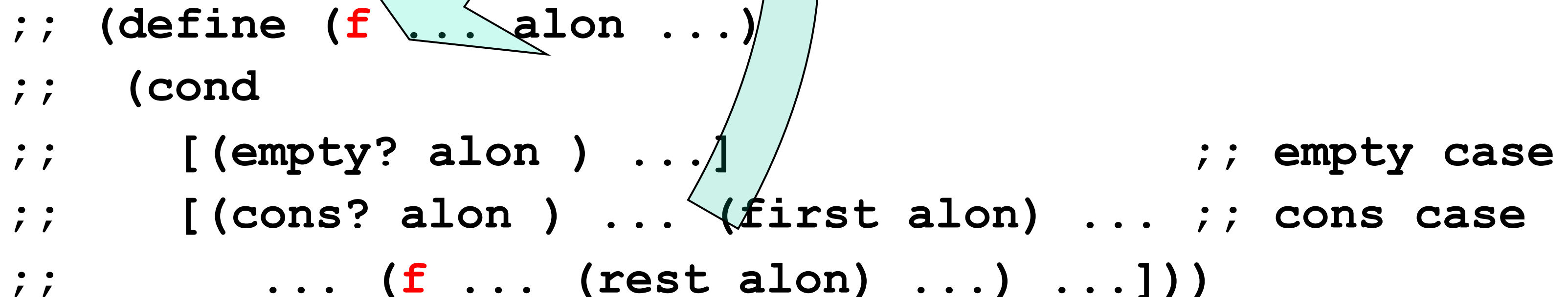
```
;; A list-of-numbers is either
;;   empty, or
;;   (cons n lon)
;; where n is a number and lon is a list-of-numbers
```

Built-in support for lists in Scheme

- **`cons`** is a built-in **`struct`** definition in Scheme, with special abbreviated names for its operations

Normal struct operation	**Equivalent list operation (what you should use)**
make-cons	**cons** Can also use **'(e1 e2 …)** as shorthand for a list, instead of nested cons operations
cons-first	**first**
cons-rest	**rest** (teaching dialects of Scheme also check that rest is a list)
cons?	**cons?**

Template for Inductive Data Type



```
;; (define (f ... alon ...)
;;   (cond
;;     [(empty? alon ) ...]                    ;; empty case
;;     [(cons? alon ) ... (first alon) ... ;; cons case
;;        ... (f ... (rest alon) ...) ...]))
```

- Processing inductive (self-referential) data requires recursion (self-reference) in the computation.

If Expressions

- Simplified notation for common conditional expressions.
- Form:

  (if *question result-1 result-2*)

  abbreviates:

```
(cond [question  result-1]
      [else      result-2])
```

- Hence,

```
(if true result-1 result-2) => result-1
(if false result-1 result-2) => result-2
```

Extended Example: Insertion Sort

- Problem: given a list-of-numbers, sort it into ascending (non-decreasing) order.
- The solution that we will develop is the sample solution in the Scheme HW Guide.
  - https://wiki.rice.edu/confluence/display/cswiki/211Guidelines

Auxiliary function: insert

```
;; Contract and purpose
;; insert: number list-of-numbers -> list-of-numbers
;; Purpose: (insert n alon), where alon is sorted in
   ascending order, returns a list containing n and the
   elements of alon also sorted in ascending order


;; Examples and Tests:
(check-expect (insert 17 empty) '(17))
(check-expect (insert 17 '(17)) '(17 17))
(check-expect (insert 4 '(1 2 3)) '(1 2 3 4))
(check-expect (insert 0 '(1 2 3)) '(0 1 2 3))
(check-expect (insert 2 '(1 1 3 4)) '(1 1 2 3 4))
```

Auxiliary function: insert (contd)

```
#| Template instantiation
   (define (insert n a-lon)
     (cond
       [(empty? a-lon) ...]
       [(cons? a-lon) ... (first a-lon) ...
                      ... (insert n (rest a-lon)) ... ]))
|#
;; Code
   (define (insert n a-lon)
     (cond
       [(empty? a-lon) (cons n empty)]
       [(cons? a-lon)
        (if (<= n (first a-lon)) (cons n a-lon)
            (cons (first a-lon) (insert n (rest a-lon))))]))
```

Main function: sort

```
;; Main function: sort

;; Contract and purpose:
;; sort: list-of-numbers -> list-of-numbers
;; Purpose: (sort alon) returns the a list with same elements
   (including duplicates) as alon but in ascending order.

;; Examples and Tests:
(check-expect (sort empty) empty)
(check-expect (sort '(0)) '(0))
(check-expect (sort '(1 2 3)) '(1 2 3))
(check-expect (sort '(3 2 1)) '(1 2 3))
(check-expect (sort '(10 -1 10 -20 5)) '(-20 -1 5 10 10))
```

Main function: sort (contd)

```
#| Template Instantiation:
   (define (sort a-lon)
     (cond
       [(empty? a-lon) ...]
       [(cons? a-lon) ... (first a-lon) ...
                      ... (sort (rest a-lon)) ... ]))
|#
;; Code:
   (define (sort a-lon)
     (cond
       [(empty? a-lon) empty]
       [(cons? a-lon) (insert (first a-lon) (sort (rest a-lon)))]))
```

The Design Recipe (Again!)

How should I go about writing programs?

1. Analyze problem and define any requisite data types
2. State contract (type) and purpose for *function* that solves the problem
3. Give examples of function use and result
4. Select and instantiate a template for the function body
5. Write the function itself
6. Test it, and confirm that tests succeeded

The order of the steps of the recipe is important

Announcements

- Monday (Jan 18th) is a holiday
  - No lecture on Monday
  - Monday labs have been rescheduled to Wednesday (Jan 20th)
- Reminder: work on HW01, due at 10am on Jan 22nd
  - See course homework guidelines for details, especially hand evaluation problems
    - https://wiki.rice.edu/confluence/display/cswiki/211Guidelines
- Next class on Jan 20th: data-directed design using other inductive types