
COMP 515: Advanced Compilation for Vector and Parallel Processors

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COMP 515

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 - <http://www.cs.rice.edu/~ken/comp515/>

Interprocedural Analysis and Optimization

Chapter 11

Introduction

- **Interprocedural Analysis**
 - Gathering information about the whole program instead of a single procedure
- **Interprocedural Optimization**
 - Program transformation modifying more than one procedure using interprocedural analysis

Overview: Interprocedural Analysis

- Examples of Interprocedural problems
- Classification of Interprocedural problems
- Solve two Interprocedural problems
 - Side-effect Analysis
 - Alias Analysis

Some Interprocedural Analysis Problems

- **Modification and Reference Side-effect**

```
COMMON X(N),Y(N) ! Static arrays
```

```
...
```

```
DO I = 1, N
```

```
S0:      CALL P
```

```
S1:      X(I) = X(I) + Y(I)
```

```
ENDDO
```

- **Can parallelize I loop if P**
 1. neither modifies nor uses X
 2. does not modify Y

Modification and Reference Side Effect

- $MOD(s)$: set of variables that may be modified as a side effect of call at s
- $REF(s)$: set of variables that may be referenced as a side effect of call at s

DO I = 1, N

S0: CALL P

S1: X(I) = X(I) + Y(I)

ENDDO

- Can vectorize S1 if $x \notin REF(S0) \wedge x \notin MOD(S0) \wedge y \notin REF(S0)$
 - TODO: replace REF by MOD for y term above

Alias Analysis

```
COMMON Y ! static variable
SUBROUTINE S(A,X,N)
  DO I = 1, N
    S0:  X = X + Y*A(I)
  ENDDO
END
```

- Could have kept X and Y in different registers and stored in X outside the loop
 - What happens when there is a call, CALL S(A,Y,N)?
 - Then Y is aliased to X on entry to S
 - Can't delay update to X in the loop any more since we don't know for sure if X and Y are aliased
 - ALIAS(p,x): set of variables that may refer to the same location as formal parameter x on entry to procedure p
-

Call Graph Construction

- Call Graph $G=(N,E)$
 - N : one vertex for each procedure
 - E : one edge for each possible call
 - Edge (p,q) is in E if procedure p may call procedure q
- Looks easy
- Construction difficult in presence of procedure parameters
- Also for virtual method calls in object-oriented languages

Call Graph Construction: Example with procedure parameter

```
SUBROUTINE S(X,P)
```

```
S0:  CALL P(X)
```

```
      RETURN
```

```
END
```

- P is a procedure parameter to S
- What values can P have on entry to S ?
- $CALL(s)$: set of all procedures that may be invoked at s
- Resembles the alias analysis problem

Live and Use Analysis

```
DO I = 1, N
  T = X(I)*C
  A(I) = T + B(I)
  C(I) = T + D(I)
ENDDO
```

- This loop can be parallelized by making T a local variable in the loop

```
PARALLEL DO I = 1, N
  LOCAL t
  t = X(I)*C
  A(I) = t + B(I)
  C(I) = t + D(I)
  IF(I.EQ.N) T = t
ENDDO
```

- Copy of local version of T to the global version of T is required to ensure correctness
- What if T was not live outside the loop?

Live and Use Analysis

- Solve Live analysis using Use Analysis
- $USE(s)$: set of variables having an upward exposed use in procedure p called at s
- If a call site, s is in a single basic block(b), x is live if either
 - x in $USE(s)$ or
 - P doesn't assign a new value to x and x is live in some control flow successor of b

Kill Analysis

```
DO I = 1, N
SO:      CALL INIT(T,I)
          T = T + B(I)
          A(I) = A(I) + T
ENDDO
```

- To parallelize the loop:
 - INIT must not create a recurrence with respect to the loop
 - T must not be upward exposed (otherwise it cannot be privatized)

Kill Analysis

```
DO I = 1, N
SO:  CALL INIT(T,I)
      T = T + B(I)
      A(I) = A(I) + T
ENDDO
```

```
SUBROUTINE INIT(T,I)
      REAL T
      INTEGER I
      COMMON X(100)
      T = X(I)
END
```

- T has to be assigned before being used on every path through INIT
- If INIT is of this form we can see that T can be privatized

Kill Analysis

- **KILL(s)**: set of variables assigned on every path through procedure p called at s and through procedures invoked in p
- T in the previous example can be privatized under the following condition

$$T \in (KILL(S0) \cap \neg USE(S0))$$

- Also we can express LIVE(s) as following

$$LIVE(s) = USE(s) \cup (\neg KILL(s) \cap \bigcup_{b \in succ(s)} LIVE(b))$$

Constant Propagation

```
SUBROUTINE S(A,B,N,IS,I1)
```

```
  REAL A(*), B(*)
```

```
  DO I = 0, N-1
```

```
    S0:      A(IS*I+I1) = A(IS*I+I1) + B(I+1)
```

```
  ENDDO
```

```
END
```

- If $IS=0$ the loop around S0 is a reduction
 - If $IS \neq 0$ the loop can be vectorized
 - **CONST(p)**: set of variables with known constant values on every invocation of p
 - Knowledge of **CONST(p)** useful for interprocedural constant propagation
-

Interprocedural Problem Classification

- **May and Must problems**
 - MOD, REF and USE are 'May' problems
 - KILL is a 'Must' problem
- **Flow sensitive and flow insensitive problems**
 - Flow sensitive: control flow info included in analysis
 - Flow insensitive: control flow info is (conservatively) ignored
- **May and Must classification can apply to call graph edges as well**

Flow Insensitive Side-effect Analysis

- **Assumptions**
 - No procedure nesting i.e., no inner functions
 - All parameters passed by reference
 - Size of the parameter list bounded by a constant,
- We will formulate and solve MOD(s) problem

Solving MOD

$$MOD(s) = DMOD(s) \cup \bigcup_{x \in DMOD(s)} ALIAS(p, x)$$

- **DMOD(s)**: set of variables which are directly modified as side-effect of call at s (ignoring aliases)

$$DMOD(s) = \{v \mid s \Rightarrow p, v \xrightarrow{s} w, w \in GMOD(p)\}$$

- **GMOD(p)**: set of global variables and formal parameters w of p that are modified, either directly or indirectly as a result of invocation of p
 - Global variables are modeled as special “parameters” in this formulation

Example: DMOD and GMOD

S0: CALL P(A,B,C)

...

SUBROUTINE P(X,Y,Z)

INTEGER X,Y,Z

X = X*Z

Y = Y*Z

END

- $GMOD(P) = \{X, Y\}$
- $DMOD(S0) = \{A, B\}$

Solving GMOD

- $GMOD(p)$ contains two types of variables
 - Variables explicitly modified in body of P : This constitutes the set $IMOD(p)$
 - Variables modified as a side-effect of some procedure invoked in p
 - Global variables are viewed as parameters to a called procedure

$$GMOD(p) = IMOD(p) \cup \bigcup_{s=(p,q)} \{z \mid z \xrightarrow{s} w, w \in GMOD(q)\}$$

- The above formulation is impractical for recursive programs

Solving GMOD

- The previous iterative method may take a long time to converge
 - Problem with recursive calls

```
SUBROUTINE P(F0,F1,F2,...,Fn)
```

```
    INTEGER X,F0,F1,F2,...,Fn
```

```
    ...
```

```
S0:    F0 = <some expr>
```

```
    ...
```

```
S1:    CALL P(F1,F2,...,Fn,X)
```

```
    ...
```

```
END
```

Solving GMOD

- Decompose $GMOD(p)$ differently to get an efficient solution in the presence of recursion
- Key: Treat side-effects to global variables and reference formal parameters separately
- $LOCAL$ refers to local variables in q

$$GMOD(p) = \boxed{IMOD^+(p)} \cup \boxed{\bigcup_{s=(p,q)} GMOD(q) \cap \neg LOCAL}$$

Solving for IMOD+

- $x \in IMOD^+(p)$ if
 - $x \in IMOD(p)$ or
 - $x \xrightarrow{s} z, z \in GMOD(q), s = (p, q)$ and x is a formal parameter of p

- Formally defined

$$IMOD^+(p) = IMOD(p) \cup \bigcup_{s=(p,q)} \{z \mid z \xrightarrow{s} w, w \in RMOD(q)\}$$

- **RMOD(p)**: set of formal parameters in p that may be modified in p , either directly or by assignment to a reference formal parameter of q as a side effect of a call of q in p

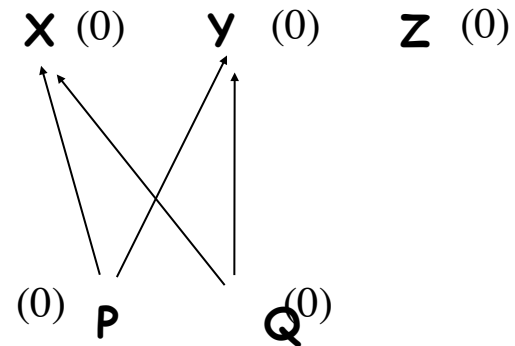
Solving for RMOD

- **RMOD(p)**: set of formal parameters in p that may be modified in p, either directly or by assignment to a reference formal parameter of q as a side effect of a call of q in p
 - Binding Graph $G_B=(N_B, E_B)$
 - One vertex for each formal parameter of each procedure
 - Directed edge from formal parameter, f1 of p to formal parameter, f2 of q if there exists a call site $s=(p,q)$ in p such that f1 is bound to f2
 - Use a marking algorithm to compute RMOD(p) (Figure 11.2)
 - Mark each vertex as false initially
 - Mark formals of P in IMOD(p) as true
 - Perform a closure operation (propagate bits)
 - Mark f1 as true if G_B has an edge from f1 to f2 and f2 is marked true
-
- Use worklist algorithm (or reverse DFS, if you prefer)

Solving for RMOD

```
SUBROUTINE A(X,Y,Z)
  INTEGER X,Y,Z
  X = Y + Z
  Y = Z + 1
END
```

```
SUBROUTINE B(P,Q)
  INTEGER P,Q,I
  I = 2
  CALL A(P,Q,I)
  CALL A(Q,P,I)
END
```

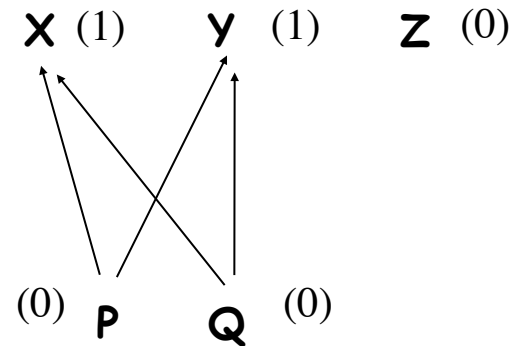


- $IMOD(A) = \{X, Y\}$
- $IMOD(B) = \{I\}$

Solving for RMOD

```
SUBROUTINE A(X,Y,Z)
  INTEGER X,Y,Z
  X = Y + Z
  Y = Z + 1
END
```

```
SUBROUTINE B(P,Q)
  INTEGER P,Q,I
  I = 2
  CALL A(P,Q,I)
  CALL A(Q,P,I)
END
```



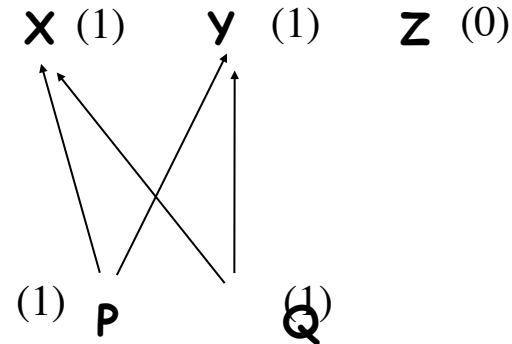
- $IMOD(A) = \{X, Y\}$
- $IMOD(B) = \{I\}$
- $Worklist = \{X, Y\}$

Solving for RMOD

```

SUBROUTINE A(X,Y,Z)
  INTEGER X,Y,Z
  X = Y + Z
  Y = Z + 1
END

```



```

SUBROUTINE B(P,Q)
  INTEGER P,Q,I
  I = 2
  CALL A(P,Q,I)
  CALL A(Q,P,I)
END

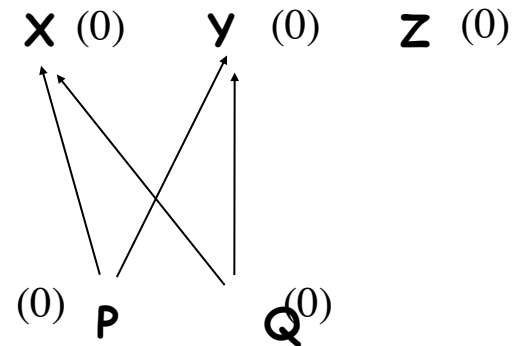
```

- $\text{RMOD}(A) = \{X, Y\}$
- $\text{RMOD}(B) = \{P, Q\}$
- **Complexity:** $O(N_B + E_B)$
 $N_B \leq \mu N$ $E_B \leq \mu E$
 $O(N + E)$

Solving for RMOD

```
SUBROUTINE A(X,Y,Z)
  INTEGER X,Y,Z
  X = Y + Z
  Y = Z + 1
END
```

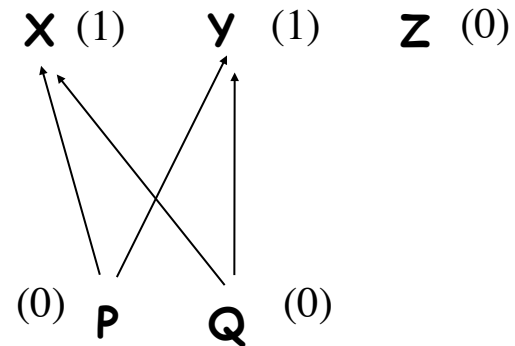
```
SUBROUTINE B(P,Q)
  INTEGER P,Q,I
  I = 2
  CALL A(P,Q,I)
  CALL A(Q,P,I)
END
```



- $IMOD(A) = \{X, Y\}$
- $IMOD(B) = \{I\}$

Solving for RMOD

```
SUBROUTINE A(X,Y,Z)
  INTEGER X,Y,Z
  X = Y + Z
  Y = Z + 1
END
```



```
SUBROUTINE B(P,Q)
  INTEGER P,Q,I
  I = 2
  CALL A(P,Q,I)
  CALL A(Q,P,I)
END
```

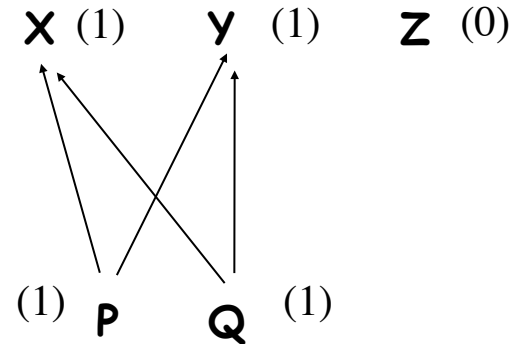
- $IMOD(A) = \{X, Y\}$
- $IMOD(B) = \{I\}$
- $Worklist = \{X, Y\}$

Solving for RMOD

```

SUBROUTINE A(X,Y,Z)
  INTEGER X,Y,Z
  X = Y + Z
  Y = Z + 1
END

```



```

SUBROUTINE B(P,Q)
  INTEGER P,Q,I
  I = 2
  CALL A(P,Q,I)
  CALL A(Q,P,I)
END

```

- $\text{RMOD}(A) = \{X, Y\}$
- $\text{RMOD}(B) = \{P, Q\}$
- **Complexity:** $O(N_B + E_B)$
 $N_B \leq \mu N$ $E_B \leq \mu E$
 $O(N + E)$

Solving for IMOD⁺

- After gathering RMOD(p) for all procedures, update RMOD(p) to IMOD⁺(p) using this equation

$$IMOD^+(p) = IMOD(p) \cup \bigcup_{s=(p,q)} \{z \mid z \xrightarrow{s} w, w \in RMOD(q)\}$$

- This can be done in $O(NV+E)$ time

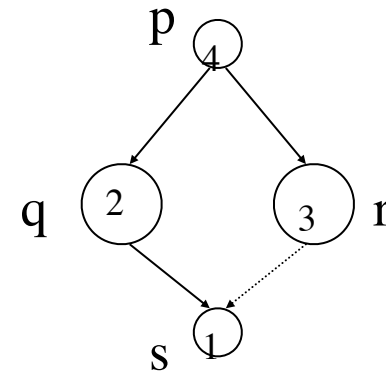
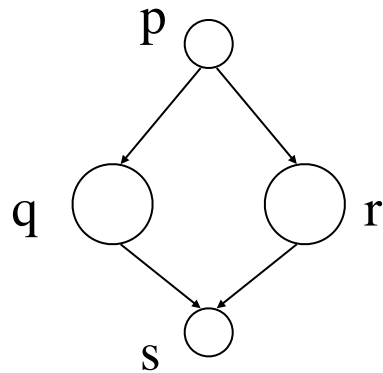
Solving for GMOD

- After gathering $IMOD^+(p)$ for all procedures, calculate $GMOD(p)$ according to the following equation

$$GMOD(p) = IMOD^+(p) \cup \bigcup_{s=(p,q)} GMOD(q) \cap \neg LOCAL$$

- This can be solved using a DFS algorithm based on Tarjan's SCR algorithm on the Call Graph

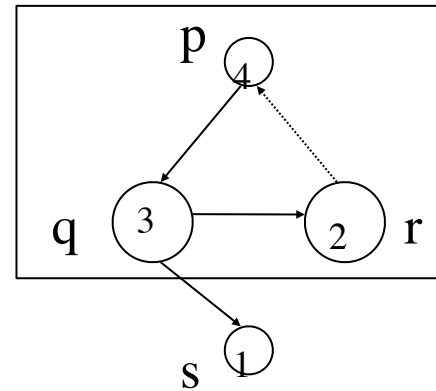
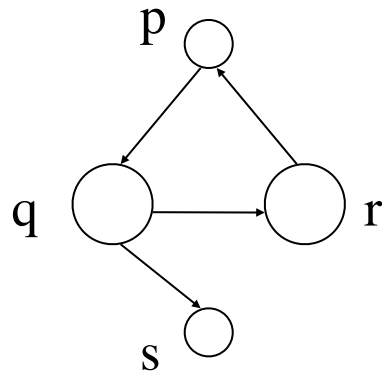
Solving for GMOD



Initialize $GMOD(p)$ to $IMOD^+(p)$ on discovery

Update $GMOD(p)$ computation while backing up

Solving for GMOD



Initialize $GMOD(p)$ to $IMOD^+(p)$ on discovery

Update $GMOD(p)$ computation while backing up

For each node u in a SCR update $GMOD(u)$ in a cycle

$O((N+E)V)$ Algorithm

Overview: Interprocedural Analysis

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