Acknowledgments

- Slides from previous offerings of COMP 515 by Prof. Ken Kennedy
Dependence Testing

Allen and Kennedy, Chapter 3 (contd)
Theorem 3.3 (Banerjee). Let $D$ be a direction vector, and $h$ be a dependence function. $h = 0$ can be solved in the region $R$ iff:

$$\sum_{i=1}^{n} H_i^-(D_i) \leq b_0 - a_0 \leq \sum_{i=1}^{n} H_i^+(D_i)$$

Proof: Immediate from Lemma 3.3 and the IMV.
Example of using Banerjee Inequality (Recap)

DO I = 1, N
  DO J = 1, M
    DO K = 1, 100
      A(I,K) = A(I+J,K) + B
    ENDDO
  ENDDO
ENDDO

Testing (I, I+J) for D = (=,<,\#):  

\[
H_1^- (=) + H_2^- (<) = -(1-0)^- N + (1-1)^+ 1 - (0^- + 1^+) (M-1) + [(0^- + 1^-) + 0^+]1 - 1 = -M \leq 0
\]

\[
\leq H_1^+ (=) + H_2^+ (<) = (1-1)^+ N - (1-1)^- 1 + (0^+ - 1^+) (M-1) - [(0^+ - 1^-) + 0^-]1 - 1 \leq -2
\]

This is impossible, so the dependency doesn’t exist.
Testing Direction Vectors

- Must test pair of statements for all direction vectors.
- Potentially exponential in loop nesting.
- Can save time by pruning:

```
• (<, <, *)
• (<, <=, *)
• (<, >, *)
• (=, *, *)
• (>*, *, *)
• (*, *, *)
```

Should be (<, <=, >) Implausible
Coupled Groups

- So far, we’ve assumed separable subscripts.
- We can glean information from separable subscripts, and use it to split coupled groups.
- Most subscripts tend to be SIV, so this works pretty well.
DO I
  DO J
    DO K
      A(J-I, I+1, J+K) = A(J-I, I, J+K)
    ENDDO
  ENDDO
ENDDO

• The delta test gives us a distance vector of (1,1,-1) for this loop nest.
• First pass: establish $\Delta I = 1$ from second dimension
• Second pass: Propagate into first dimension to obtain $\Delta J = 1$
• Third pass: Propagate into third dimension to obtain $\Delta K = -1$
• WORKSHEET: write the dependence equations for this example, and confirm that (1,1,-1) is a correct solution
Delta Test

• Constraint vector $C$ for a subscript group, contains one constraint for each index in group.

• The Delta test derives and propagates constraints from SIV subscripts.

• Constraints are also propagated from Restricted Double Index Variable (RDIV) subscripts, those of the form

\[ < a_i i + c_1, a_j j + c_2 > \]

• See Figure 3.13 in textbook for Delta test algorithm
Basic dependence algorithm (for a given direction vector)

Figure out what sort of subscripts we have
Partition subscripts into coupled groups
for each separable subscript
    test it using appropriate test
    if no dependence, we’re done
for each coupled group
    use delta test
    if no dependence, we’re done
return dependence otherwise

- For more advanced dependence tests, see the Omega Project [http://www.cs.umd.edu/projects/omega/](http://www.cs.umd.edu/projects/omega/) and Polyhedral compiler frameworks
Preliminary Transformations

Chapter 4 of Allen and Kennedy
Overview

• Why do we need preliminary transformations?
• To create canonical representations of loop nests that simplify dependence testing
  — Requirements of dependence testing
    - Stride 1
    - Normalized loop
    - Linear subscripts
    - Subscripts composed of functions of loop induction variables
  — Higher dependence test accuracy
  — Easier implementation of dependence tests
An Example

- Programmer optimized code
  — Confusing to smart compilers

INC = 2
KI = 0
DO I = 1, 100
  DO J = 1, 100
    KI = KI + INC
    U(KI) = U(KI) + W(J)
  ENDDO
S(I) = U(KI)
ENDDO
An Example

• Applying Induction-Variable Substitution (IVS)
  — Replace references to induction variables with functions of loop index for the purpose of dependence analysis

```
INC = 2
KI = 0
DO I = 1, 100
  DO J = 1, 100
    ! Deleted: KI = KI + INC
    U(KI + J*INC) = U(KI + J*INC) + W(J)
  ENDDO
  KI = KI + 100 * INC
ENDDO
S(I) = U(KI)
```

• In practice, induction variable information is often stored as “look-aside” information without actually transforming the code
  — Depends on whether optimizing back-end will strength-reduce the multiply operations
An Example

- **Second application of IVS**
  - Remove all references to KI

\[
\begin{align*}
\text{INC} &= 2 \\
\text{KI} &= 0 \\
\text{DO } I &= 1, 100 \\
&\quad \text{DO } J = 1, 100 \\
&\quad \quad U(\text{KI} + (I-1)*100*\text{INC} + J*\text{INC}) = \\
&\quad \quad U(\text{KI} + (I-1)*100*\text{INC} + J*\text{INC}) + W(J) \\
&\quad \quad \text{ENDDO} \\
&\quad ! \text{Deleted: KI = KI + 100 * INC} \\
&\quad S(I) = U(\text{KI} + I * (100*\text{INC})) \\
&\quad \text{ENDDO} \\
&\quad \text{KI = KI + 100 * 100 * INC}
\end{align*}
\]
An Example

- **Applying Constant Propagation**
  - *Substitute the constants*

  INC = 2
  ! Deleted: KI = 0
  DO I = 1, 100
    DO J = 1, 100
      U(I*200 + J*2 - 200) = U(I*200 + J*2 -200) + W(J)
      ENDDO
    S(I) = U(I*200)
    ENDDO
  KI = 20000
An Example

- **Applying Dead Code Elimination**
  - Removes all unused code

```plaintext
DO I = 1, 100
  DO J = 1, 100
    U(I*200 + J*2 - 200) = U(I*200 + J*2 - 200) + W(J)
  ENDDO
  S(I) = U(I*200)
ENDDO
```
Information Requirements

- Transformations need knowledge
  - Loop Stride
  - Loop-invariant quantities
  - Constant-values assignment
  - Usage of variables
Loop Normalization

• Transform loop so that
  – The new stride becomes +1 (more important)
  – The new lower bound becomes +1 (less important)

• To make dependence testing as simple as possible

• Serves as information gathering phase
Loop Normalization

- Caveat
  - Un-normalized:
    DO I = 1, M
    DO J = I, N
    A(J, I) = A(J, I - 1) + 5
    ENDDO
    ENDDO
    Has a direction vector of (<,=)

  - Normalized:
    DO I = 1, M
    DO J = 1, N - I + 1
    A(J + I - 1, I) = A(J + I - 1, I - 1) + 5
    ENDDO
    ENDDO
    Has a direction vector of (<,>)
Loop Normalization

• Caveat
  – Consider interchanging loops
    - $(<,=)$ becomes $(=,<)$ OK
    - $(<,>)$ becomes $(>,<)$
      Problem (as we will study later)
        Handled by another transformation (loop skewing)
  – What if the step size is symbolic?
    - Prohibits dependence testing
    - Workaround: use step size 1 (if we know step size is positive)
      Less precise, but allow dependence testing
Definition-use Graph

- Traditionally called Definition-use Chains
- Provides the map of variables usage
- Heavily used by preliminary transformations
Definition-use Graph

- Definition-use graph is a graph that contains an edge from each definition point in the program to every possible use of the variable at run time.
- uses(b): the set of all variables used within the block b that have no prior definitions within the block.
- defsout(b): the set of all definitions within block b that are not killed within the block.
- killed(b): the set of all definitions that define variables killed by other definitions within block b.

\[
reaches(b) = \bigcup_{p \in P(b)} (\text{defsout}(p) \cup (\text{reaches}(p) \cap \neg \text{killed}(p)))
\]
Dead Code Elimination

• Removes all dead code

• What is Dead Code?
  — Code whose results are never used in any ‘Useful statements’

• What are Useful statements?
  — Are they simply output statements?
  — Output statements, input statements, control flow statements, and their required statements

• Makes code cleaner

• Note that Dead Code is different from Unreachable Code
  — Unreachable code is code that can never be reached e.g., code for which all control conditions always evaluate to false
procedure eliminateDeadCode(P);

    // P is the procedure in which constants are to be propagated
    // Assume the availability of def-use chains for all the statements in P

    let worklist := \{absolutely useful statements\};

    while worklist \neq \emptyset do begin
        x := an arbitrary element of worklist;
        mark x useful;
        worklist := worklist \setminus \{x\};
        for all \((y,x) \in \text{defuse}\) do
            if y is not marked useful then worklist := worklist \cup \{y\};
        end
    delete every statement that is not marked useful;

    end eliminateDeadCode
Homework #3 (Written Assignment)

1. Solve exercise 3.6 in book
   — This is case 4 of Lemma 3.3
   — Read Definitions 3.1, 3.2, 3.3 and Lemmas 3.1, 3.2, 3.3 before starting

• Due in class on Thursday, Oct 3rd

• Honor Code Policy: All submitted homeworks are expected to be the result of your individual effort. You are free to discuss course material and approaches to problems with your other classmates, the teaching assistants and the professor, but you should never misrepresent someone else’s work as your own. If you use any material from external sources, you must provide proper attribution.