COMP 515: Advanced Compilation for Vector and Parallel Processors

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Transformation Frameworks

- Goal: develop a unified transformation framework in which legality testing and code generation for different transformations can be unified
  - Textbook approach: catalog of (AST-based) transformations
    - Pro: Generality
    - Con: each transformation needs special-case handling
  - Lecture 19: polyhedral transformations
    - Pro: more general than unimodular transformations (includes many cases of loop distribution and fusion)
    - Con: limited to transformation of "static control parts" (SCoP's)
  - Lecture 18: IBM ASTI optimizer
    - Pro: more general than unimodular and some cases of polyhedral
    - Pro: cost-based framework for automatic selection of transformations
    - Con: no unified framework for combining AST-based transformations beyond iteration-reordering, e.g., loop distribution & fusion

Transformation Framework Case Studies

1. IBM ASTI Optimizer
   - Automatic Selection of High Order Transformations in the IBM XL Fortran Compilers", V. Sarkar, IBM Journal of Res. & Dev., Vol. 41, No. 3, May 1997.
2. PolyOpt: Polyhedral + AST Optimizer
   - Oil and Water Can Mix: An Integration of Polyhedral and AST-based Transformations. Jun Shirako, Louis-Noel Pouchet, Vivek Sarkar. IEEE Conference on High Performance Computing, Networking, Storage and Analysis (SC'14), November 2014.

High-Order Transformations

Traditional optimizations operate on a low-level intermediate representation that is close to the machine level

High-order transformations operate on a high-level intermediate representation that is close to the source level

Examples of high-order transformations: loop transformations, data alignment and padding, inline expansion of procedure calls, ...

Selection of High-Order Transformations

Improperly selected high-order transformations can degrade performance to levels worse than unoptimized code.

Traditional optimizations rarely degrade performance.

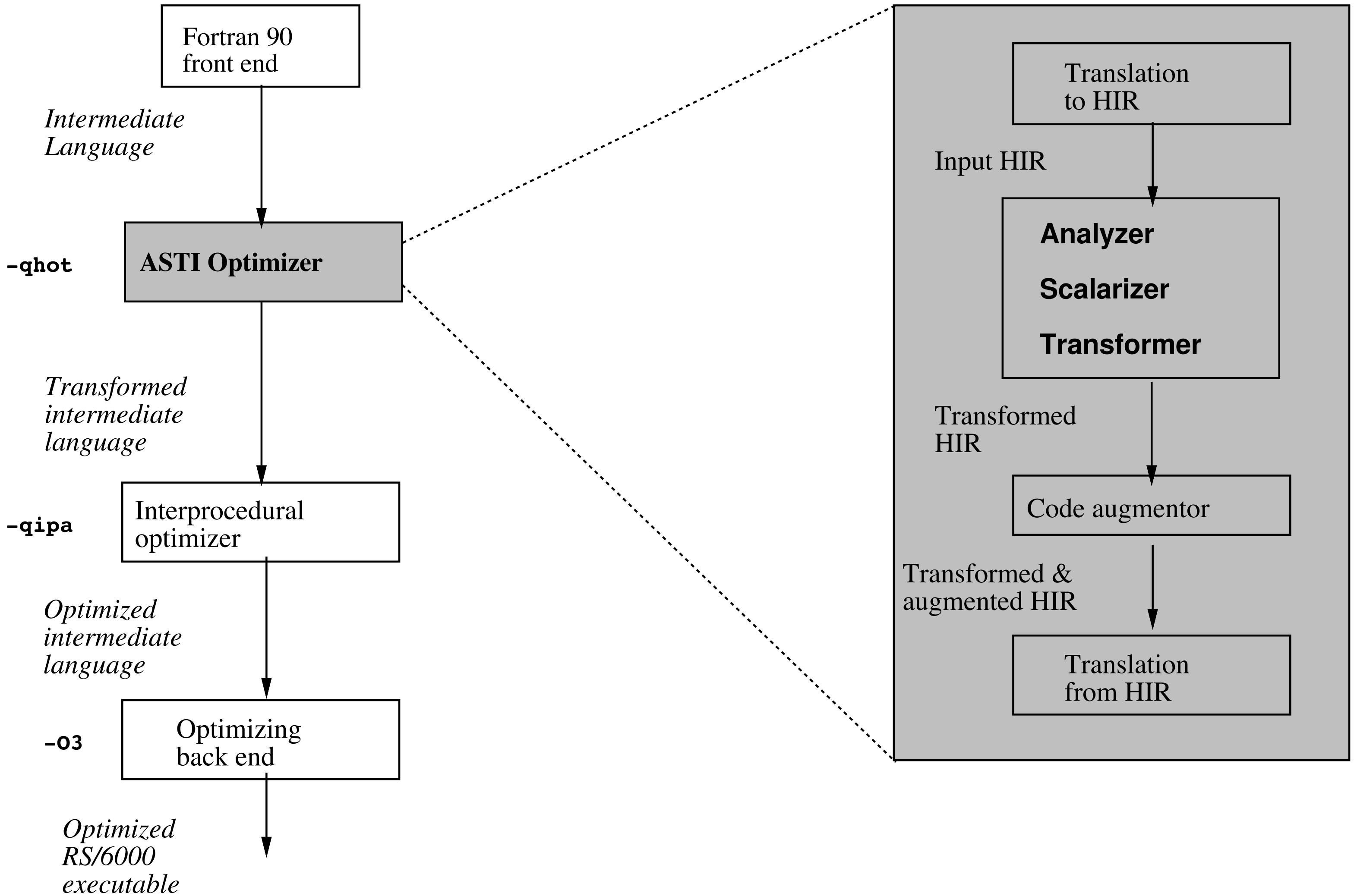
$\Rightarrow$ automatic selection has to be performed more carefully for high-order transformations than for traditional optimizations

This Work

- Automatic selection of high-order transformations in the IBM XL Fortran compilers
- Quantitative approach to program optimization using cost models
- High-order transformations selected for *uniprocessor* target include: loop distribution, fusion, interchange, reversal, skewing, tiling, unrolling, and scalar replacement of array references
- Design and initial product implementation completed during 1991–1993

Reference: "Automatic Selection of High Order Transformations in the IBM XL Fortran Compilers", V. Sarkar, IBM Journal of Res. & Dev., Vol. 41, No. 3, May 1997. (To appear).

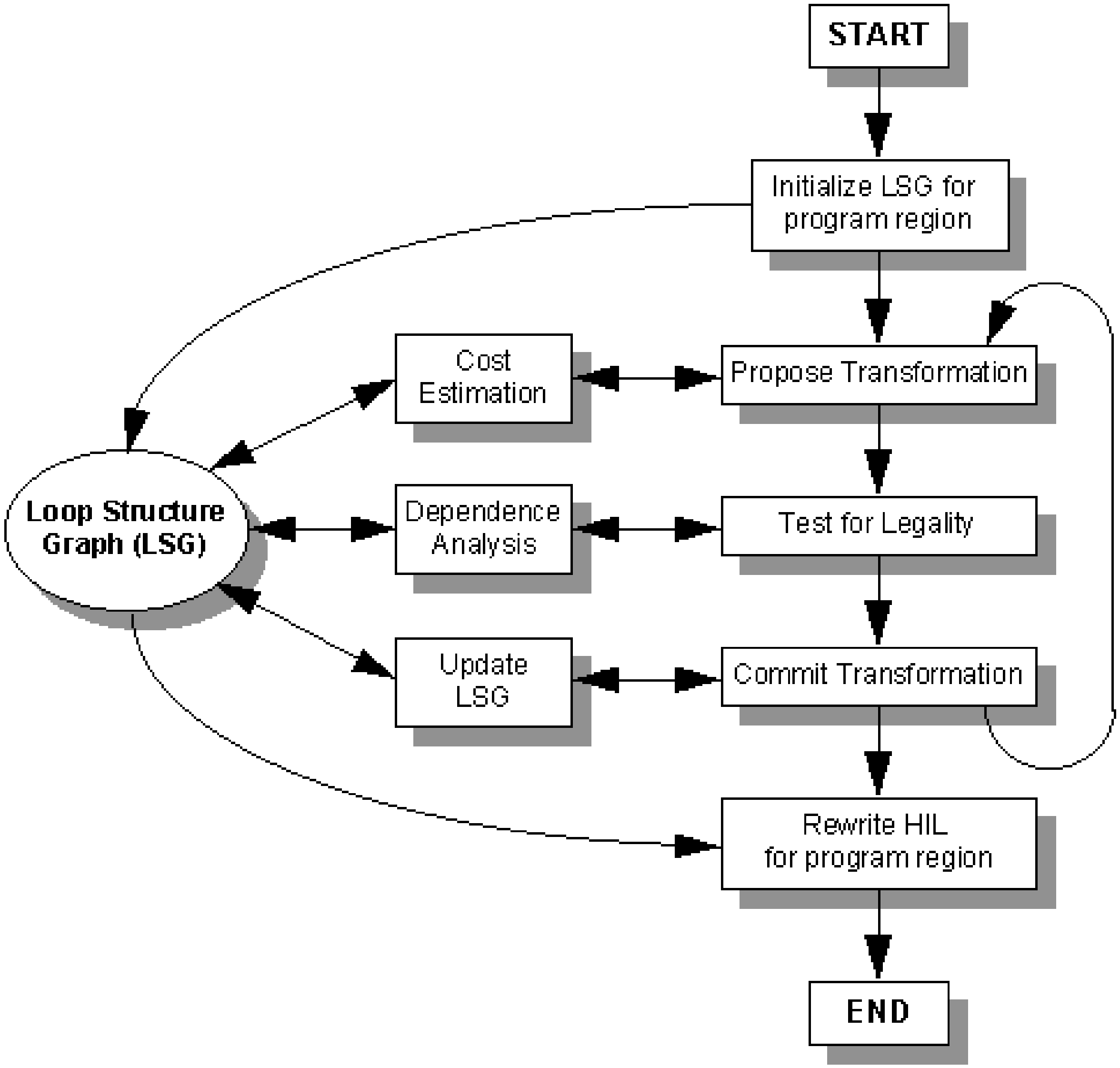
Structure of XL Fortran Product Compiler (Version 4)



Quantitative Approach to Program Optimization

- Compiler optimization is viewed as optimization problems based on quantitative cost models
- Cost models driven by compiler estimates of execution time costs, memory costs, execution frequencies (obtained either by compiler analysis or from execution profiles)
- Cost model depends on computer architecture and computer system parameters
- Individual program transformations used in different ways to satisfy different optimization goals

High level structure of the ASTI Transformer



Steps performed by ASTI Transformer

1. Initialization
2. Loop distribution
3. Identification of perfect loop nests
4. Reduction recognition
5. Locality optimization
6. Loop fusion
7. Loop–invariant scalar replacement
8. Loop unrolling and interleaving
9. Local scalar replacement
10. Transcription — generate transformed HIR

Memory Cost Analysis

Consider an innermost perfect nest of h loops:

```
do i_1 = ...
   ...
      do i_h = ...
         ...
      end do
   ...
end do
```

The job of memory cost analysis is to estimate
$DL_{total}(t_1, \ldots, t_h) = \#$ distinct cache lines, and
$DP_{total}(t_1, \ldots, t_h) = \#$ distinct pages
accessed by a (hypothetical) *tile* of $t_1 \times \ldots \times t_h$ iterations.

Motivation for Memory Cost Functions

Assume that DL_{total} and DP_{total} are small enough so that no collision and capacity misses occur within a tile i.e.,
$DL_{total}(t_1, \ldots, t_h) \leq$ effective cache size
$DP_{total}(t_1, \ldots, t_h) \leq$ effective TLB size

The memory cost is then estimated as follows:

$$\begin{aligned} COST_{total} &= \text{(cache miss penalty)} \times DL_{total} + \\ &\quad \text{(TLB miss penalty)} \times DP_{total} \end{aligned}$$

Our objective is to minimize the memory cost per iteration which is given by the ratio, $COST_{total}/(t_1 \times \ldots \times t_h)$.

Matrix Multiply-Transpose Example

```
real*8 a(n,n), b(n,n), c(n,n)

. . .

do i1 = 1, n
   do i2 = 1, n
      do i3 = 1, n
         a(i1,i2) = a(i1,i2) + b(i2,i3) * c(i3,i1)
      end do
   end do
end do
```

Memory Cost Analysis for Matrix Multiply-Transpose Example

Assume cache line size, $L = 32$ bytes:

$$
\begin{aligned}
DL_{total}(t_1, t_2, t_3) &\approx \lceil 8t_1/L \rceil t_2 + \lceil 8t_2/L \rceil t_3 + \lceil 8t_3/L \rceil t_1 \\
&\approx (1 + 8(t_1 - 1)/L)\, t_2 + (1 + 8(t_2 - 1)/L)\, t_3 + \\
&\quad (1 + 8(t_3 - 1)/L)\, t_1 \\
&= (0.25t_1 + 0.75)\, t_2 + (0.25t_2 + 0.75)\, t_3 + \\
&\quad (0.25t_3 + 0.75)\, t_1
\end{aligned}
$$

Algorithm for selecting an optimized loop ordering

1. Build a symbolic expression for

$$F(t_1, \ \ldots \ , t_h) = \frac{COST_{total}(t_1, \ \ldots \ , t_h)}{t_1 \ \times \ \ldots \ \times \ t_h}$$

2. Evaluate the h partial derivatives (slopes) of function F, $\delta F / \delta t_k$, at $(t_1, \ \ldots \ , t_h) = (1, \ \ldots \ , 1)$

   A negative slope identifies a loop that carries temporal/spatial locality

3. Desired ordering is to place loop with most negative slope in innermost position, and so on.

Matrix Initialization example

```
   do 10 i1 = 1, n
      do 10 i2 = 1, n
10       a(i1,i2) = 0
```

For a PowerPC 604 processor:

$$\begin{aligned}
DL_{total}(t_1, t_2) &= (0.25t_1 + 0.75)t_2 \\
DP_{total}(t_1, t_2) &= (0.001953t_1 + 0.998047)t_2 \\
\Rightarrow COST_{total}(t_1, t_2) &= 17 \times DL_{total}(t_1, t_2) + 21 \times DP_{total}(t_1, t_2) \\
&= (4.25t_1t_2 + 12.75t_2) + (0.04t_1t_2 + 20.96t_2) \\
\Rightarrow F(t_1, t_2) &= \frac{COST_{total}}{t_1t_2} = \left(4.25 + \frac{12.75}{t_1}\right) + \left(0.04 + \frac{20.96}{t_1}\right) \\
\Rightarrow \frac{\delta F}{\delta t_1} &= \frac{-33.71}{t_1^2} \text{ is } < 0 \text{ and } \frac{\delta F}{\delta t_2} = 0
\end{aligned}$$

Desired loop ordering is i_2, i_1

Transformation Framework Case Studies

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Oil and Water Can Mix: An Integration of Polyhedral and AST-based Transformations

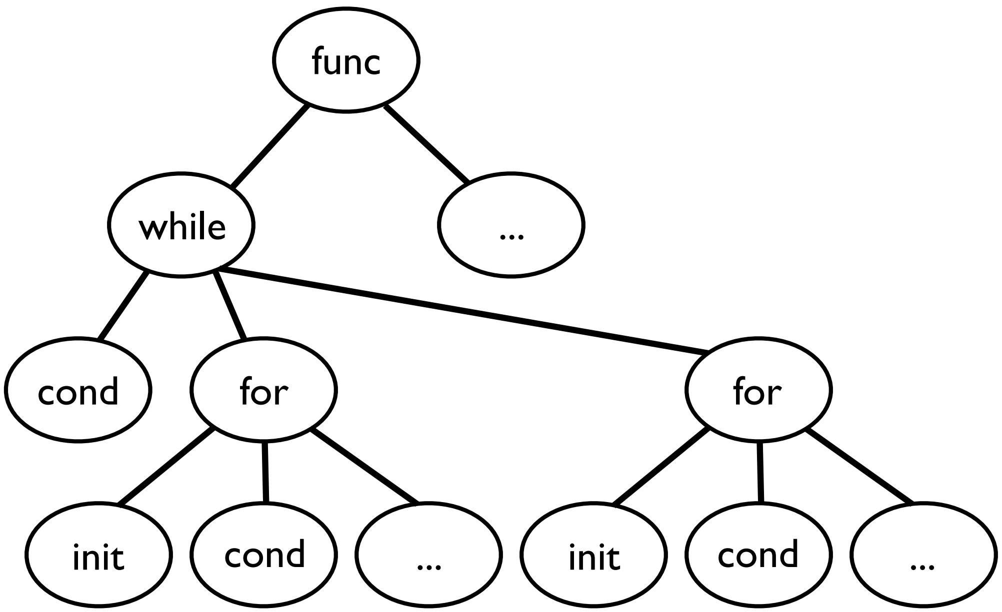
SC14 - New Orleans, Louisiana

November 18th, 2014

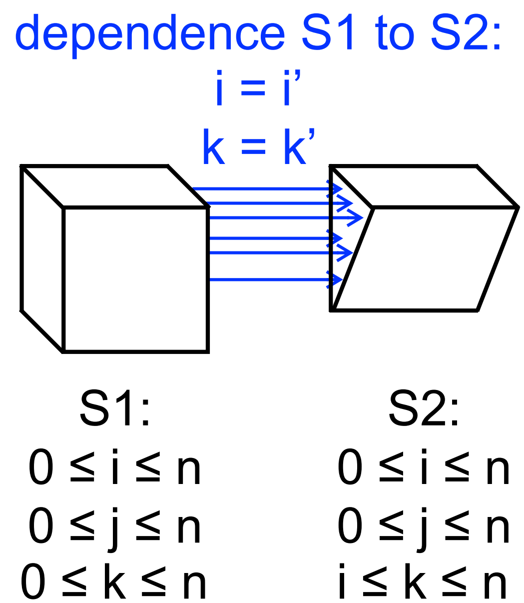
Jun Shirako, Louis-Noel Pouchet, Vivek Sarkar

Two Views of Program Representations

AST (Abstract Syntax Tree) view



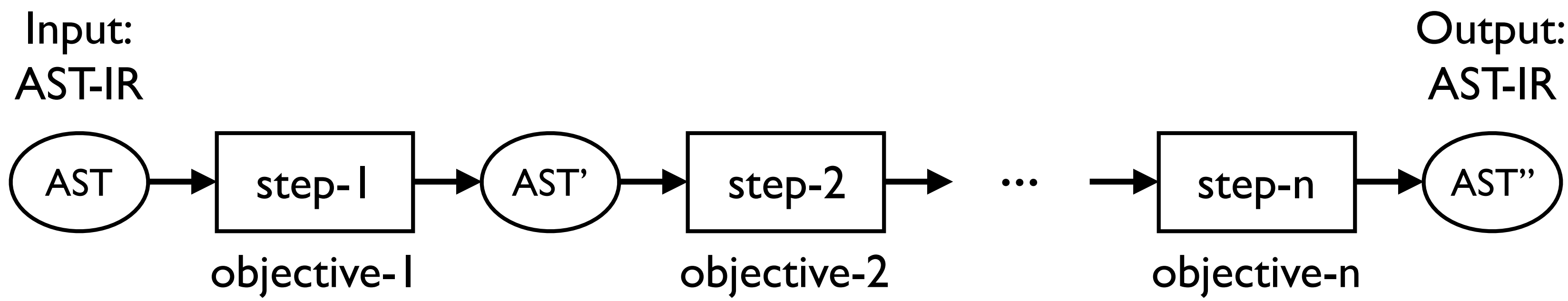
Polyhedral view



- AST captures all input programs
- Multiple steps modify AST while keeping the semantics

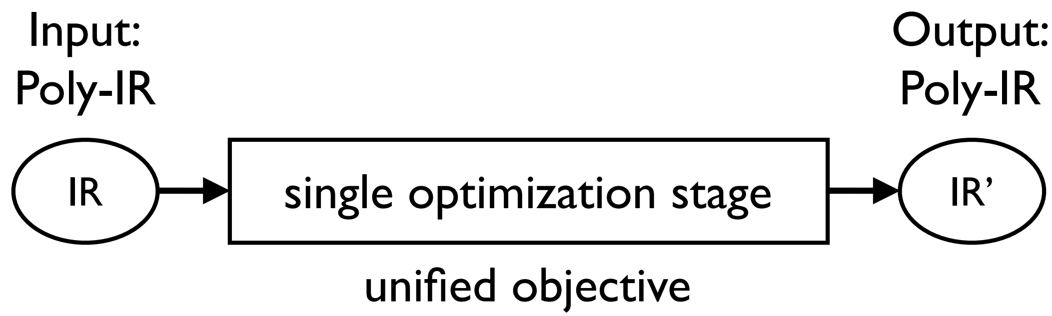
- Limited to loops whose bounds and accesses are affine expressions
- Single mathematical operation computes optimal solution

AST-based Loop Transformation Framework



- Sequence of individual loop transformations on Abstract Syntax Tree
  - Including : fusion, distribution, permutation, skewing, tiling, unroll-and-jam
  - Each step focuses on specific optimization objective:
    - Parallelism (doall, reduction, pipeline)
    - Temporal and spatial data locality
    - Vectorization efficiency
  - Analysis and cost model customized for each transformation
  - Phase-ordering problem (which comes before/after which)
    - Numerous transformations are complementary to each other

Mathematical Approach to Unified Transformation

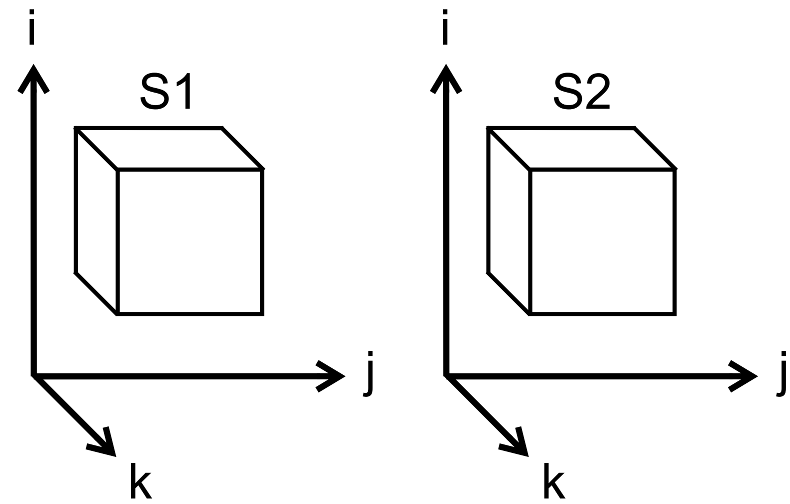


- Polyhedral model
  - Algebraic framework for affine program representation and transformation
  - Ability to handle everything in single stage
    - Unified view that captures arbitrary loop structures
    - Generalizes loop transformations as form of affine transform
  - Complexity due to unification/generalization
    - Hard to model cost functions for unified transformations
      - Multiple objectives to be combined in a single cost model

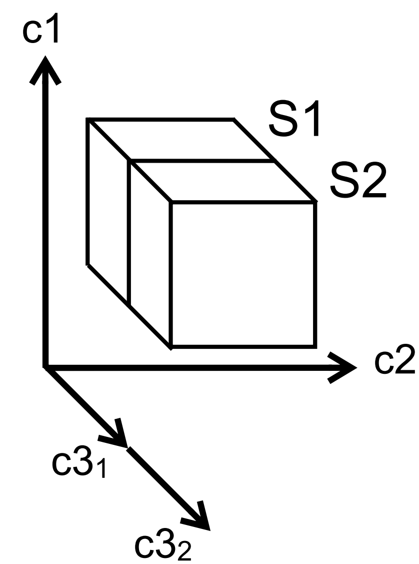
Cost Model Example in Polyhedral Approaches

```
    // Input: sequence of two matmults
    for (i = 0; i < N; i++)
      for (j = 0; j < N; j++)
        for (k = 0; k < N; k++)
S1:       tmp[i][j] += A[i][k] * B[k][j];

    for (i = 0; i < N; i++)
      for (j = 0; j < N; j++)
        for (k = 0; k < N; k++)
S2:       D[i][j] += C[i][k] * tmp[k][j];
```

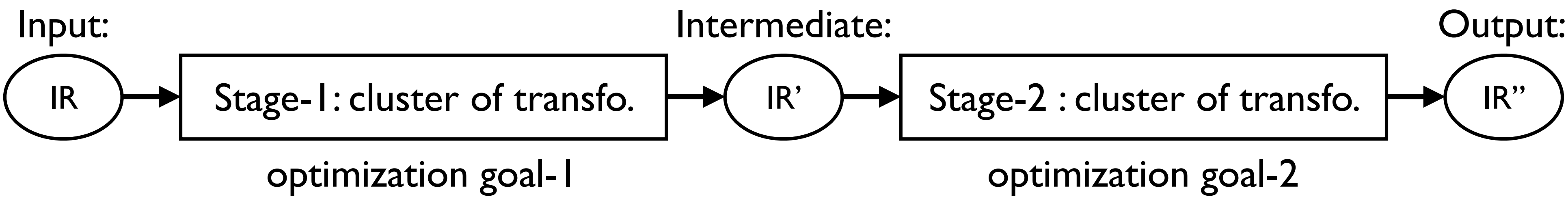



```
    // Output: Minimum reuse distance
    #pragma omp parallel for private(c2, c3)
    for (c1 = 0; c1 < N; c1++) {
      for (c2 = 0; c2 < N; c2++) {
        for (c3 = 0; c3 < N; c3++)
S1:       tmp[c2][c1] += A[c2][c3] * B[c3][c1];
        for (c3 = 0; c3 < N; c3++)
S2:       D[c3][c1] += C[c3][c2] * tmp[c2][c1];
    } }
```



- Objective : Minimization of reuse distance
  - Better temporal data locality
  - Outer parallelism by pushing dependences inside
  - Poor spatial data locality : not modeled in this objective

Mathematical Approach to Unified Transformation



- Challenge : Combining multiple objectives for unified transformations
  - Objectives can conflict, e.g., temporal locality (fuse loop) vs. vectorization (distribute)
- *Our approach --- decouple the optimization problem into two stages with different cost functions:*
  - Global - i.e., inter-loop-nest
    - Good candidate for polyhedral approach
      - Unified view that captures arbitrary loop structures (perfect & imperfect nests)
  - Local - i.e., per-loop-nest
    - Good candidate for AST-based approach
      - Well-defined sequence of transformations on perfect loop nest

Integrating Polyhedral and AST-based Transformations

- Poly+AST : two-stage approach to integration
  - Stage-1 : Polyhedral transformations
    - Finds optimal loop structures to provide sufficient data locality
      - Restricted form of affine transform
      - Extension of memory cost model for polyhedral model
    - Output : locality-optimized loop nests
  - Stage-2 : AST-based transformations
    - Input : loop nests and dependences from stage-1
    - Sequence of individual transformations per loop nest (w/ different objectives)
      - Loop skewing (increase tilability)
      - Parallelization (outermost doall / reduction / doacross)
      - Loop tiling (enhance locality and granularity of parallelism)
      - Intra-tile optimization (e.g., register-tiling, if-optimization, ...)

Outline

- Introduction
- Stage-1 : Cache-aware polyhedral transformations
- Stage-2 : AST-based transformations
- Experimental results vs. stage-of-the-art polyhedral compiler
- Conclusions

Polyhedral Representation of Program

```
    for (i = 0; i < N; i++)
     for (j = 0; j < N; j++)
      for (k = 0; k < N; k++)
S1:    tmp[i][j] += A[i][k] * B[k][j];

    for (i = 0; i < N; i++)
     for (j = 0; j < N; j++)
      for (k = 0; k < N; k++)
S2:    D[i][j] += C[i][k] * tmp[k][j];
```

$(i, j, k) \in \mathcal{D}^{S1}$:

$0 \le i \le N\text{-}1$
$0 \le j \le N\text{-}1$
$0 \le k \le N\text{-}1$

$(i, j, k) \in \mathcal{D}^{S2}$:

$0 \le i \le N\text{-}1$
$0 \le j \le N\text{-}1$
$0 \le k \le N\text{-}1$

$\langle (i, j, k), (i', j', k') \rangle \in \mathcal{D}^{S1 \rightarrow S2}$:

$0 \le i \le N\text{-}1$
$0 \le j \le N\text{-}1$
$0 \le k \le N\text{-}1$
$0 \le i' \le N\text{-}1$
$0 \le j' \le N\text{-}1$
$0 \le k' \le N\text{-}1$
$i = k'$
$j = j'$

- Iteration domain
  - $\mathcal{D}^{S_i}$: Set of iteration instances $\boldsymbol{i} = (i_1, i_2, \ldots, i_n)$ of S_i
    - Statement S_i is enclosed in n loops
- Dependence polyhedron
  - $\mathcal{D}^{S_i \rightarrow S_j}$: Captures dependence from S_i to S_j
    - $\langle \boldsymbol{s}, \boldsymbol{t} \rangle \in \mathcal{D}^{S_i \rightarrow S_j} \Leftrightarrow \boldsymbol{t} \in \mathcal{D}^{S_j}$ depends on $\boldsymbol{s} \in \mathcal{D}^{S_i}$

General Affine Program Transformation

$$\Theta^{S_i}(\boldsymbol{i}) = \begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} & \ldots & \alpha_{1,d} & c_1 \\ \alpha_{2,1} & \alpha_{2,2} & \ldots & \alpha_{2,d} & c_2 \\ \vdots & \vdots & & \vdots & \vdots \\ \alpha_{n,1} & \alpha_{n,2} & \ldots & \alpha_{n,d} & c_n \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_d \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_{1,1} i_1 + \alpha_{1,2} i_2 + \ldots + \alpha_{1,d} i_d + c_1 \\ \alpha_{2,1} i_1 + \alpha_{2,2} i_2 + \ldots + \alpha_{2,d} i_d + c_2 \\ \vdots \\ \alpha_{n,1} i_1 + \alpha_{n,2} i_2 + \ldots + \alpha_{n,d} i_d + c_n \end{pmatrix}$$

$\boldsymbol{i} = (i_1, i_2, \ldots, i_d)^T$: iteration instances of statement S_i

- Multi-dimensional affine transform
  - Θ^{S_i} associates $\boldsymbol{i}$ with a *timestamp* - i.e., logical execution date (yy/mm/dd)
  - Can model any composition of loop transformations including:

    Loop fusion, distribution, permutation, skewing, tiling
- Legality requirements
  - For all dependence polyhedra : $\Theta^{S_j}(\boldsymbol{t}) \succ \Theta^{S_i}(\boldsymbol{s})$, $\langle \boldsymbol{s}, \boldsymbol{t} \rangle \in \mathcal{D}^{S_i \rightarrow S_j}$

Stage-1 : Cache-aware Polyhedral Transformations

- Restricted form of affine transformations
  - To focus on optimal loop structure to provide sufficient locality
  - Weaker constraints can generate simple (i.e., easy-to-optimize) codes
- Subsumes the following:
  - Loop fusion, distribution and code motion
    - Group statements with locality into a loop
  - Loop permutation
    - Optimal loop order to optimize locality
  - Loop reversal and index-set shifting
    - Increase the opportunities of fusion/permutation
  - No loop skewing (but supported in AST stage)
    - Changes array access pattern, e.g., a[i][j] to a[i+j][j]
    - Can miss spatial locality / affect memory cost analysis

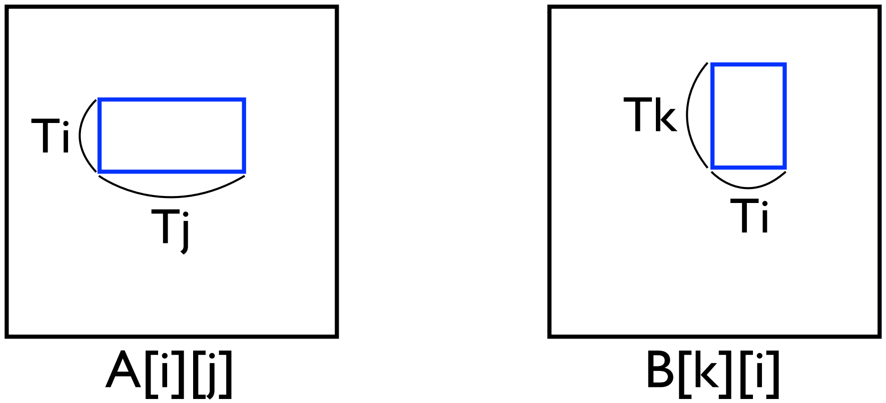
Proposed Restricted Affine Transformation

$$\Theta^{Si}(\boldsymbol{i}) = \begin{pmatrix} 0 & 0 & \dots & 0 & \beta_1 \\ \alpha_{1,1} & \alpha_{1,2} & \dots & \alpha_{1,d} & c_1 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \beta_k \\ \alpha_{k,1} & \alpha_{k,2} & \dots & \alpha_{k,d} & c_k \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \beta_d \\ \alpha_{d,1} & \alpha_{d,2} & \dots & \alpha_{d,d} & c_d \\ 0 & 0 & \dots & 0 & \beta_{d+1} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_d \\ 1 \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \alpha_{1,x}\, i_x + c_1 \\ \vdots \\ \beta_k \\ \alpha_{k,y}\, i_y + c_k \\ \vdots \\ \beta_d \\ \alpha_{1,z}\, i_z + c_d \\ \beta_{d+1} \end{pmatrix} \quad \forall k,\ \sum_{j=1}^{d} |\alpha_{k,j}| = 1$$

- Restricted forms
  - Odd row : constant offset β_k
  - Even row : linear expression of index where coefficient $\alpha_{k,x} = \pm 1$
- Symbols ⇔ transformations
  - offset β_k ⇔ fusion / distribution / code motion
  - index i_x ⇔ permutation
  - coefficient $\alpha_{k,x}$ ⇔ reversal (apply loop reversal when $\alpha_{k,x} = -1$)
  - offset c_k ⇔ index-set shifting

Cost Model to Guide Polyhedral Transfo.

```
for ti = 0, N-1, Ti
  for tj = 0, M-1, Tj
    for tk = 0, K-1, Tk
      for i = ti, ti+Ti-1
        for j = tj, tj+Tj-1
          for k = tk, tk+Tk-1
            A[i][j] += B[k][i];
```



$$DL(Ti,Tj,Tk) = DL_A(Ti,Tj,Tk) + DL_B(Ti,Tj,Tk) = Ti \times \lceil Tj / L \rceil + Tk \times \lceil Ti / L \rceil$$

$$mem\_cost(T_1, T_2, ..., T_d) = COST_{LINE} * DL(T_1, T_2, ..., T_d) \;/\; (T_1 * T_2 * ... * T_d)$$

- DL (Distinct Line) model
  - Assumes loop tiling to fit data within cache/TLB
  - Number of Distinct cache Lines accessed within a tile
    - Total cache miss counts per tile
- Average (per-iteration) memory cost
  - Defined as [total cache miss penalty per tile] / [tile size]

Profitability Analysis via DL Memory Cost

- Most profitable loop permutation order
  - Partial derivative of memory cost w.r.t. T_k :

$$\frac{\partial \text{mem\_cost}(T_1, T_2, \ldots, T_d)}{\partial T_k}$$

    - Reduction rate of memory cost when increasing T_k → Priority of permutation
      - Loop_k with most negative value → to be innermost position
    - Best loop order = descending order of $\partial \text{mem\_cost}(T_1, T_2, \ldots, T_d) / \partial T_k$
- Profitability of loop fusion
  - Comparing $\text{mem\_cost}(T_1, T_2, \ldots, T_d)$ before/after fusion
    - Memory cost decreased → fusion is profitable

      * tentative tile size used; final tile size selected later phase
  - Other criteria, e.g., parallelism, are also considered

Affine Transformation Algorithm

Input : S : set of statements S_i,
$PoDG$: polyhedral dependence graph,
k : current nest level, or dimension,
$niter^{Si}$: # iterators not yet scheduled in Θ^{Si}

begin

$PoDG'$:= subset of $PoDG$ w/o satisfied dependence;
$SccSet$:= compute SCCs of $PoDG'$;
/* Intra-SCC transformation (permutation) */
for each $SCC_a \in SccSet$ **do**
  compute permutation at level k and get constraints on reversal ($\alpha_{k,*}$) and shifting (c_k);

/* Inter-SCC transformation (fusion / distribution) */
$FuseSet$:= compute β_k and get constraints on reversal and shifting;
for each $Fuse_a \in FuseSet$ **do**
  solve constraints on reversal and shifting and compute $\alpha_{k,*}$ and c_k;
  if $\exists S_i \in Fuse_a : niter^{Si} \geq 1$ **then**
    recursively process the next level - i.e., k+1;

end

Output : Dimensions k ... m of schedule Θ^{Si}

Running Example : 2mm

```
// Input: sequence of two matmults
for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    for (k = 0; k < N; k++)
S1:   tmp[i][j] += A[i][k] * B[k][j];

for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    for (k = 0; k < N; k++)
S2:   D[i][j] += C[i][k] * tmp[k][j];
```

```
// Output: Best permutation order
for (c1 = 0; c1 < N; c1++)    // c1 = i
  for (c2 = 0; c2 < N; c2++)    // c2 = k
    for (c3 = 0; c3 < N; c3++)    // c3 = j
S1:   tmp[c1][c3] += A[c1][c2] * B[c2][c3];

for (c1 = 0; c1 < N; c1++)    // c1 = i
  for (c2 = 0; c2 < N; c2++)    // c2 = k
    for (c3 = 0; c3 < N; c3++)    // c3 = j
S2:   D[c1][c3] += C[c1][c2] * tmp[c2][c3];
```

	tmp/D[i][j]	A/C[i][k]	B/tmp[k][j]
i	N/A	N/A	temporal
j	spatial	temporal	spatial
k	temporal	spatial	N/A

- Optimization policy
  - Permute loops as close to the DL best order as possible
  - Fuse loops if legality and profitability criteria are met

Connection between Polyhedral and AST-based Stages

- Output of polyhedral stage
  - Locality-optimized loop nests
    - Permuted with legal & profitable loop order
    - Fused statements with locality into a loop
  - Dependence information
    - $\langle \boldsymbol{s}, \boldsymbol{t} \rangle \in \mathcal{P}_e^{S_i \rightarrow S_j}$: relationship between source and target instances ***s*** and ***t***
    - Extracted as dependence vector - i.e., $\boldsymbol{d} = \boldsymbol{t} - \boldsymbol{s}$
- Input of AST-based stage
  - $loop_k$: a loop that is nested at level $k \in \{1 \ldots n\}$
  - $\Delta^{loop_k} = \{\boldsymbol{d}^1, \boldsymbol{d}^2, \ldots, \boldsymbol{d}^n\}$:
    - Set of dependences whose source and target statements are within $loop_k$
    - Free from affine constraints in AST-based stage

Stage-2 : AST-based Transformation

- Dependence vectors : base of analysis
  - Legality : loop skewing, loop tiling, register tiling, ...
  - Detection of parallelism
- Sequence of transformations in stage-2
  - Loop skewing
    - In order to increase permutability (i.e., applicability of tiling) and parallelism
  - Coarse-grain parallelization
    - Doall / reduction / doacross parallelism
  - Loop tiling
    - Enhance computation granularity and data locality
  - Intra-tile optimizations
    - Register-tiling (i.e., multi-dimensional unrolling)

Parallelism in Poly+AST Framework

- Loop permutation order
  - To optimize spatial and temporal data locality
  - Outermost loop is not always doall
    - Also leverage other parallelism : reduction and doacross (pipeline parallelism)

- Reduction parallelism

```
#pragma omp for reduction(+: S[0:N-1])
for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    S[j] += alpha * X[i][j];
```

- Doall-only approach

```
#pragma omp for
for (j = 0; j < N; j++)
  for (i = 0; i < N; i++)
    S[j] += alpha * X[i][j];
```

- Doacross parallelism (OpenMP 4.5)

```
#pragma omp for ordered(2)
for (i = 1; i < N-1; i++) {
  for (j = 0; j < N; j++) {
#pragma omp ordered depend(sink: i-1,j)
    C[i][j] = 0.33 * (C[i-1][j]
          + C[i][j] + C[i+1][j]);
#pragma omp ordered depend(src: i,j)
} }
```

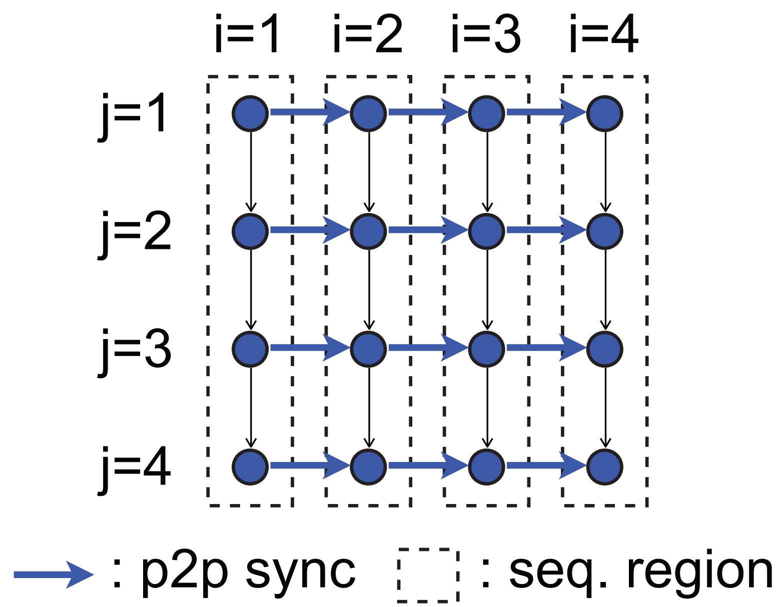
- Doall-only approach

```
#pragma omp for
for (j = 0; j < N; j++)
  for (i = 1; i < N-1; i++)
    C[i][j] = 0.33 * (C[i-1][j]
          + C[i][j] + C[i+1][j]);
```

Pipeline Parallelism vs. Wavefront Doall

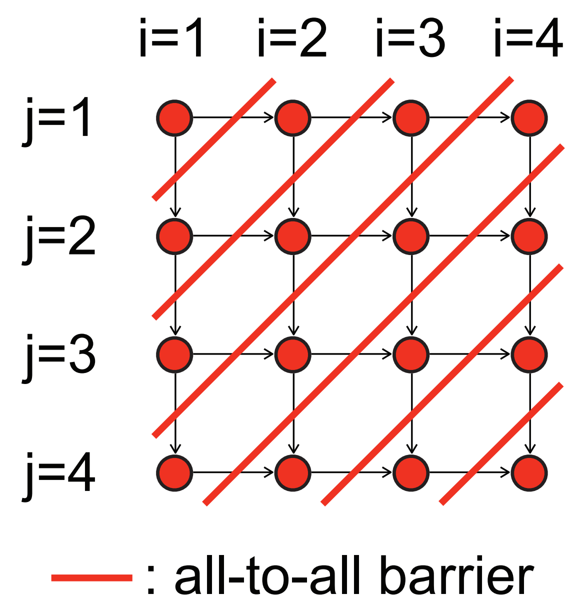
- Pipeline parallelism (OpenMP extension)

```
#pragma omp parallel for ordered(2)
for (i = 1; i < N-1; i++) {
  for (j = 1; j < N-1; j++) {
#pragma omp ordered depend(sink: i-1,j)
                    depend(sink: i,j-1)
    A[i][j] = A[i-1][j] + a[i][j-1];
#pragma omp ordered depend(src: i,j)
} }
```



- Wavefront doall with skewing

```
#pragma omp parallel
for (i = 2; i <= 2*N-4; i++) {
#pragma omp for
  for (j = max(1,i-N+2);
       j < min(N-2,i-1); j++) {
    A[i-j][j] = A[i-j-1][j] + a[i-j][j-1];
} }
```



Another Example : Jacobi-1d stencil

```
    // Input (imperfect nest)
    for (t = 0; t < time_steps; t++) {
      for (i = 1; i < n-1; i++)
S1:     b[i] = 0.33 * (a[i-1] + a[i] + a[i+1]);
      for (i = 1; i < n-1; i++)
S2:     a[i] = b[i];
    }

    // Stage-1: polyhedral transformation (perfect nest)
    for (c1 = 0; c1 <= time_steps-1; c1++) {
      for (c3 = 1; c3 <= n-1; c3++) {
S1:     if (c3 <= n-2) b[c3] = 0.33 * (a[c3-1] + a[c3] + a[c3+1]);
S2:     if (c3 >= 2) a[c3-1] = b[c3-1];
    } }

    // Stage-2: skewing & parallelization
    //   - Loop nest is fully permutable
    //   - Doacross parallelization by OpenMP extensions
    #pragma omp parallel for private(c3) ordered(2)
    for (c1 = 0; c1 < time_steps; c1++) {
      for (c3 = 2*c1+1; c3 < 2*c1+n; c3++) {
    #pragma omp ordered depend(sink: c1-1,c3) depend (sink: c1,c3-1)
S1:     if (i <= n-2) b[-2*c1+c3] = 0.33*(a[-2*c1+c3-1]+a[-2*c1+c3]+a[-2*c1+c3+1]);
S2:     if (i >= 2) a[-2*c1+c3-1] = b[-2*c1+c3-1];
    #pragma omp ordered depend(source: c1,c3)
    } }
```

Another Example : Jacobi-1d stencil

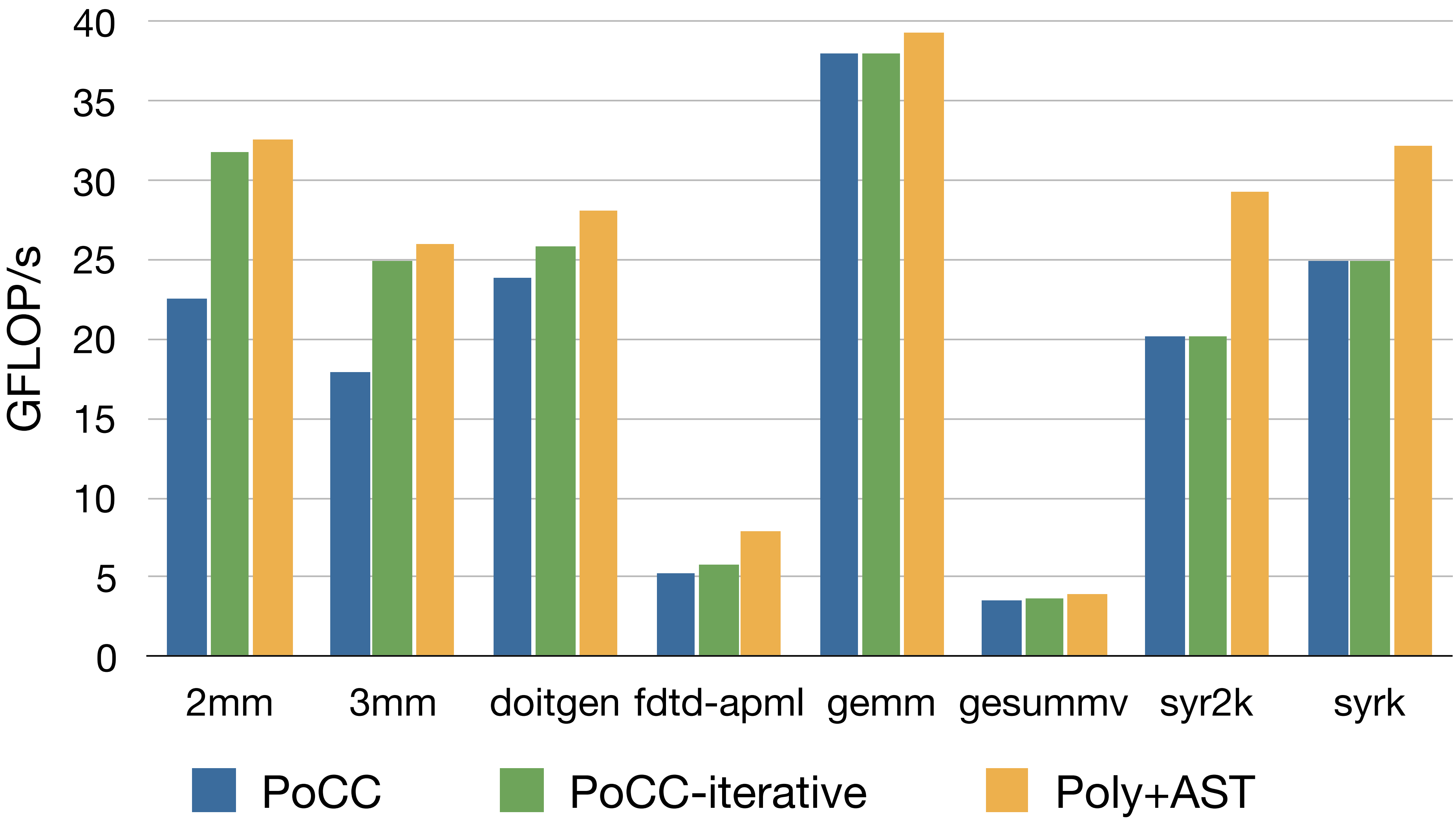
```
    // Stage-2: loop tiling
    #pragma omp parallel for private(c3,c5,i) ordered(2)
    for (c1 = ...) {
      for (c3 = ...) {
    #pragma omp ordered depend(sink: c1-1,c3) depend(sink: c1,c3-1)
        ...
        for (c5 = ...) {
          if (...) B[1] = 0.33 * (A[1-1] + A[1] + A[1+1]);
          for (c7 = ...) {
S1:         b[-2*c5+c7] = 0.33 * (a[-2*c5+c7-1] + a[-2*c5+c7] + a[-2*c5+c7+1]);
S2:         a[-2*c5+c7-1] = b[-2*c5+c7-1];
          }
          if (...) A[n-2] = B[n-2];
        }
        ...
    #pragma omp ordered depend(source: c1,c3)
    } }

    // Stage-2: register tiling (innermost by factor = 2)
          ...
          for (c7 = ...; c7 <= (...)-1; c7+=2) {
S1:         b[-2*c5+c7] = 0.33 * (a[-2*c5+c7-1]+a[-2*c5+c7]+a[-2*c5+c7+1]);
S2:         a[-2*c5+c7-1] = b[-2*c5+c7-1];
S1':        b[-2*c5+c7+1] = 0.33 * (a[-2*c5+c7+1-1]+a[-2*c5+c+1]+a[-2*c5+c7+1+1]);
S2':        a[-2*c5+c7+1-1] = b[-2*c5+c7+1-1];
          }
          ...
```

Experimental Setting

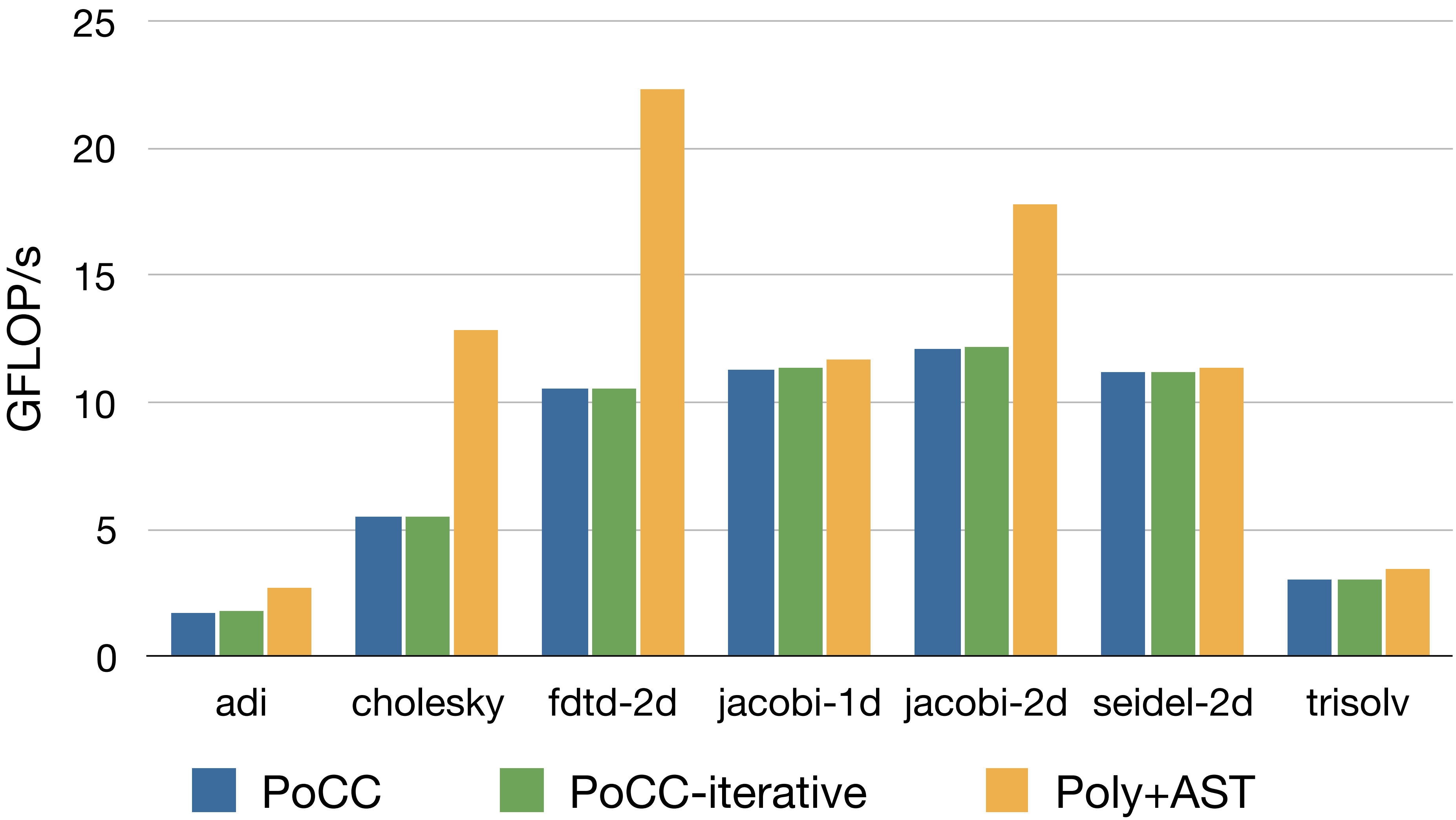
- Platforms
  - Two quad-core 2.8GHz Intel Core i7 (Nehalem) with Intel C compiler 12.0
  - Four eight-core 3.86GHz IBM Power7 with IBM XLC compiler 11.1
- Benchmarks
  - PolyBench-C 3.2 (22 benchmarks, standard/large dataset)
- Comparisons
  - PoCC : research polyhedral compiler [http://www.cs.ucla.edu/~pouchet/software/pocc]
    - PLuTo heuristic for parallelism, locality, tiling and intra-tile optimizations
    - Doall parallelism (convert doacross into wavefront doall)
  - PoCC-iterative : Iterative compilation approach [Pouchet-SC'10]
    - PoCC + empirical search for outermost fusion/distribution
  - Poly+AST : proposed integration approach
    - Doall / doacross / reduction parallelism
- Additional results in paper, e.g., ICC and XLC

GFLOP/s on Nehalem (doall dominant)



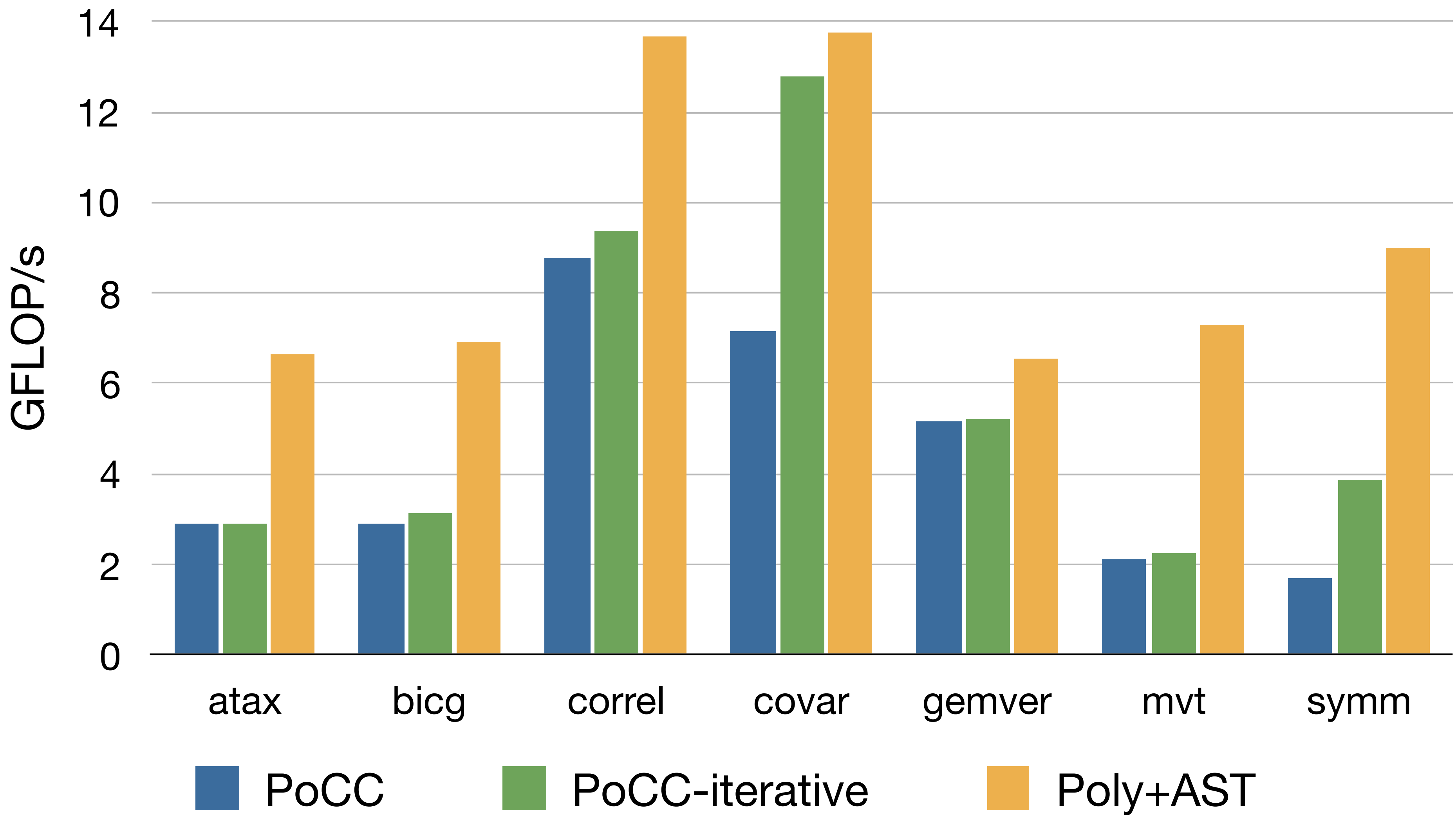
- PoCC ≤ PoCC-iterative ≤ Poly+AST
  - PoCC-iterative : empirical search for fusion/distribution
  - Poly+AST (polyhedral stage) : DL model for fusion/dist. and permutation

GFLOP/s on Nehalem (doacross-parallel dominant)



- PoCC = PoCC-iterative ≤ Poly+AST
  - adi / cholesky / fdtd-2d : loop structures (e.g., fusion, perm., index-shifting)
  - jacobi-2d : DOACROSS parallelization vs. wavefront doall by skewing

GFLOP/s on Nehalem (with reduction parallelism)



- PoCC ≤ PoCC-iterative < Poly+AST
  - Reduction support to increase flexibility of loop permutation
    - Loop order w/ better locality while keeping outermost parallelism

Transformed Codes by PoCC and Poly+AST

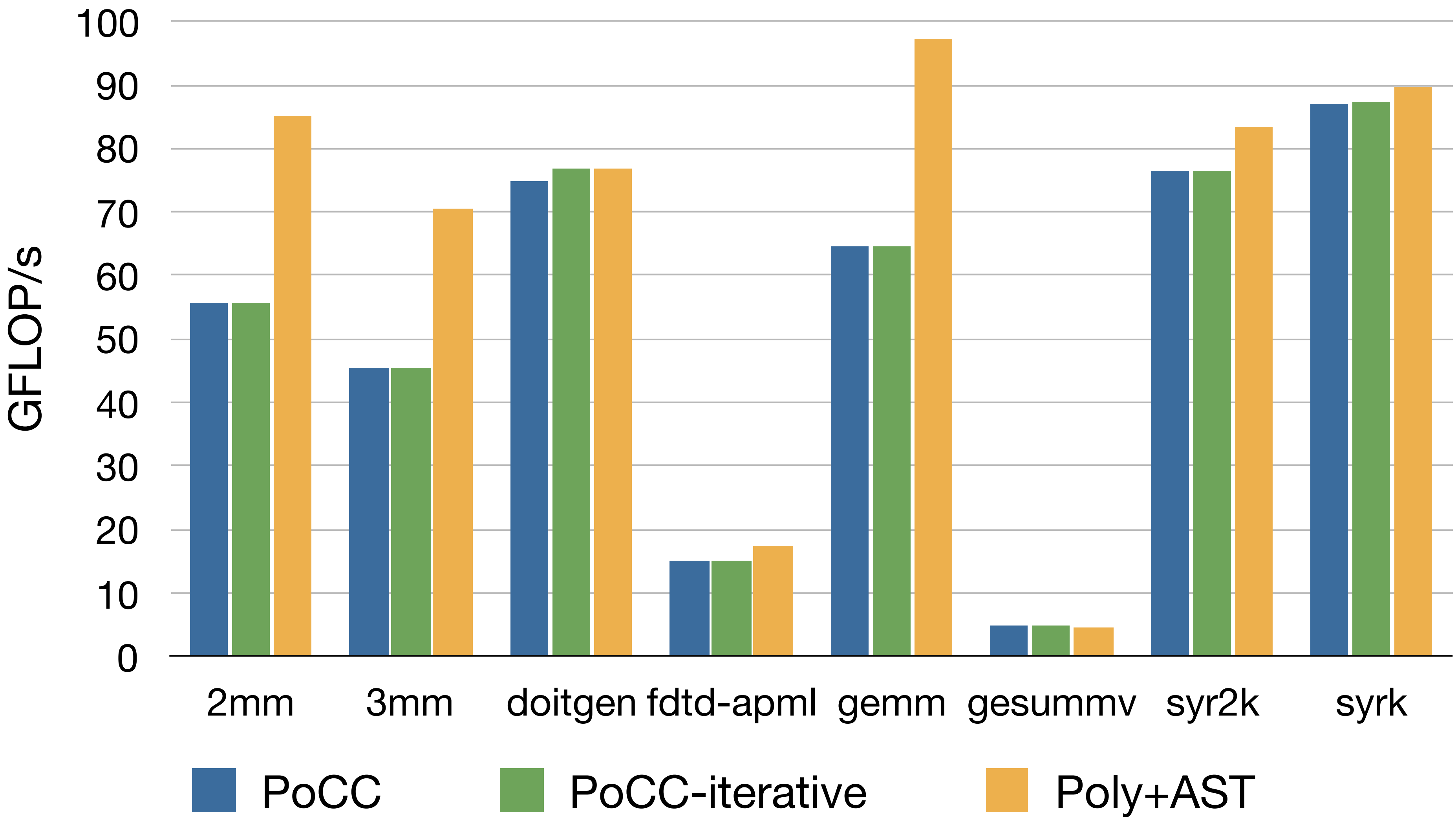
```
    // PoCC optimized (omitting tiling and intra-tile optimizations)
    #pragma omp parallel for private(c2, c3)
    for (c1 = 2; c1 <= NJ-1; c1++) {
      for (c2 = 0; c2 <= NI-1; c2++) {
        for (c3 = 0; c3 <= c1+NI-1; c3++) {
S1:       if (c3 <= c1-2) acc[c2][c1] += B[c3][c1] * A[c3][c2];
S2:       if (c2 <= c1-2 && c3 >= c1) C[c2][c1] += alpha * A[c2][-c1+c3] * B[-c1+c3][c1];
S3:       if (c3 == c1+c2) C[c2][c1] = beta * C[c2][c1] + alpha * A[c2][c2] * B[c2][c1] ...
    } } }
```

doall accessing inner array dimensions; poor spatial locality

```
    // Poly+AST optimized (omitting tiling and intra-tile optimizations)
    #pragma omp parallel for private(c3, c5) reduction(+: acc[0:NI-1][2:NJ-1])
    for (c1 = 0; c1 <= NJ-3; c1++) {
      for (c3 = 0; c3 <= NI-1; c3++) {
        for (c5 = c1 + 2; c5 <= NJ-1; c5++) {
S1:       acc[c3][c5] += B[c1][c5] * A[c1][c3];
    } } }
    #pragma omp parallel for private(c3, c5)
    for (c1 = 0; c1 <= MAX(NI-1, NJ-3); c1++) {
      for (c3 = 0; c3 <= NI-1; c3++) {
        for (c5 = 0; c5 <= NJ-1; c5++) {
S2:       if (c5 >= c1+2) C[c1][c5] += alpha * A[c1][c3] * B[c3][c5];
S3:       if (c3 == c1) C[c1][c5] = beta * C[c1][c5] + alpha * A[c1][c1] * B[c1][c5] ...
    } } }
```

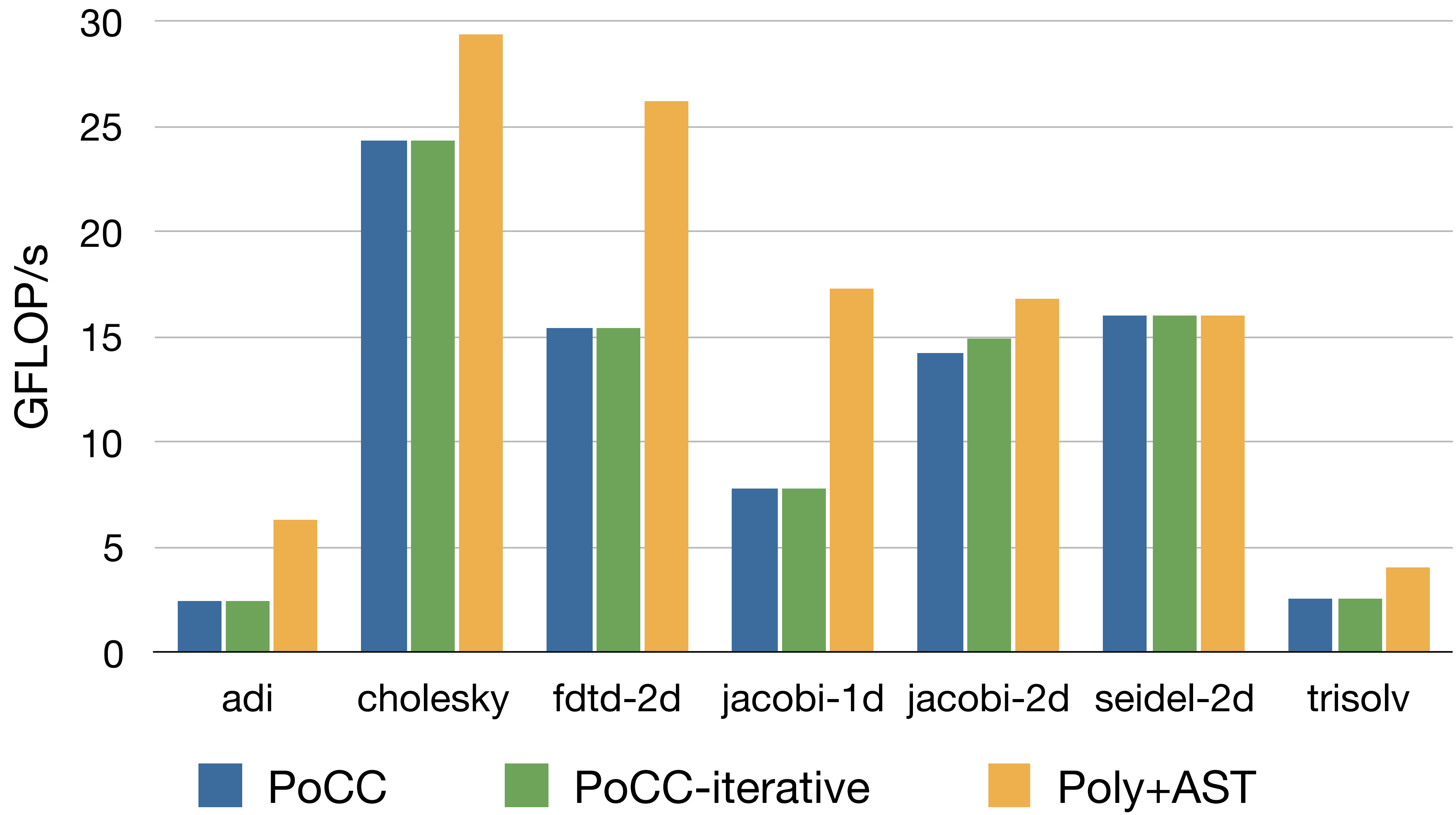
reduction / doall accessing outer array dimensions; better spatial locality

GFLOP/s on Power7 (doall dominant)



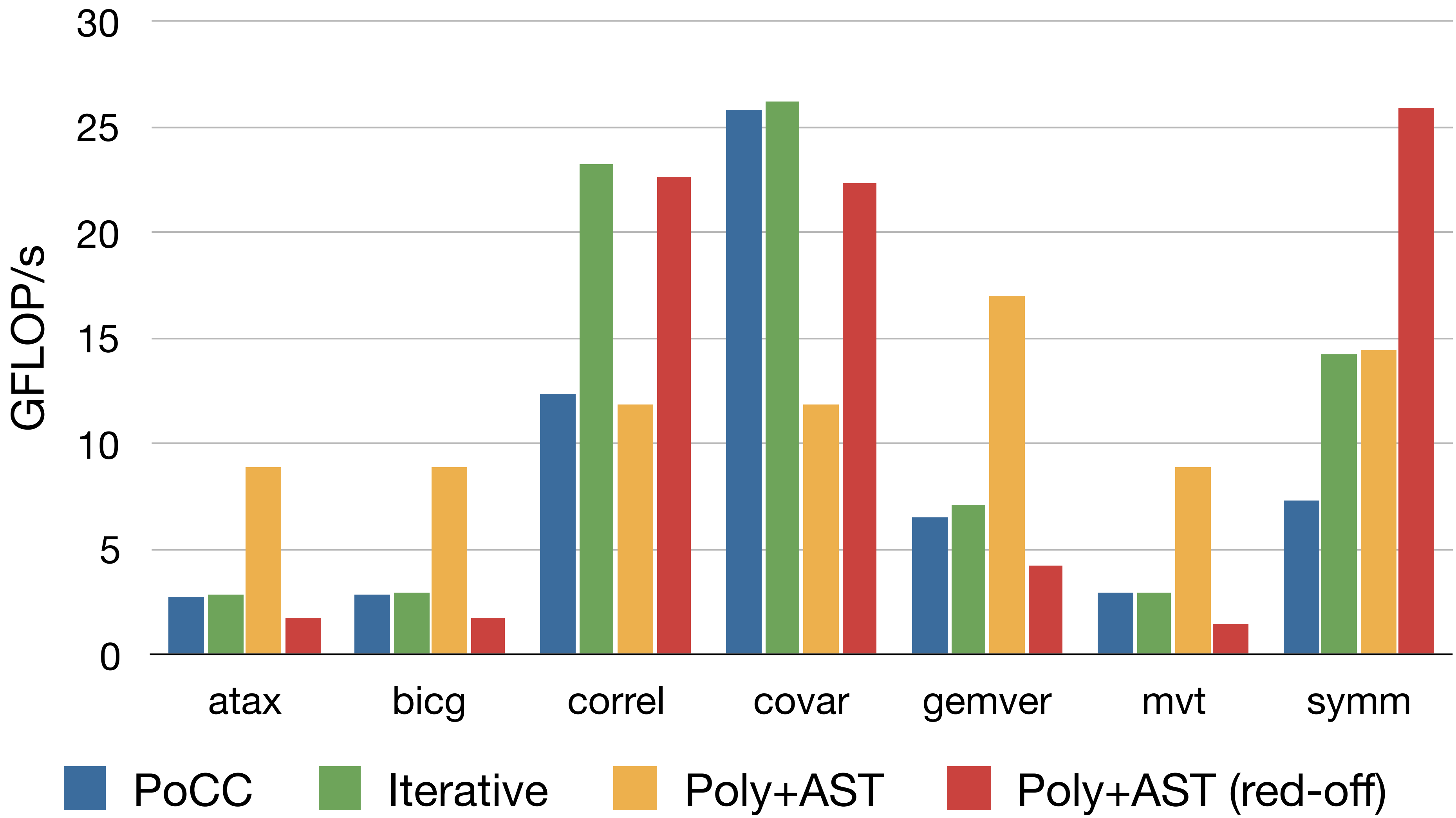
- PoCC = PoCC-iterative ≤ Poly+AST
  - Good selection of loop structures (e.g., fusion/distribution and permutation)

GFLOP/s on Power7 (doacross-parallel dominant)



- PoCC = PoCC-iterative ≤ Poly+AST
  - Efficiency of DOACROSS has more impact (32-core Power7 vs. 8-core Nehalem)

GFLOP/s on Power7 (with reduction parallelism)



- Reduction reduces performance (correl, covar and symm)
  - Sequential aggregation for final results is scalability bottleneck
  - Future work : parallel aggregation

Take-home Message

- AST-based transformations
  - Sequence of individual loop transformations
  - Difficulty in composing the optimal sequence (i.e., phase-ordering)
- Polyhedral model
  - Unification & generalization of loop transformations
  - Difficulty in modeling cost functions for whole unified transformations
- Integration of both
  - Decoupling the optimization problem into two stages
    - Polyhedral model as first stage, AST-based as second stage
  - Simpler & customized cost modeling within stage
  - Each stage leverage its strengths
  - Geometric mean speedup vs. PoCC (polyhedral optimizer)
    - 1.62x on 8-core Nehalem / 1.49x on 32-core Power7