COMP 515: Advanced Compilation for Vector and Parallel Processors

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https://wiki.rice.edu/confluence/display/PARPROG/COMP515

Homework #3 (REMINDER)

1. **Solve exercise 5.6 in book**
   - Your solution should be legal for all values of K (note that the value of K is invariant in loop I)

Exercise 5.6: What vector code should be generated for the following loop?

```
DO I = 1, 100
    A(I) = B(K) + C(I)
    B(I+1) = A(I) + D(I)
END DO
```

- **Due on Oct 8th**

Coarse-Grain Parallelism (contd)

Chapter 6 of Allen and Kennedy

- **Acknowledgment: Slides from previous offerings of COMP 515 by Prof. Ken Kennedy**
  - **http://www.cs.rice.edu/~ken/comp515/**

Scalar Privatization (Recap)

- The analog of scalar expansion is privatization.
- Temporaries can be given separate namespaces for each iteration.

```
    DO I = 1,N
S1     T = A(I)
S2     A(I) = B(I)
S3     B(I) = T
    ENDDO
```

```
    PARALLEL DO I = 1,N
      PRIVATE t
S1     t = A(I)
S2     A(I) = B(I)
S3     B(I) = t
    ENDDO
```

Array Privatization

- Array variables can be privatized as well (but the underlying analysis can be more complicated than for scalars)

```
    DO I = 1,100
S0      T(1)=X
L1      DO J = 2,N
S1        T(J) = T(J-1)+B(I,J)
S2        A(I,J) = T(J)
        ENDDO
    ENDDO
```

⇨

```
  PARALLEL DO I = 1,100
      PRIVATE t(N)
S0    t(1) = X
L1    DO J = 2,N
S1       t(J) = t(J-1)+B(I,J)
S2       A(I,J)=t(J)
      ENDDO
   ENDDO
```

Loop Distribution

- As we saw in Chapter 5, loop distribution can convert loop-carried dependences to loop-independent dependences.
- Consequently, it often creates opportunity for outer-loop parallelism.
- However, we must add extra barriers to keep distributed loops from executing out of order, so the overhead may override the parallel savings.

Loop Alignment

- Many carried dependencies are due to array alignment issues.
- If we can align all references, then dependencies would *go* away, and loop-level parallelism can be exposed.
- This is also related to Software Pipelining

```
DO I = 2,N
   A(I) = B(I)+C(I)
   D(I) = A(I-1)*2.0
ENDDO
```

→

```
DO I = 1,N   ! Aligned loop
   IF (I .GT. 1) A(I) = B(I)+C(I)
   IF (I .LT. N) D(I+1) = A(I)*2.0
ENDDO
```

→

```
D(2) = A(1)*2.0
DO I = 2,N-1   ! Pipelined loop
   A(I) = B(I)+C(I)
   D(I+1) = A(I)*2.0
ENDDO
A(N) = B(N)+C(N)
```

Code Replication

- **If two dependences between the same statements have different dependence distances, then alignment doesn't help.**
- **We can fix the second case by replicating code:**

```
DO I = 1,N
   A(I+1) = B(I)+C
   X(I) = A(I+1)+A(I)
ENDDO
```

```
DO I = 1,N
   A(I+1) = B(I)+C
   ! Replicated Statement
   IF (I .EQ 1) THEN
      t = A(I)
   ELSE
      t = B(I-1)+C
   END IF
   X(I) = A(I+1)+t
ENDDO
```

Strip Mining

- Converts available parallelism into a form more suitable for the hardware
- Assume THRESHOLD = minimum iters for parallel loop (due to overhead reasons)

```
        DO I = 1, N
            A(I) = A(I) + B(I)
        ENDDO
==>
        k = MAX(THRESHOLD, CEIL (N / P))
        PARALLEL DO I = 1, N, k
                DO i = I, MIN(I + k-1, N)
                        A(i) = A(i) + B(i)
                ENDDO
        END PARALLEL DO
```

Loop Fusion

- Loop distribution was a method for separating parallel parts of a loop.
- Our solution in Section 5 attempted to find the maximal loop distribution.
- The maximal distribution often finds parallelizable components too small for efficient coarse-grain parallelism.
- Two obvious solutions:
  - Strip mine large loops to create larger granularity (with an outer parallel loop and inner sequential loop)
  - Perform maximal distribution, and then fuse together parallelizable loops.
  - Both solutions can be combined as well.

Fusion Safety: Fusion-Preventing Loop-Independent Dependences

Definition: A loop-independent dependence between statements S1 and S2 in loops L1 and L2 respectively is fusion-preventing if fusing L1 and L2 causes the dependence to be carried by the combined loop in the opposite direction.

Example of an illegal loop fusion:

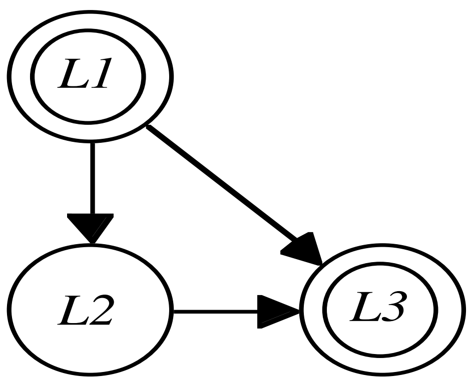
```
    DO I = 1,N
S1      A(I) = B(I)+C
    ENDDO
    DO I = 1,N
S2      D(I) = A(I+1)+E
    ENDDO
```

⇨

```
    DO I = 1,N
S1      A(I) = B(I)+C
S2      D(I) = A(I+1)+E
    ENDDO
```

Fusion Safety: Ordering Constraint

- **We shouldn't fuse loops if the fusion will result in an illegal ordering of the dependence graph.**
- **Ordering Constraint: Two loops can't be legally fused if there exists a path of loop-independent dependencies between them containing a loop or statement not being fused with them i.e., if fusion will result in a cycle in the resulting loop-independent dependences**



Fusing L1 with L3 violates the ordering constraint. {L1,L3} must occur both before and after the node L2, which is not possible.

Fusion Profitability

Parallel loops should generally not be merged with sequential loops.

Definition: An edge between two statements in loops L1 and L2 respectively is said to be parallelism-inhibiting if after merging L1 and L2, the dependence is carried by the combined loop.

```
      DO I = 1,N
S1       A(I+1) = B(I) + C
      ENDDO
      DO I = 1,N
S2       D(I) = A(I) + E
      ENDDO
```

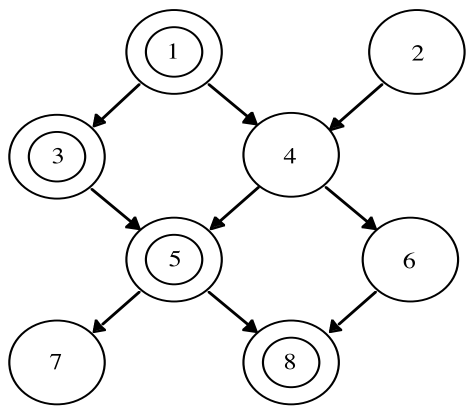
```
      DO I = 1,N
S1       A(I+1) = B(I) + C
S2       D(I) = A(I) + E
      ENDDO
```

Typed Fusion

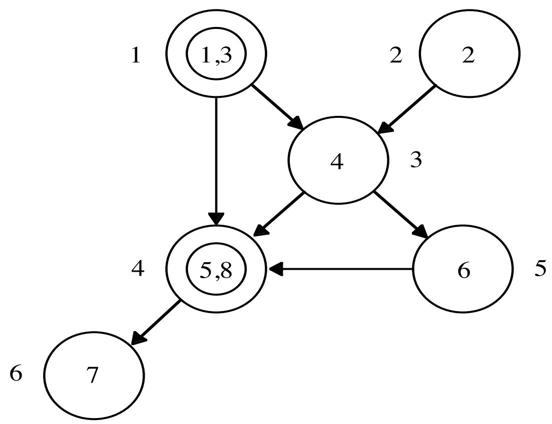
- We start by classifying loops into two types: parallel and sequential.
- We next gather together all edges that inhibit efficient fusion, (i.e., that connect a sequential and a parallel loops) and call them "bad edges".
- Given a graph of loop-independent dependences (V,E), we want to obtain a graph (V',E') by merging vertices of V subject to the following constraints:
  – Bad Edge Constraint: vertices joined by a bad edge aren't fused.
  – Ordering Constraint: vertices joined by path containing non-parallel vertex aren't fused

Typed Fusion Example

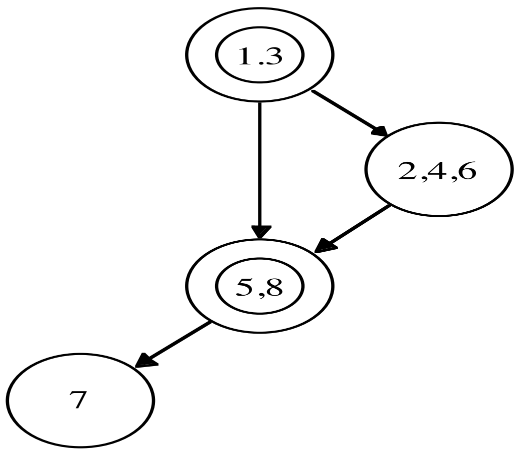
Original loop graph



After fusing parallel loops



After fusing sequential loops



Optimal Weighted Loop Fusion for Parallel Programs

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Proceedings of the 1997 ACM Symposium on Parallel Algorithms and Architectures (SPAA), Pages 282-291

Motivation

- Loop fusion is an important program transformation for scientific applications and stream-based applications

- The weighted loop fusion optimization problem is NP-hard

- Approximation solutions are hard because weights can be arbitrary and the program graph has no special structure

- Problem sizes are not very large in practice ($\leq$ 100 nodes)

- Computers are 1000$\times$ faster now compared to when NP-hardness was introduced

$\Rightarrow$ Our goal is to find optimal solutions in a tractable way

Loop Dependence Graph (LDG)

- The LDG is a directed **acyclic** dependence graph (built for a region of acyclic control flow across loop nests).

- **Node** = loop nest.

- **Edge** = data dependence from source node to destination node.

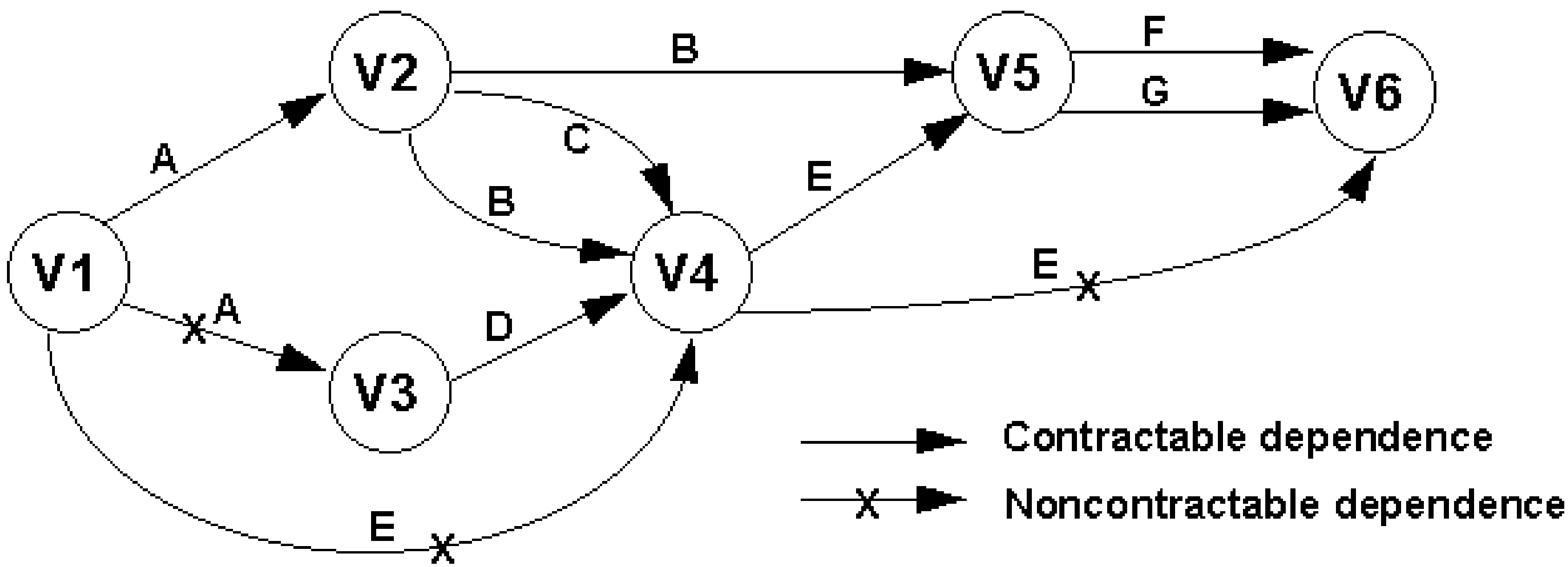
- Each LDG edge is marked as **contractable** or **noncontractable**.

  The source and destination loop nests of a noncontractable edge cannot be fused (otherwise a data dependence will be violated or a parallel loop will be serialized).

Example program

```
      PARDO 10 I = 1, N
10        A(I) =  E(I-1)
      PARDO 20 I = 1, N
          B(I) = A(I)*2 + 3
20        C(I) = B(I) + 99
      PARDO 30 I = 1, N
30        D(I) = A(N-I+1) + A(I)
      PARDO 40 I = 1, N
40        E(I) = B(I) + C(I) * D(I)
      PARDO 50 I = 1, N
          F(I) = B(I)*4 + 2
50        G(I) = E(I)*8 - F(I)
      PARDO 60 I = 1, N
60        H(I) = F(I) + G(I) * E(I-1)
```

LDG for example program



Fusion Partition

Fusion partition = partition of LDG nodes into disjoint fusion clusters

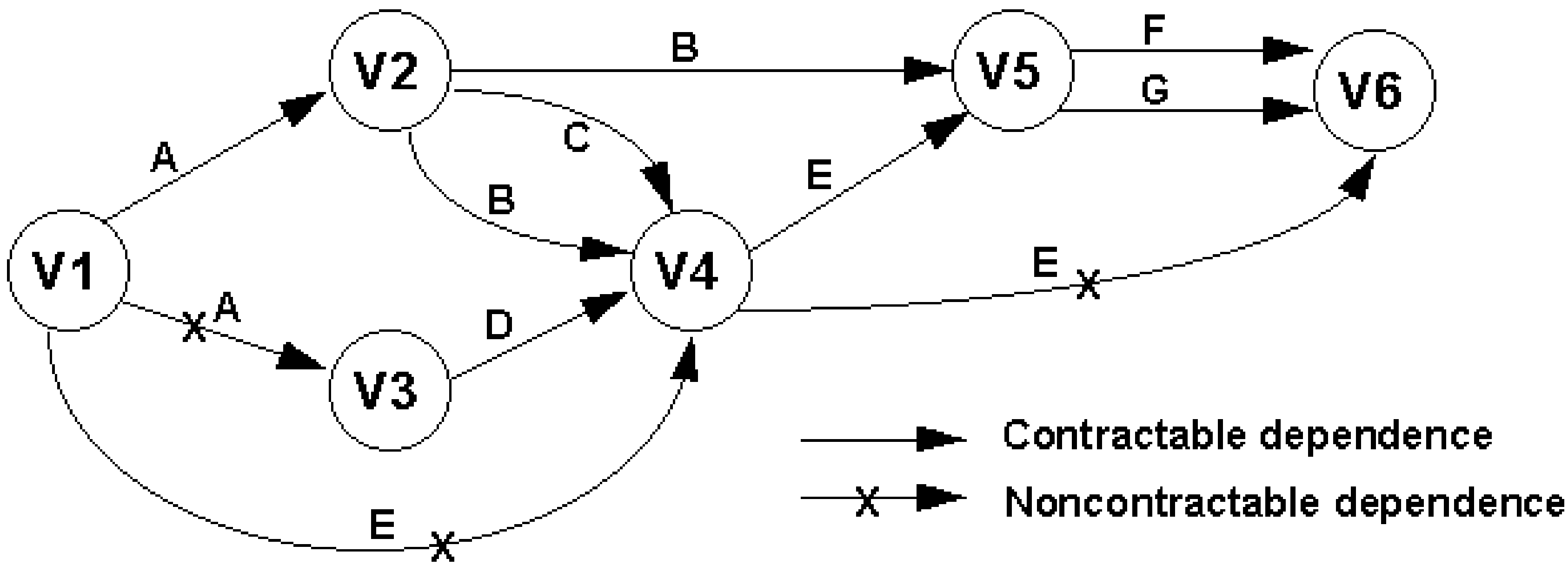
Fusion cluster = set of loop nests to be fused together

A fusion partition is **legal** if and only if:

1. The source and destination nodes of each **noncontractable** edge are placed in distinct clusters, and

2. The reduced graph is acyclic

Examples of Fusion Partitions



Cost of a Fusion Partition

$$\begin{aligned}\text{Define } w_{ij} &= \textbf{weight} \text{ of pair of nodes, } i \text{ and } j \\ &= \text{cost savings obtained by fusing nodes } i \text{ and } j\end{aligned}$$

$\Rightarrow$ cost of fusion partion P is given by

$$F(P) = \sum_{P(i) \neq P(j)} w_{ij}$$

where $P(i)$ = cluster number for node i in fusion partition P.

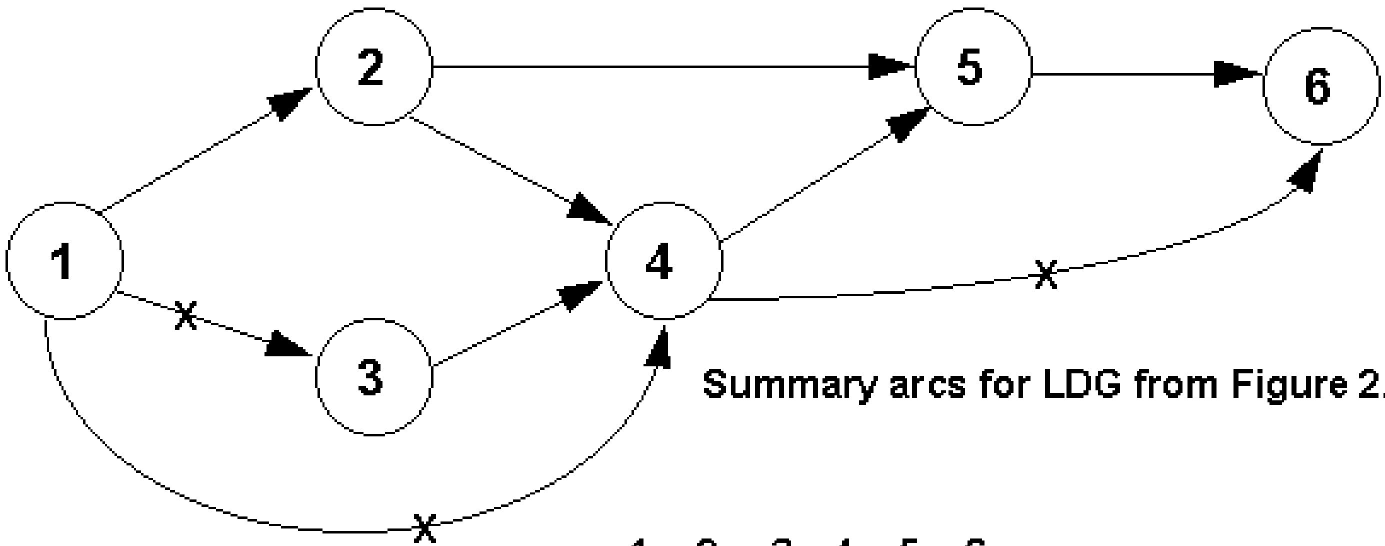
Cost of fusion partition = sum of inter-cluster w_{ij} weights

Computing Weights

Examples of computing weights in different applications of weighted loop fusion:

- **Common Loads:** set w_{ij} = number of common values in load instructions in loop nests i and j
- **Cache Locality:** set w_{ij} = number of common cache lines accessed by loop nests i and j
- **Remote data accesses:** set w_{ij} = number (and size) of common remote data access in loop nests i and j

Example of Cost of a Fusion Partition



Summary arcs for LDG from Figure 2.

	1	2	3	4	5	6
1	0	1	0	0	0	0
2	1	0	1	2	1	0
3	0	1	0	1	0	0
4	0	2	1	0	2	0
5	0	1	0	2	0	2
6	0	0	0	0	2	0

Table of weights, $w_{ij} = w_{ji}$

Problem Statement for Optimal Weighted Loop Fusion

Given an LDG and weights for pairs of nodes, find a legal fusion partition P with minimum cost, $F(P)$.

The optimized fusion configuration is obtained by fusing all loops belonging to the same cluster, and generating the fused loops in a topological sort order of the reduced LDG.

A Simple Integer Programming Formulation

- Introduce (0,1)-variables x_{ij} such that $x_{ij} = 0$ means that LDG nodes i and j are placed in the same cluster

- **Contractability constraint:** $x_{ij} = 1$ if there is a noncontractable LDG edge from node i to node j

- **Transitivity constraint:** $x_{ik} \leq x_{ij} + x_{jk}$

- Introduce integer variables π_i such that π_i represents a topologically sorted cluster numbering of LDG nodes

- **Equivalence constraint:** If $x_{ij} = 0$ then $\pi_i = \pi_j$. This constraint can be rewritten as $-n \cdot x_{ij} \leq \pi_j - \pi_i \leq n \cdot x_{ij}$.

- **Acyclicity constraint:** If there is an LDG edge from i to j, then $x_{ij} \leq \pi_j - \pi_i$

A Simple Integer Programming Formulation (contd.)

$$
\begin{array}{lrll}
\underset{x,\pi}{\text{Minimize}} & \displaystyle\sum_{(i,j)\in B\cup C} w_{ij}x_{ij} & & \\
\text{subject to} & x_{ik} \le x_{ij} + x_{jk} & \forall \text{ nodes } i,j,k \\
& x_{ij} \le \pi_j - \pi_i & \forall \text{ arcs } (i,j) \\
& -n\cdot x_{ij} \le \pi_j - \pi_i & \forall \text{ nodes } i,j \\
& \pi_j - \pi_i \le n\cdot x_{ij} & \forall \text{ nodes } i,j \\
& x_{ij} = 1 & \forall \text{ noncontractable arcs } (i,j) \\
& x_{ij} \in \{0,1\} & \forall \text{ nodes } i,j
\end{array}
$$

where $C \subseteq A$ is the set of *contractable* arcs and B is the set of unordered node pairs with nonzero weights

This formulation has $O(|N|^2)$ **variables and** $O(|N|^3|)$ **constraints**

More Efficient Formulation

Observations:

- The $x_{ik} \leq x_{ij} + x_{jk}$ triangular inequalities can be dropped without changing the set of feasible solutions.

- We only need to maintain x_{ij} variables for $(i, j) \in B \cup C$

- We only need the $x_{ij} \leq \pi_j - \pi_i$ inequalities for contractable arcs i.e., for $(i, j) \in C$

- We only need the $-n \cdot x_{ij} \leq \pi_j - \pi_i$ inequalities for unordered node pairs with nonzero weights i.e., for $\{i, j\} \in B$

More Efficient Formulation (contd.)

$$
\begin{aligned}
&\underset{x,\pi}{\text{Minimize}} \quad \sum_{(i,j)\in B\cup C} w_{ij}x_{ij} \\
&\text{subject to} \quad x_{ij} \le \pi_j - \pi_i \le n \cdot x_{ij} && \forall \text{ contractable arcs } (i,j) \\
&\qquad -n \cdot x_{ij} \le \pi_j - \pi_i \le n \cdot x_{ij} && \forall \text{ unordered node pairs } (i,j) \\
&&& \text{with nonzero weight} \\
&\qquad \pi_j - \pi_i \ge 1 && \forall \text{ noncontractable arcs } (i,j) \\
&\qquad x_{ij} \in \{0,1\} && ((i,j) \in B \cup C) \ .
\end{aligned}
$$

This formulation has $(|N| + |B| + |C|)$ **variables and** $(2|C| + 2|B| + |NC|)$ **constraints.**

Example of More Efficient Formulation

$$\underset{x,\pi}{\text{Minimize}} \quad x_{23} + (x_{12} + 2x_{24} + x_{25} + x_{34} + 2x_{45} + 2x_{56})$$

subject to

$$\begin{aligned}
x_{12} &\le \pi_2 - \pi_1 \le 6 \cdot x_{12} \\
x_{24} &\le \pi_4 - \pi_2 \le 6 \cdot x_{24} \\
x_{25} &\le \pi_5 - \pi_2 \le 6 \cdot x_{25} \\
x_{34} &\le \pi_4 - \pi_3 \le 6 \cdot x_{34} \\
x_{45} &\le \pi_5 - \pi_4 \le 6 \cdot x_{45} \\
x_{56} &\le \pi_6 - \pi_5 \le 6 \cdot x_{56} \\
-6 \cdot x_{23} &\le \pi_3 - \pi_2 \le 6 \cdot x_{23}
\end{aligned}$$

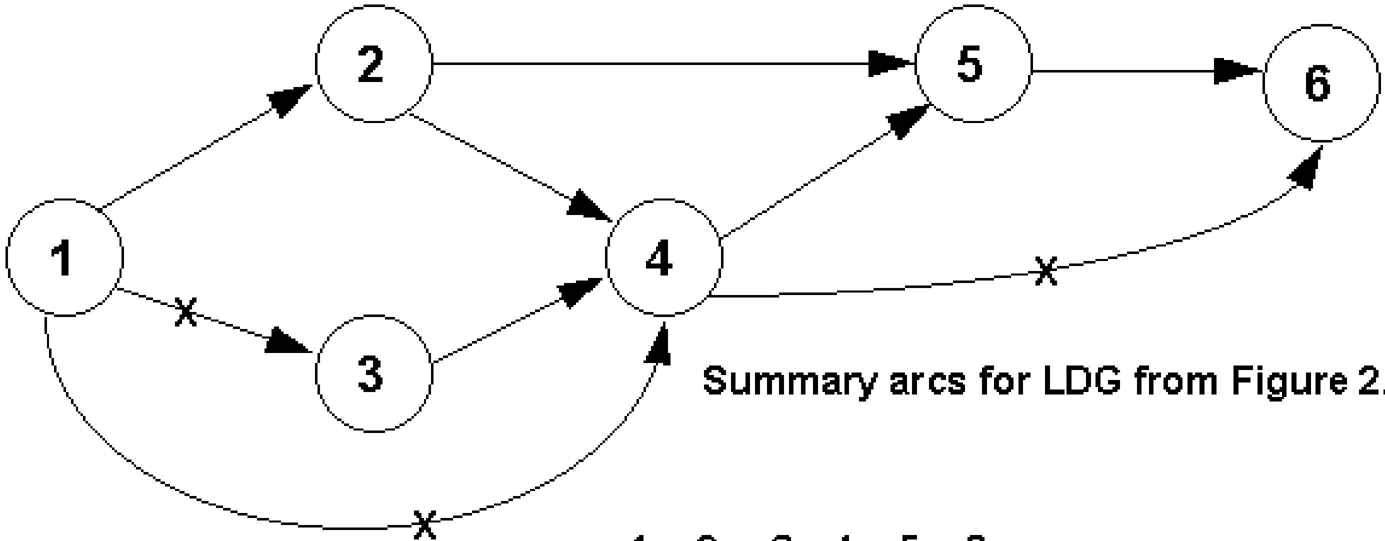
$$\begin{aligned}
\pi_3 - \pi_1 &\ge 1 \\
\pi_4 - \pi_1 &\ge 1 \\
\pi_6 - \pi_4 &\ge 1
\end{aligned}$$

Efficient formulation has 12 variables and 17 constraints

(Simple formulation has 18 variables and 99 constraints)

Optimal Weighted Loop Fusion solution for Example

$P = 1 \mid 2, 3, 4, 5 \mid 6$ (cost = 3)



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5	0	1	0	2	0	2
6	0	0	0	0	2	0

Table of weights, $w_{ij} = w_{ji}$

Optimal Weighted Loop Fusion solution for Example (contd.)

```
      PARDO 10 I = 1, N
10        A(I) =  E(I-1)

      PARDO 20 I = 1, N
          B(I) = A(I)*2 + 3
          C(I) = B(I) + 99
          D(I) = A(N-I+1) + A(I)
          E(I) = B(I) + C(I) * D(I)
          F(I) = B(I)*4 + 2
50        G(I) = E(I)*8 - F(I)

      PARDO 60 I = 1, N
60        H(I) = F(I) + G(I) * E(I-1)
```

Preliminary Experimental Results

Source of LDG	n	$\|B\|$	$\|A\|$	$\|C\|$	Total # iters	Time for total # iters (OSL)
Example	6	1	9	6	15	0.080 seconds
034.mdljdp2	12	0	12	9	19	0.050 seconds
Synthetic	100	0	90	80	60	0.140 seconds

Execution times for solving optimal weighted loop fusion problems using the IBM Optimization Subroutine Library on a 33MHz RS/6000 model 220 workstation.

Extensions

- **Cost term proportional to no. of clusters** — extend objective function to

$$F(P) = \sum_{P(i) \neq P(j)} w_{ij} + \mathrm{S} \times \text{(no. of clusters)}$$

  where S is the synchronization cost incurred per cluster

- **Conformability classes** — equivalence classes such that only nodes from within the same class are allowed to be fused.

  Conformability classes can be used to model non-loop statements, loops with nonconformable bounds, loops with premature exits, etc.

Extensions (contd.)

- **Hierarchical fusion** — apply algorithm recursively on LDG for the body of each fused loop.

- **Control dependences** — extend LDG to be an acyclic PDG. Each control dependence edge is noncontractable.

- **Branch-and-bound method** — compute bounds using linear programming relaxation described in paper.

  Branch-and-bound method automatically stores the best feasible solution seen till current point in time, and can be more efficient than using an optimization library.

Related Work

- **[Allen & Cocke '72]** Introduced loop fusion transformation.
- **[Goldberg & Paige '84]** Showed how loop fusion can be used to optimize stream processing in database queries.
- **[Callahan '87]** Greedy merge algorithm for *unweighted* loop fusion (minimizing the number of clusters).
- **[Gao et al '92]** Heuristic solution to weighted loop fusion using repeated applications of max-flow min-cut algorithm
- **[Kennedy & McKinley '93]** NP-hardness proof for weighted loop fusion. Experimental results show 4–17% improvement in uniprocessor execution times with heuristic algorithm.

Conclusions

- Presented an integer programming formulation for weighted loop fusion

- Size of formulation is linear in size of LDG and weights

- Preliminary execution time measurements show that optimal weighted loop fusion is tractable to solve in practice

Future Work

- Extend prototype implementation by making calls to optimization subroutine library from within the compiler
- Compare performance of heuristic and optimal solutions on Fortran 90 programs for SMPs
- Extend weighted loop fusion model by adding capacity constraints
- Extend branch-and-bound algorithm with incremental recomputation of edge weights
- Investigate development of tractable optimal algorithms for other NP-hard problems in compiler optimization