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# COMP 515: Advanced Compilation for Vector and Parallel Processors

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# COMP 515 Projects

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- Yuhan Peng, Maggie Tang
  - DFGL transformations and OpenCL generation
- Prasanth Chatarasi
  - Polyhedral extensions for data race detection
- Lucas Martinelli, Jonathan Sharman,
  - Exploration of dependences and transformations in higher level OO languages, with a focus on C++ language and libraries (RAJA, Kokkos)
- Jack Feser
  - Exploration of ILP solvers for dependence analysis
- Pete Curry, Lung Li
  - OpenCL transformations for Digital Signal Processors
- Zhipeng Wang
  - Memory Hierarchy Management for iterative graph structures.

# Simple Vectorization Algorithm (Recap)

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procedure vectorize (L, D)

// L is the maximal loop nest containing the statement.

// D is the dependence graph for statements in L.

1. find the set  $\{S_1, S_2, \dots, S_m\}$  of maximal strongly-connected regions in the dependence graph D restricted to L (Tarjan);
2. construct  $L_p$  from L by reducing each  $S_i$  to a single node and compute  $D_p$ , the dependence graph naturally induced on  $L_p$  by D;
3. let  $\{p_1, p_2, \dots, p_m\}$  be the m nodes of  $L_p$  numbered in an order consistent with  $D_p$  (use topological sort);
4.     for i = 1 to m do begin  
        if  $p_i$  is a dependence cycle then  
           generate a DO-loop nest around the statements in  $p_i$ ;  
        else  
           directly rewrite  $p_i$  in Fortran 90, vectorizing it with respect to every loop containing it;  
        end  
   end  
end vectorize

# Problems With Simple Vectorization

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```
DO I = 1, N
    DO J = 1, M
S1         A(I+1,J) = A(I,J) + B
    ENDDO
ENDDO
```

- Dependence from  $S_1$  to itself with  $d(i, j) = (1, 0)$
- Key observation: Since dependence is at level 1, we can vectorize the inner loop!
- Can be converted to:

```
DO I = 1, N
S1     A(I+1,1:M) = A(I,1:M) + B
ENDDO
```

- The simple algorithm does not capitalize on such opportunities

# Advanced Vectorization Algorithm (Recursive “codegen” procedure)

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```
procedure codegen(R, k, D);
// R is the region for which we must generate code.
// k is the minimum nesting level of possible parallel loops.
// D is the dependence graph among statements in R..
1. find the set  $\{S_1, S_2, \dots, S_m\}$  of maximal strongly-connected regions in the dependence graph D
   restricted to R;
2. construct  $R_p$  from R by reducing each  $S_i$  to a single node and compute  $D_p$ , the dependence graph
   naturally induced on  $R_p$  by D;
3. let  $\{p_1, p_2, \dots, p_m\}$  be the m nodes of  $R_p$  numbered in an order consistent with  $D_p$  (topological sort);
4. for i = 1 to m do begin
   if  $p_i$  is cyclic then begin
       generate a level-k DO statement;
       let  $D_i$  be the dependence graph consisting of all dependence edges in D that are at level k+1
         or greater and are internal to  $p_i$ ;
       codegen ( $p_i, k+1, D_i$ );
       generate the level-k ENDDO statement;
   end
   else
       generate a vector statement for  $p_i$  in  $r(p_i)-k+1$  dimensions, where  $r(p_i)$  is the number of loops
         containing  $p_i$ ;
   end
end

codegen(L, 1, D); // Root call for recursive “codegen” procedure
```

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# Advanced Vectorization Algorithm

---

```
DO I = 1, 100
S1      X(I) = Y(I) + 10
DO J = 1, 100
S2      B(J) = A(J,N)
          DO K = 1, 100
S3      A(J+1,K)=B(J)+C(J,K)
          ENDDO
S4      Y(I+J) = A(J+1, N)
ENDDO
ENDDO
```

- codegen called at the outermost level
- S<sub>1</sub> will be vectorized, and moved later due to topological sort

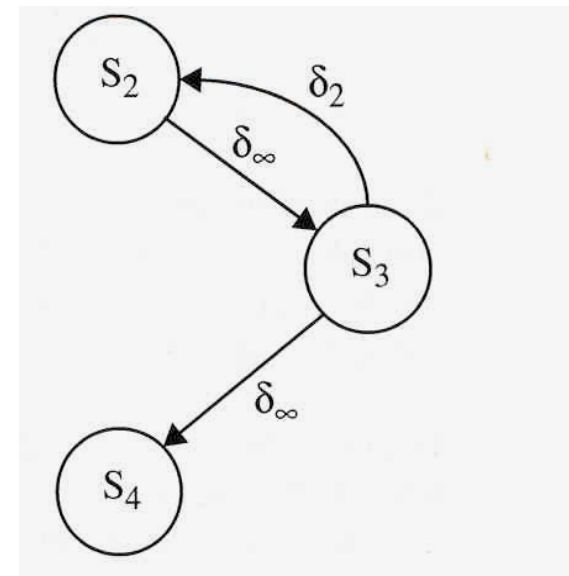
```
DO I = 1, 100
      codegen({S2, S3, S4}, 2, D)
ENDDO
X(1:100) = Y(1:100) + 10
```

# Advanced Vectorization Algorithm

- `codegen ({S2, S3, S4}, 2, D)`
- level-1 dependences are stripped off

```
DO I = 1, 100
  DO J = 1, 100
    codegen({S2, S3}, 3, D)
  ENDDO
S4 Y(I+1:I+100) = A(2:101,N)
ENDDO

X(1:100) = Y(1:100) + 10
```

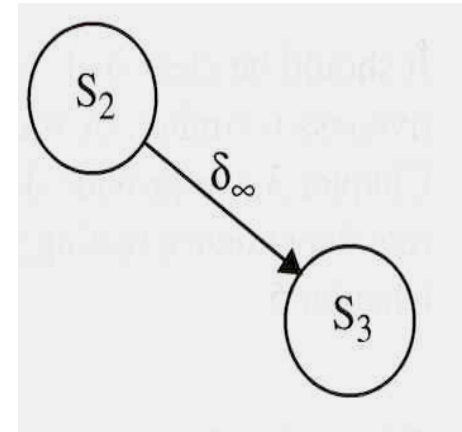


# Advanced Vectorization Algorithm

- codegen ( $\{S_2, S_3\}, 3, D$ )
- level-2 dependences are stripped off

```
DO I = 1, 100
  DO J = 1, 100
    B(J) = A(J,N)
    A(J+1,1:100)=B(J)+C(J,1:100)
  ENDDO
  Y(I+1:I+100) = A(2:101,N)
ENDDO
X(1:100) = Y(1:100) + 10
```

```
DO I = 1, 100
S1      X(I) = Y(I) + 10
  DO J = 1, 100
S2      B(J) = A(J,N)
          DO K = 1, 100
S3      A(J+1,K)=B(J)
          +C(J,K)
          ENDDO
S4      Y(I+J) = A(J+1, N)
  ENDDO
ENDDO
```

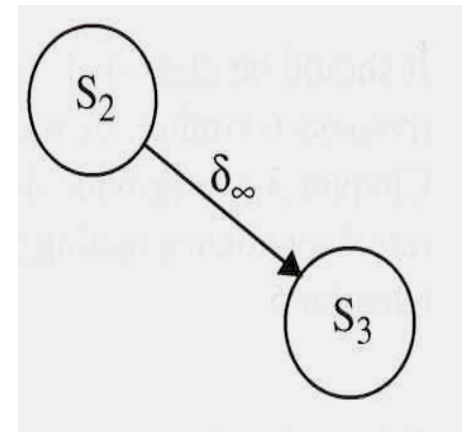




# Advanced Vectorization Algorithm (shown as distributed parallel loops)

```
DO I = 1, 100
  DO J = 1, 100
    S2:    B(J) = A(J,N)
          DOALL K = 1, 100
    S3:    A(J+1,K) = B(J) + C(J,K)
          ENDDO
        ENDDO
      DOALL J = 1, 100
    S4:    Y(I+J) = A(J+1,N)
          END DO
        ENDDO
      DOALL I = 1, 100
    S1:    X(I) = Y(I) + 10
          END DO
```

```
DO I = 1, 100
  S1      X(I) = Y(I) + 10
  DO J = 1, 100
    S2      B(J) = A(J,N)
            DO K = 1, 100
    S3      A(J+1,K) = B(J)
            +C(J,K)
            ENDDO
    S4      Y(I+J) = A(J+1, N)
          ENDDO
        ENDDO
```



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# Enhancing Fine-Grained Parallelism

Chapter 5 of *Allen and Kennedy*

# Fine-Grained Parallelism

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Techniques to enhance fine-grained (vector) parallelism:

- Loop Interchange
- Scalar Expansion
- Scalar Renaming
- Array Renaming

# Loop Shifting (Permutation)

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- **Motivation:** Identify loops which can be moved and interchange them to “optimal” nesting levels
- **Theorem 5.3** In a perfect loop nest, if loops at level  $i, i+1, \dots, i+n$  carry no dependence, it is always legal to shift these loops inside of loop  $i+n+1$ . Furthermore, these loops will not carry any dependences in their new position.

# Loop Shifting

```
DO I = 1, N
  DO J = 1, N
    DO K = 1, N
      S      A(I,J) = A(I,J) + B(I,K)*C(K,J)
    ENDDO
  ENDDO
ENDDO
```

**I J K**  
**(=, =, <)**

- **S** has true, anti and output dependences on itself, hence codegen will fail as recurrence exists at innermost level
- Use loop shifting to shift loops I and J inside loop K:

```
DO K = 1, N
  DO I = 1, N
    DO J = 1, N
      S      A(I,J) = A(I,J) + B(I,K)*C(K,J)
    ENDDO
  ENDDO
ENDDO
```

# Loop Shifting

---

```
DO K= 1, N
  DO I = 1, N
    DO J = 1, N
S      A(I, J) = A(I, J) + B(I, K) * C(K, J)
    ENDDO
  ENDDO
ENDDO
```

K I J  
(<, =, =)

**codegen vectorizes to:**

```
DO K = 1, N

  A(1:N, 1:N) = A(1:N, 1:N) + SPREAD(B(1:N, K), 2) * SPREAD(C(K, 1:N), 1)

ENDDO
```

# Loop Selection

- **Loop Shifting doesn't always find the best loop to move. Consider:**

```
DO I = 1, N
  DO J = 1, M
S      A(I+1, J+1) = A(I, J) + A(I+1, J)
  ENDDO
ENDDO
```

- **Direction matrix:**  $\begin{pmatrix} < & < \\ = & < \end{pmatrix}$
- **Loop shifting algorithm will fail to uncover vector loops; however, interchanging the loops can lead to:**

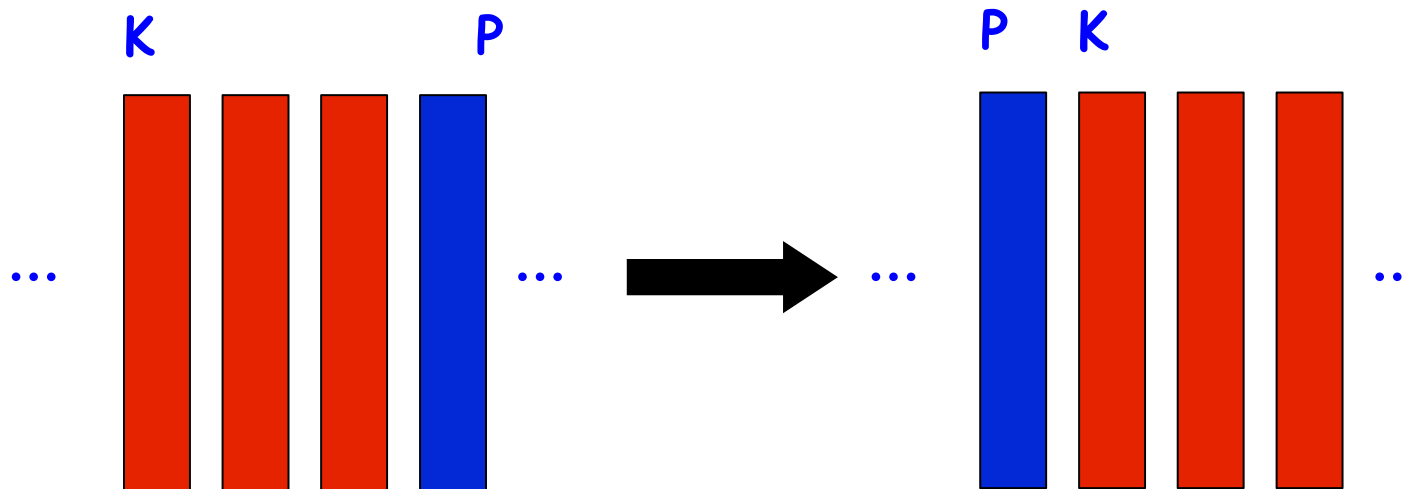
```
DO J = 1, M
  A(2:N+1, J+1) = A(1:N, J) + A(2:N+1, J)
ENDDO
```

- **Need a more general algorithm**  $\begin{pmatrix} < & < \\ < & = \end{pmatrix}$

# Loop Selection

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- Loop selection:
  - Select a loop at nesting level  $p \geq k$  that can be safely moved outward to level  $k$  and shift the loops at level  $k, k+1, \dots, p-1$  inside it





# Fully Permutable Loop Nest

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- A contiguous set of  $k \geq 1$  loops,  $i_j, \dots, i_{j+k-1}$  is fully permutable if all permutations of  $i_j, \dots, i_{j+k-1}$  are legal
- Data dependence test: Loops  $i_j, \dots, i_{j+k-1}$  are fully permutable if for each dependence vector  $(d_1, \dots, d_n)$  carried at levels  $j \dots j+k-1$ , each of  $d_j, \dots, d_{j+k-1}$  is non-negative
- Fundamental result (to be discussed later in course): a set of  $k$  fully permutable loops can be transformed using only Interchange, Reversal and Skewing transformations into an equivalent set of  $k$  loops where  $k-1$  of the loops are parallel

# Scalar Expansion and its use in Removing Anti and Output Dependences

```

DO I = 1, N
S1   T = A(I)
S2   A(I) = B(I)
S3   B(I) = T
      ENDDO
  
```

- **Scalar Expansion:**

```

DO I = 1, N
S1   T$(I) = A(I)
S2   A(I) = B(I)
S3   B(I) = T$(I)
      ENDDO
      T = T$(N)
  
```

- **leads to:**

```

S1   T$(1:N) = A(1:N)
S2   A(1:N) = B(1:N)
S3   B(1:N) = T$(1:N)

      T = T$(N)
  
```

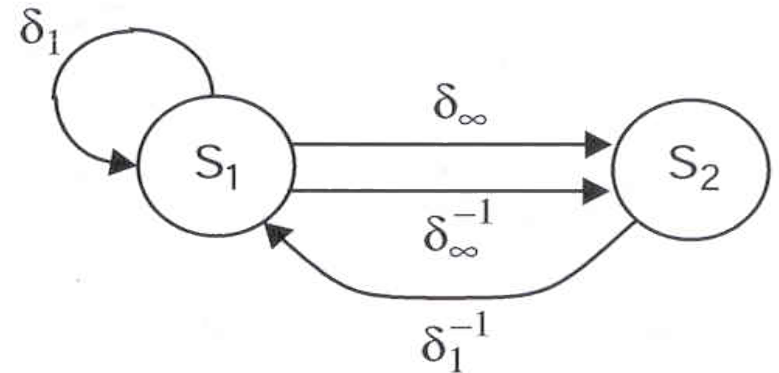
# Scalar Expansion

- However, scalar expansion (or any other form of storage duplication) is not useful in removing true dependences. Consider:

```
DO I = 1, N
  T = T + A(I) + A(I+1)
  A(I) = T
ENDDO
```

- **Scalar expansion gives us:**

```
T$(0) = T
DO I = 1, N
S1    T$(I) = T$(I-1) + A(I) + A(I+1)
S2    A(I) = T$(I)
ENDDO
T = T$(N)
```



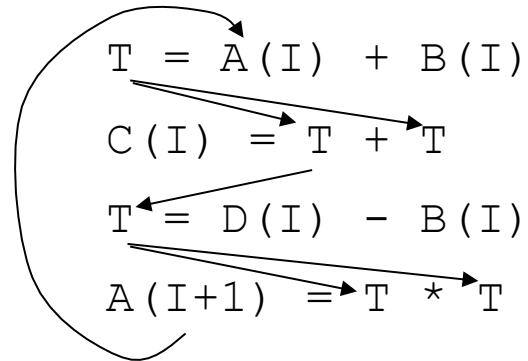
# Scalar Expansion: Safety

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- Scalar expansion is always safe
- When is it useful?
  - Brute force approach: Expand all scalars, vectorize, shrink all unnecessary expansions.
  - However, we want to predict when expansion is useful i.e., when scalar expansion can enable a dependence edge to be deleted
- Dependences due to reuse of memory location vs. reuse of values
  - Dependences due to reuse of **values** must be preserved (true dependences)
  - Dependences due to reuse of **memory location** can be deleted by expansion (anti & output dependences)
    - This is also why functional languages are easier to parallelize, at the cost of increased memory overhead

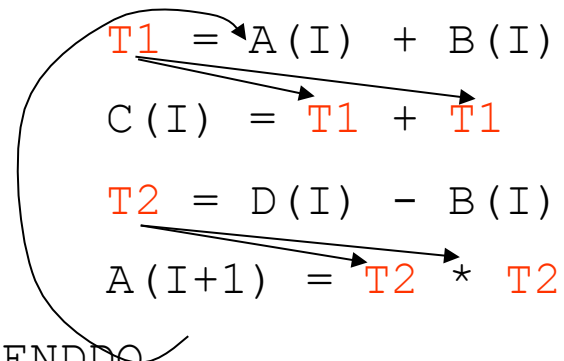
# Scalar Renaming

```
DO I = 1, 100
S1   T = A(I) + B(I)
S2   C(I) = T + T
S3   T = D(I) - B(I)
S4   A(I+1) = T * T
ENDDO
```



- Renaming scalar T:

```
DO I = 1, 100
S1   T1 = A(I) + B(I)
S2   C(I) = T1 + T1
S3   T2 = D(I) - B(I)
S4   A(I+1) = T2 * T2
ENDDO
```



# Scalar Renaming

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- will lead to:

$$S_3 \quad T2\$(1:100) = D(1:100) - B(1:100)$$

$$S_4 \quad A(2:101) = T2\$(1:100) * T2\$(1:100)$$

$$S_1 \quad T1\$(1:100) = A(1:100) + B(1:100)$$

$$S_2 \quad C(1:100) = T1\$(1:100) + T1\$(1:100)$$

$$T = T2\$(100)$$

# Homework #3 (Written Assignment)

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## 1. Solve exercise 5.6 in book

—Your solution should be legal for all values of  $K$  (note that the value of  $K$  is invariant in loop  $I$ )

Exercise 5.6: What vector code should be generated for the following loop?

```
DO I = 1, 100
```

```
  A(I) = B(K) + C(I)
```

```
  B(I+1) = A(I) + D(I)
```

```
END DO
```

- Due in class on Thursday, Oct 8<sup>th</sup>