
COMP 515: Advanced Compilation for Vector and Parallel Processors

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Enhancing Fine-Grained Parallelism (contd)

Chapter 5 of Allen and Kennedy

Scalar Expansion and its use in Removing Anti and Output Dependences (Recap)

```
DO I = 1, N
S1   T = A(I)
S2   A(I) = B(I)
S3   B(I) = T
      ENDDO
```

- **Scalar Expansion:**

```
DO I = 1, N
S1   T$(I) = A(I)
S2   A(I) = B(I)
S3   B(I) = T$(I)
      ENDDO
      T = T$(N)
```

- **leads to:**

```
S1   T$(1:N) = A(1:N)
S2   A(1:N) = B(1:N)
S3   B(1:N) = T$(1:N)
      T = T$(N)
```

Scalar Renaming (Recap)

```
DO I = 1, 100
S1   T = A(I) + B(I)
S2   C(I) = T + T
S3   T = D(I) - B(I)
S4   A(I+1) = T * T
ENDDO
```

- **Renaming scalar T:**

```
DO I = 1, 100
S1   T1 = A(I) + B(I)
S2   C(I) = T1 + T1
S3   T2 = D(I) - B(I)
S4   A(I+1) = T2 * T2
ENDDO
```

Scalar Renaming (Recap)

- **will lead to:**

$$S_3 \quad T2\$(1:100) = D(1:100) - B(1:100)$$

$$S_4 \quad A(2:101) = T2\$(1:100) * T2\$(1:100)$$

$$S_1 \quad T1\$(1:100) = A(1:100) + B(1:100)$$

$$S_2 \quad C(1:100) = T1\$(1:100) + T1\$(1:100)$$

$$T = T2\$(100)$$

Scalar Renaming: Profitability

- Scalar renaming will break recurrences in which a loop-independent output dependence or anti-dependence is a critical element of a cycle
- Relatively cheap to use scalar renaming
- Usually done by compilers when calculating live ranges for register allocation

Array Renaming

```

DO I = 1, N
S1   A(I) = A(I-1) + X
S2   Y(I) = A(I) + Z
S3   A(I) = B(I) + C
ENDDO

```

- $S_1 \delta_\infty S_2$ $S_2 \delta_\infty^{-1} S_3$ $S_3 \delta_1 S_1$ $S_1 \delta_\infty^0 S_3$

- **Rename $A(I)$ to $A'(I)$:**

```

DO I = 1, N
S1   A'(I) = A(I-1) + X
S2   Y(I) = A'(I) + Z
S3   A(I) = B(I) + C
ENDDO

```

- **Dependencies remaining:** $S_1 \delta_\infty S_2$ and $S_3 \delta_1 S_1$

Array Renaming: Profitability

- Examining dependence graph and determining minimum set of critical edges to break a recurrence is NP-complete!
- Solution: determine edges that are removed by array renaming and analyze effects on dependence graph
- procedure `array_partition`:
 - Assumes no control flow in loop body
 - Identifies collections of references to arrays which refer to the same value
 - Identifies deletable output dependences and antidependences
- Use this procedure to generate code
 - Minimize amount of copying back to the “original” array at the beginning and the end

Seen So Far...

- Uncovering potential vectorization in loops by
 - Loop Interchange
 - Scalar Expansion
 - Scalar and Array Renaming
- Safety and Profitability of these transformations

What's next ...

- More transformations to expose more fine-grained parallelism
 - Node Splitting
 - Recognition of Reductions
 - Index-Set Splitting
 - Run-time Symbolic Resolution
 - Loop Skewing
- Unified framework to generate vector code
- Note: these transformations are useful for generating other forms of parallel and locality-optimized code as well (beyond vectorization)

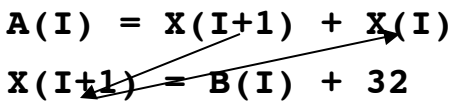
} Today's lecture

} Next lecture

Node Splitting

- Sometimes Renaming fails

```
DO I = 1, N
  S1:      A(I) = X(I+1) + X(I)
  S2:      X(I+1) = B(I) + 32
ENDDO
```



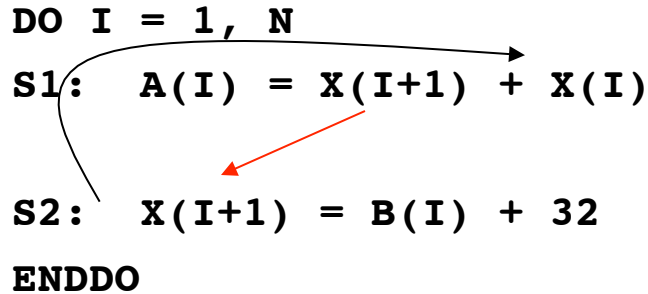
- Recurrence kept intact by renaming algorithm

```
DO I = 1, N
  S0:      X'(I) = X(I)
  S1:      A(I) = X(I+1) + X'(I)
  S2:      X'(I+1) = B(I) + 32
ENDDO
```

- **NOTE:** renaming $X(I)$ and $X(I+1)$ to $Z(I)$ and $Z(I+1)$ can help!

Node Splitting

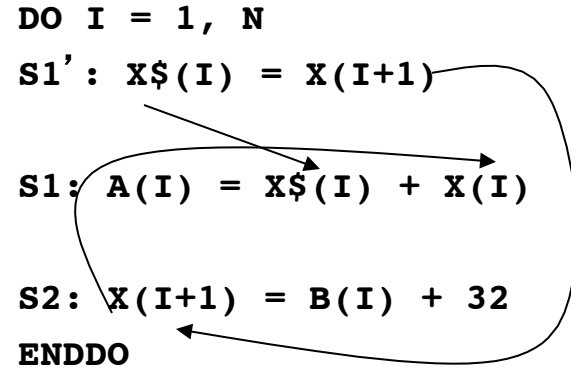
```
DO I = 1, N
S1:  A(I) = X(I+1) + X(I)
S2:  X(I+1) = B(I) + 32
ENDDO
```



- Break critical antidependence
- Make copy of read from which antidependence emanates

```
DO I = 1, N
S1': X$ = X(I+1)
S1:  A(I) = X$ + X(I)
S2:  X(I+1) = B(I) + 32
ENDDO
```

```
DO I = 1, N
S1': X$(I) = X(I+1)
S1:  A(I) = X$(I) + X(I)
S2:  X(I+1) = B(I) + 32
ENDDO
```



- Recurrence broken
 - Vectorized to
- ```
S1': X$(1:N) = X(2:N+1)
S2: X(2:N+1) = B(1:N) + 32
S1: A(1:N) = X$(1:N) + X(1:N)
```

# Node Splitting Algorithm

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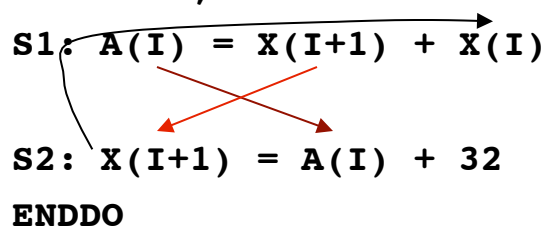
- Takes a constant loop independent antidependence  $D$
- Add new assignment  $x: T\$ = \text{source}(D)$
- Insert  $x$  before  $\text{source}(D)$
- Replace  $\text{source}(D)$  with  $T\$$
- Make changes in the dependence graph

# Node Splitting: Profitability

- Not always profitable

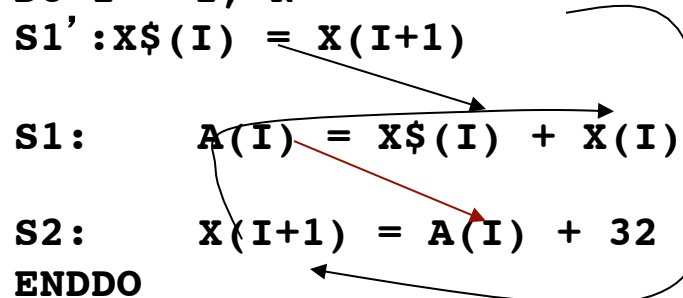
- For example

```
DO I = 1, N
S1: A(I) = X(I+1) + X(I)
S2: X(I+1) = A(I) + 32
ENDDO
```



- Node Splitting gives

```
DO I = 1, N
S1': X$(I) = X(I+1)
S1: A(I) = X$(I) + X(I)
S2: X(I+1) = A(I) + 32
ENDDO
```



- Recurrence still not broken
- Antidependence was not critical

# Node Splitting

---

- Determining minimal set of critical antidependences is NP-hard
- Perfect job of Node Splitting difficult
- Heuristic:
  - Select antidependences
  - Delete it to see if acyclic
  - If acyclic, apply Node Splitting

# Recognition of Reductions

---

- Sum Reduction, Min/Max Reduction, Count Reduction

- Vector ---> Single Element

```
S = 0.0
```

```
DO I = 1, N
```

```
 S = S + A(I)
```

```
ENDDO
```

- Not directly vectorizable



# Recognition of Reductions

---

- Assuming commutativity and associativity

```
S = 0.0
DO k = 1, 4
 SUM(k) = 0.0
 DO I = k, N, 4
 SUM(k) = SUM(k) + A(I)
 ENDDO
 S = S + SUM(k)
ENDDO
```

- Distribute k loop

```
S = 0.0
DO k = 1, 4
 SUM(k) = 0.0
ENDDO
DO k = 1, 4
 DO I = k, N, 4
 SUM(k) = SUM(k) + A(I)
 ENDDO
ENDDO
DO k = 1, 4
 S = S + SUM(k)
ENDDO
```

# Recognition of Reductions

---

- **After Loop Interchange**

```
DO I = 1, N, 4
 DO k = I, min(I+3,N)
 SUM(k-I+1) = SUM(k-I+1) + A(I)
 ENDDO
ENDDO
```

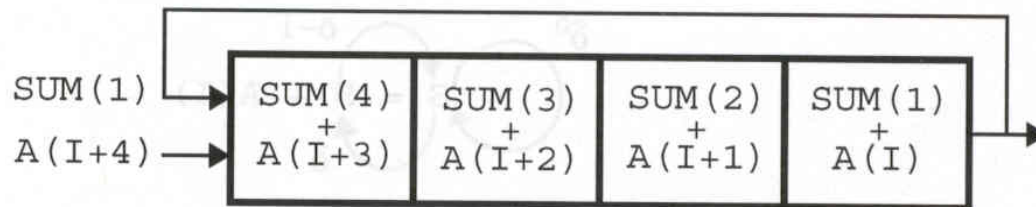
- **Vectorize**

```
DO I = 1, N, 4
 SUM(1:4) = SUM(1:4) + A(I:I+3)
ENDDO
```

# Recognition of Reductions

- Useful for vector machines with 4 stage pipeline, and fine-grain SIMD parallelism on modern processors (MMX, AltiVec)
- Recognize Reduction and Replace by the efficient version

Pipeline for Sum Reduction.



# Recognition of Reductions

---

- Properties of Reductions
  - Reduce Vector/Array to one element
  - No use of Intermediate values
  - Reduction operates on vector and nothing else
- Reduction recognized by
  - Presence of self true, output and anti dependences
  - Absence of other true dependences

# Index-set Splitting

---

- Subdivide loop into different iteration ranges to achieve partial parallelization
  - **Threshold Analysis** [Strong SIV, Weak Crossing SIV]
  - **Loop Peeling** [Weak Zero SIV]
  - **Section Based Splitting** [Variation of loop peeling]

# Threshold Analysis

```
DO I = 1, 20
 A(I+20) = A(I) + B
ENDDO
```

*Vectorize to..*

```
A(21:40) = A(1:20) + B
```

```
DO I = 1, 100
 A(I+20) = A(I) + B
ENDDO
```

*Strip mine to..*

```
DO I = 1, 100, 20
```

```
 DO ii = I, I+19
 A(ii+20) = A(ii) + B
 ENDDO
```

```
ENDDO
```

*Vectorize this*



# Threshold Analysis

---

- **Crossing thresholds**

```
DO I = 1, 100
 A(100-I) = A(I) + B
ENDDO
```

*Strip mine to...*

```
DO I = 1, 100, 50
 DO ii = I, I+49
 A(101-ii) = A(ii) + B
 ENDDO
ENDDO
```

*Vectorize to...*

```
DO I = 1, 100, 50
 A(101-I:51-I) = A(I:I+49) + B
ENDDO
```

# Loop Peeling

---

- Source of dependence is a single iteration

```
DO I = 1, N
 A(I) = A(I) + A(1)
ENDDO
```

*Loop peeled to..*

```
A(1) = A(1) + A(1)
DO I = 2, N
 A(I) = A(I) + A(1)
ENDDO
```

*Vectorize to..*

```
A(1) = A(1) + A(1)
A(2:N) = A(2:N) + A(1)
```



# Section-based Splitting

```
DO I = 1, N
 DO J = 1, N/2
S1: B(J,I) = A(J,I) + C
 ENDDO
 DO J = 1, N
S2: A(J,I+1) = B(J,I) + D
 ENDDO
ENDDO
```

- J Loop bound by recurrence due to B
- Only a portion of B is responsible for it

- Partition second loop into loop that uses result of S1 and loop that does not

```
DO I = 1, N
 DO J = 1, N/2
S1: B(J,I) = A(J,I) + C
 ENDDO
 DO J = 1, N/2
S2: A(J,I+1) = B(J,I) + D
 ENDDO
 DO J = N/2+1, N
S3: A(J,I+1) = B(J,I) + D
 ENDDO
ENDDO
```

# Section-based Splitting

---

```
DO I = 1, N
 DO J = 1, N/2
S1: B(J,I) = A(J,I) + C
 ENDDO
 DO J = 1, N/2
S2: A(J,I+1) = B(J,I) + D
 ENDDO
 DO J = N/2+1, N
S3: A(J,I+1) = B(J,I) + D
 ENDDO
ENDDO
```

- S3 now independent of S1 and S2

- Loop distribute to

```
DO I = 1, N
 DO J = N/2+1, N
S3: A(J,I+1) = B(J,I) + D
 ENDDO
ENDDO
DO I = 1, N
 DO J = 1, N/2
S1: B(J,I) = A(J,I) + C
 ENDDO
 DO J = 1, N/2
S2: A(J,I+1) = B(J,I) + D
 ENDDO
ENDDO
```

# Section-based Splitting

```
DO I = 1, N
 DO J = N/2+1, N
S3: A(J,I+1) = B(J,I) + D
 ENDDO
ENDDO
```

- **Vectorized to**  
 $A(N/2+1:N, 2:N+1) = B(N/2+1:N, 1:N) + D$

```
DO I = 1, N
 DO J = 1, N/2
S1: B(J,I) = A(J,I) + C
 ENDDO
 DO J = 1, N/2
S2: A(J,I+1) = B(J,I) + D
 ENDDO
ENDDO
```

```
DO I = 1, N
 B(1:N/2, I) = A(1:N/2, I) + C
 A(1:N/2, I+1) = B(1:N/2, I) + D
ENDDO
```

# Homework #3 (REMINDER)

---

## 1. Solve exercise 5.6 in book

— Your solution should be legal for all values of  $K$  (note that the value of  $K$  is invariant in loop  $I$ )

Exercise 5.6: What vector code should be generated for the following loop?

```
DO I = 1, 100
```

```
 A(I) = B(K) + C(I)
```

```
 B(I+1) = A(I) + D(I)
```

```
END DO
```

- Due in class on Thursday, Oct 8<sup>th</sup>