COMP 515: Advanced Compilation for Vector and Parallel Processors

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https://wiki.rice.edu/confluence/display/PARPROG/COMP515

Dependence Testing

Allen and Kennedy, Chapter 3 (up to Section 3.3.2)

The General Problem

```
DO i1 = L1, U1
  DO i2 = L2, U2
            …
    DO in = Ln, Un
 S1                  A(f1(i1,…,in),…,fm(i1,…,in)) = …
 S2                  … = A(g1(i1,…,in),…,gm(i1,…,in))
    ENDDO
    ...
  ENDDO
ENDDO
```

Under what conditions is the following true for iterations α and β ?

$$f_i(\alpha) = g_i(\beta) \text{ for all } i, 1 \leq i \leq m$$

Note that the number of equations equals the rank of the array,

and the number of variables is twice the number of loops that enclose both array references (two iteration vectors)

Basics: Complexity

A subscript equation is said to be

- ZIV if it contains no index (zero index variable)
- SIV if it contains only one index (single index variable)
- MIV if it contains more than one index (multiple index variables)

For Example:

```
A(5,I+1,j) = A(1,I,k) + C

    First subscript equation is ZIV

    Second subscript equation is SIV

    Third subscript equation is MIV
```

Terminology: Indices and Subscripts

Index: **Index variable for some loop surrounding a pair of references**

Subscript: **A PAIR of subscript positions in a pair of array references (corresponds to dependence equation for that dimension)**

For Example:

```
A(I,j) = A(I,k) + C
    ‹I,I› is the first subscript
    ‹j,k› is the second subscript
```

Basics: Separability

- A subscript is separable if its indices do not occur in other subscripts
- If two different subscripts contain the same index they are coupled

For Example:

```
A(I+1,j) = A(k,j) + C
        Both subscripts are separable
A(I,j,j) = A(I,j,k) + C
        Second and third subscripts are
coupled
```

Basics: Coupled Subscript Groups

- Why are they important?

  Ignoring coupled subscripts may lead to imprecision in dependence testing

e.g., is there a loop-carried dependence on A in the following loop?

```
  DO I = 1, 100
S1      A(I+1,I) = B(I) + C
S2      D(I) = A(I,I) * E
  ENDDO
```

Basics: Conservative Testing

- Consider only linear subscript expressions
- Finding integer solutions to system of linear Diophantine Equations is NP-hard
  - Another possible class project: explore use of ILP solvers for dependence testing!
- Most common approximation is Conservative Testing

  See if you can assert

  "No dependence exists between two subscripted references of the same array"

  If in doubt, you can always say "dependence exist"
- Never incorrect, may be less than optimal

Dependence Testing: Overview

1. Partition subscripts of a pair of array references into separable and coupled groups (easy)
2. Classify each subscript as ZIV, SIV or MIV (easy)
3. For each separable subscript apply single subscript test. If not done goto next step
4. For each coupled group apply multiple subscript test
5. If still not done, merge all direction vectors computed in the previous steps into a single set of direction vectors

Step 3: Applying Single Subscript Tests

- ZIV Test
- SIV Test
  - Strong SIV Test
  - Weak SIV Test
    - Weak-zero SIV
    - Weak Crossing SIV
- SIV Tests in Complex Iteration Spaces

ZIV Test

```
DO j = 1, 100
      A(e1) = A(e2) + B(j)
ENDDO
```

e1,e2 are constants or loop invariant expressions

If (e1-e2)!=0 No Dependence exists

Program analyses that can improve the accuracy of this test include constant propagation, value numbering, and symbolic “definitely different” analysis (inferring that e1 = e2 + nonzero-constant)

Strong SIV Test

- Strong SIV subscripts are of the form

$$\langle ai + c_1, ai + c_2 \rangle$$

where $a \neq 0$

- For example the following are strong SIV subscripts

$$\langle i + 1, i \rangle$$

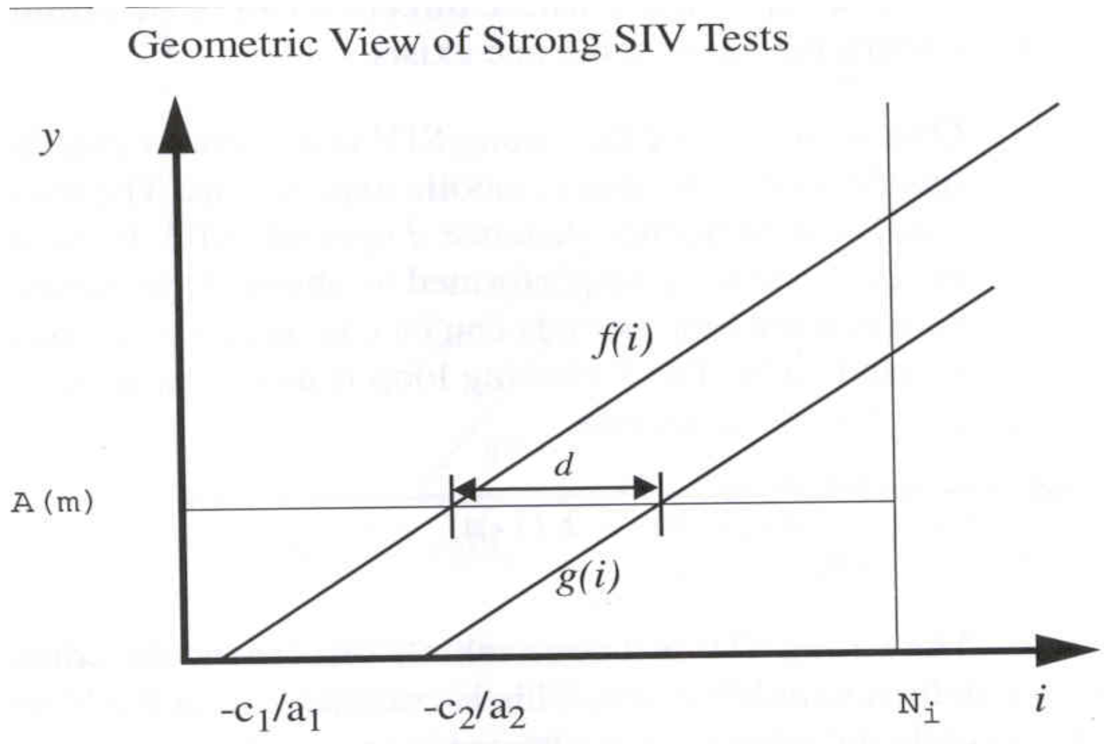
$$\langle 4i + 2, 4i + 4 \rangle$$

- Strong subscripts are also referred to as "uniformly generated"

Strong SIV Test Example

```
DO k = 1, 100
   DO j = 1, 100
S1       A(j+1,k) = ...
S2            ... = A(j,k) + 32
   ENDDO
ENDDO
```

Strong SIV Test



$$d = i' - i = \frac{c_1 - c_2}{a}$$

Dependence exists if there is an integer value of d within loop bounds,

$$|d| \leq U - L$$

Weak SIV Tests

- Weak SIV subscripts are of the form

$$\langle a_1 i + c_1, a_2 i + c_2 \rangle$$

where $a_1 \neq 0$ (without loss of generality)

- For example the following are weak SIV subscripts

$$\langle i + 1, 5 \rangle$$

$$\langle 2i + 1, i + 5 \rangle$$

$$\langle 2i + 1, -2i \rangle$$

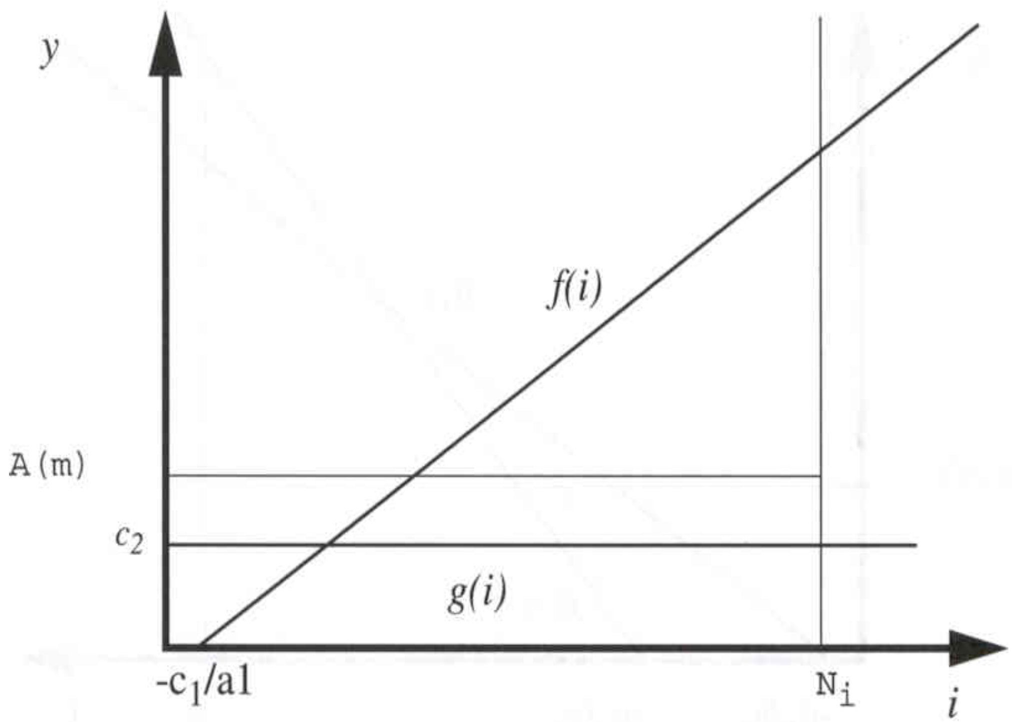
Weak-zero SIV Test

- Special case of Weak SIV where one of the coefficients (a_2) of the index is zero
- The test consists merely of checking whether the solution is an integer and is within loop bounds

$$i = \frac{c_2 - c_1}{a_1}$$

Weak-zero SIV Test

Geometric View of Weak-zero SIV Subscripts



Weak-crossing SIV Test

- Special case of Weak SIV where the coefficients of the index are equal in magnitude but opposite in sign i.e., $a_2 = -a_1$
- The test consists merely of checking whether the solution index

  is 1. within loop bounds and is

  2. either an integer or has a non-integer

  part equal to 1/2

$$i = \frac{c_2 - c_1}{2a_1}$$

Weak-crossing SIV Test: Checking Loop Bounds

- Consider the dependence equation

  a1 * i' + c1 = a2 * i" + c2

  → a1 * i' + c1 = -a1 * i" + c2

  → i' + i" = (c2 – c1) / a1

Two tests:

1) Is i' + i" in range?

- Min (i' + i") = 2*L
- Max (i' + i") = 2*U

→ Check if (c2 – c1) / a1 is < 2*L or > 2*U

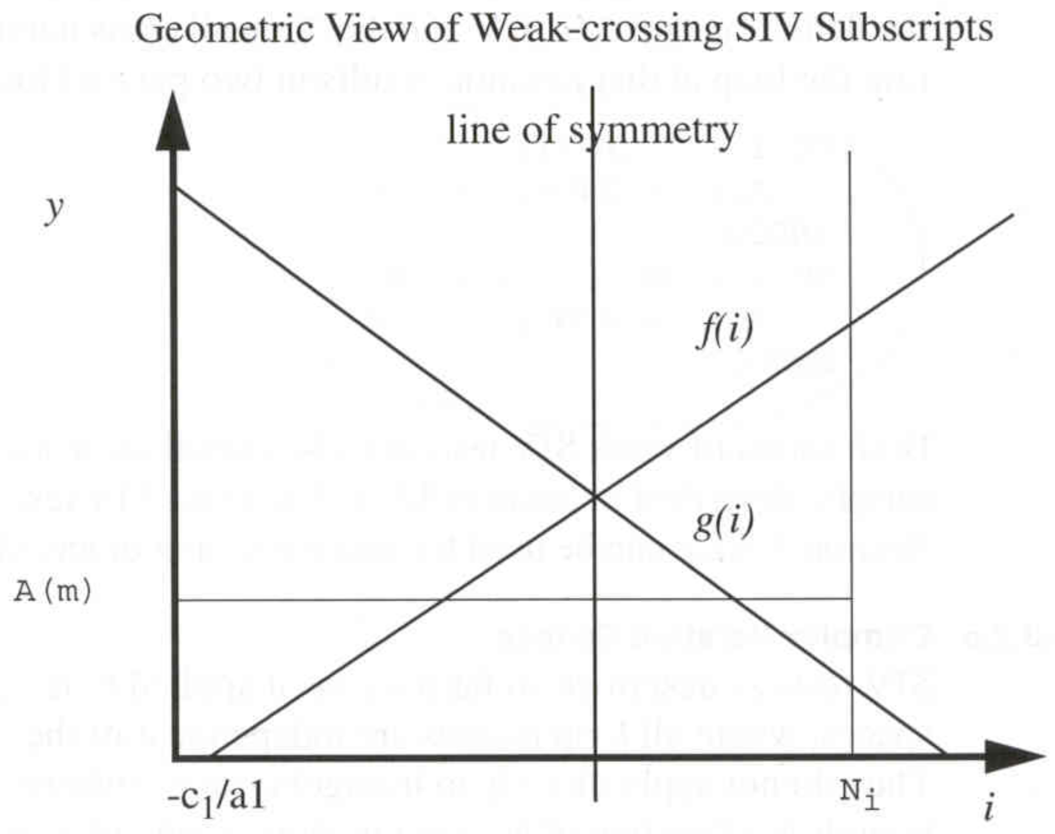
→ Check if (c2 – c1) / (2*a1) is < L or > U

2) Is i' + i" an integer?

→ Is (c2 – c1) / a1 an integer?

→ Is (c2 – c1) / (2*a1) an integer, or does it have a non-integer part equal to 1/2?

Weak-crossing SIV Test



Weak-crossing SIV & Loop Splitting

```
              DO i = 1, N
S1            A(i) = A(N-i+1) + C
              ENDDO
```

This loop can be split into...

```
              DO i = 1,(N+1)/2
                     A(i) = A(N-i+1) + C
              ENDDO
              DO i = (N+1)/2 + 1, N
                     A(i) = A(N-i+1) + C
              ENDDO
```

Worksheet (can be done in groups)

Name: ______________________ Netid: ______________________

```
DO i = 1, N
   Y(i, N) = Y(1, N) + Y(N, N)
ENDDO
```

Consider the above loop:

1) Perform the Weak-zero SIV test on this loop, and indicate if a dependence exists.

2) If so, can you suggest alternate loop bounds for which no dependence would exist?