COMP 322: Fundamentals of Parallel Programming

Lecture 21: Linearizability of Concurrent Objects

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Announcements

- Graded midterm exams can be picked up from Amanda Nokleby in Duncan Hall room 3137
- Homework 5 (written assignment) has been posted
  - Deadline: 5pm on Friday, March 18th
- Homework 6 (HJ programming assignment) will be given on March 18th
- Homework 7 (Concurrent Java programming assignment) will be given on April 1st (really!)

Acknowledgments for Today's Lecture

- Lecture 21 handout
- Maurice Herlihy and Nir Shavit. The art of multiprocessor programming. Morgan Kaufmann, 2008.
  - Optional text for COMP 322
  - Slides and code examples extracted from http://www.elsevierdirect.com/companion.jsp?ISBN=9780123705914

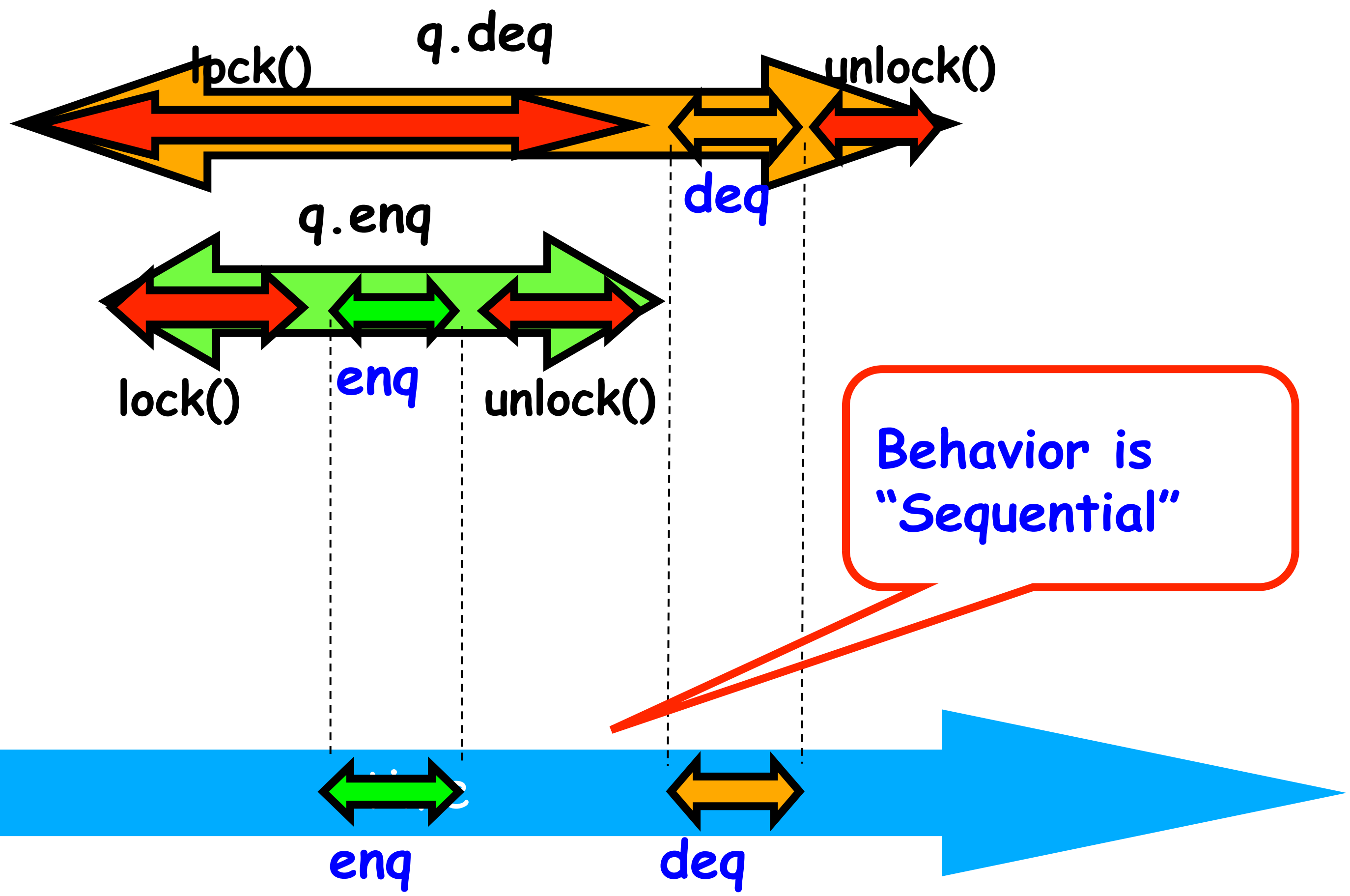
Concurrent Objects

- A concurrent object is an object that can correctly handle methods invoked in parallel by different tasks or threads
  - Originally referred to as monitors
  - Also informally referred to as "thread-safe objects"
- For simplicity, it is usually assumed that the body of each method in a concurrent object is itself sequential
  - Assume that method does not create child async tasks
- Implementations of methods can be *serial* (e.g., enclose each method in an isolated statement like a critical section) or *concurrent* (e.g., ConcurrentHashMap, ConcurrentLinkedQueue and CopyOnWriteArraySet)
- A desirable goal is to develop method implementations that are concurrent while being as close to the semantics of the serial version as possible

The Big Question!

- Consider a simple FIFO (First In, First Out) queue as a canonical example of a concurrent object
  - Method q.enq(o) inserts object o at the tail of the queue
    - Assume that there is unbounded space available for all enq() operations to succeed
  - Method q.deq() removes and returns the item at the head of the queue.
    - Throws EmptyException if the queue is empty.
- What does it mean for a *concurrent* object like a FIFO queue to be correct?
  - What *is* a concurrent FIFO queue?
  - FIFO means strict temporal order
  - Concurrent means ambiguous temporal order

Describing the concurrent via the sequential



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Informal definition of Linearizability

1. A *linearizable execution* is one in which the semantics of a set of method calls performed in parallel on a concurrent object is equivalent to that of some legal linear sequence of those method calls.

2. A *linearizable concurrent object* is one for which all possible executions are linearizable.

Table 1: Example execution of a monitor-based implementation of FIFO queue q

Is this a linearizable execution?

Time	Task A	Task B
0	Invoke `q.enq(x)`	
1	Work on `q.enq(x)`	
2	Work on `q.enq(x)`	
3	Return from `q.enq(x)`	
4		Invoke `q.enq(y)`
5		Work on `q.enq(y)`
6		Work on `q.enq(y)`
7		Return from `q.enq(y)`
8		Invoke `q.deq()`
9		Return `x` from `q.deq()`

Yes! Equivalent to "q.enq(x) ; q.enq(y) ; q.deq():x"

Table 2: Example execution of method calls on a concurrent FIFO queue q

Is this a linearizable execution?

Time	Task A	Task B
0	Invoke `q.enq(x)`	
1	Work on `q.enq(x)`	Invoke `q.enq(y)`
2	Work on `q.enq(x)`	Return from `q.enq(y)`
3	Return from `q.enq(x)`	
4		Invoke `q.deq()`
5		Return `x` from `q.deq()`

Yes! Equivalent to "q.enq(x) ; q.enq(y) ; q.deq():x"

- Would the execution be linearizable if q.deq() returned y instead of x?

Table 3: Example of a non-linearizable execution on a concurrent FIFO queue q

Is this a linearizable execution?

Time	Task A	Task B
0	Invoke `q.enq(x)`	
1	Return from `q.enq(x)`	
2		Invoke `q.enq(y)`
3	Invoke `q.deq()`	Work on `q.enq(y)`
4	Work on `q.deq()`	Return from `q.enq(y)`
5	Return y from `q.deq()`	

- No! q.enq(x) must precede q.enq(y) in all linear sequences of method calls invoked on q. It is illegal for the q.deq() operation to return y.

Alternate definition of Linearizability

- Assume that each method call takes effect "instantaneously" at some distinct point in time between its invocation and return.
- Execution is linearizable if we can choose instantaneous points that are consistent with a sequential execution in which methods are executed at those points

Table 2: Example execution of method calls on a concurrent FIFO queue q

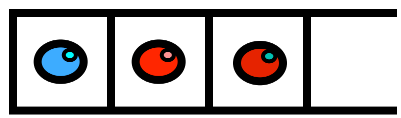
Is this a linearizable execution?

Time	Task *A*	Task *B*
0	Invoke `q.enq(x)`	
1	Work on `q.enq(x)`	Invoke `q.enq(y)`
2	Work on `q.enq(x)`	Return from `q.enq(y)`
3	Return from `q.enq(x)`	
4		Invoke `q.deq()`
5		Return `x` from `q.deq()`

Yes! Equivalent to "q.enq(x) ; q.enq(y) ; q.deq():x"

- Would the execution be linearizable if q.deq() returned y instead of x?

An Example



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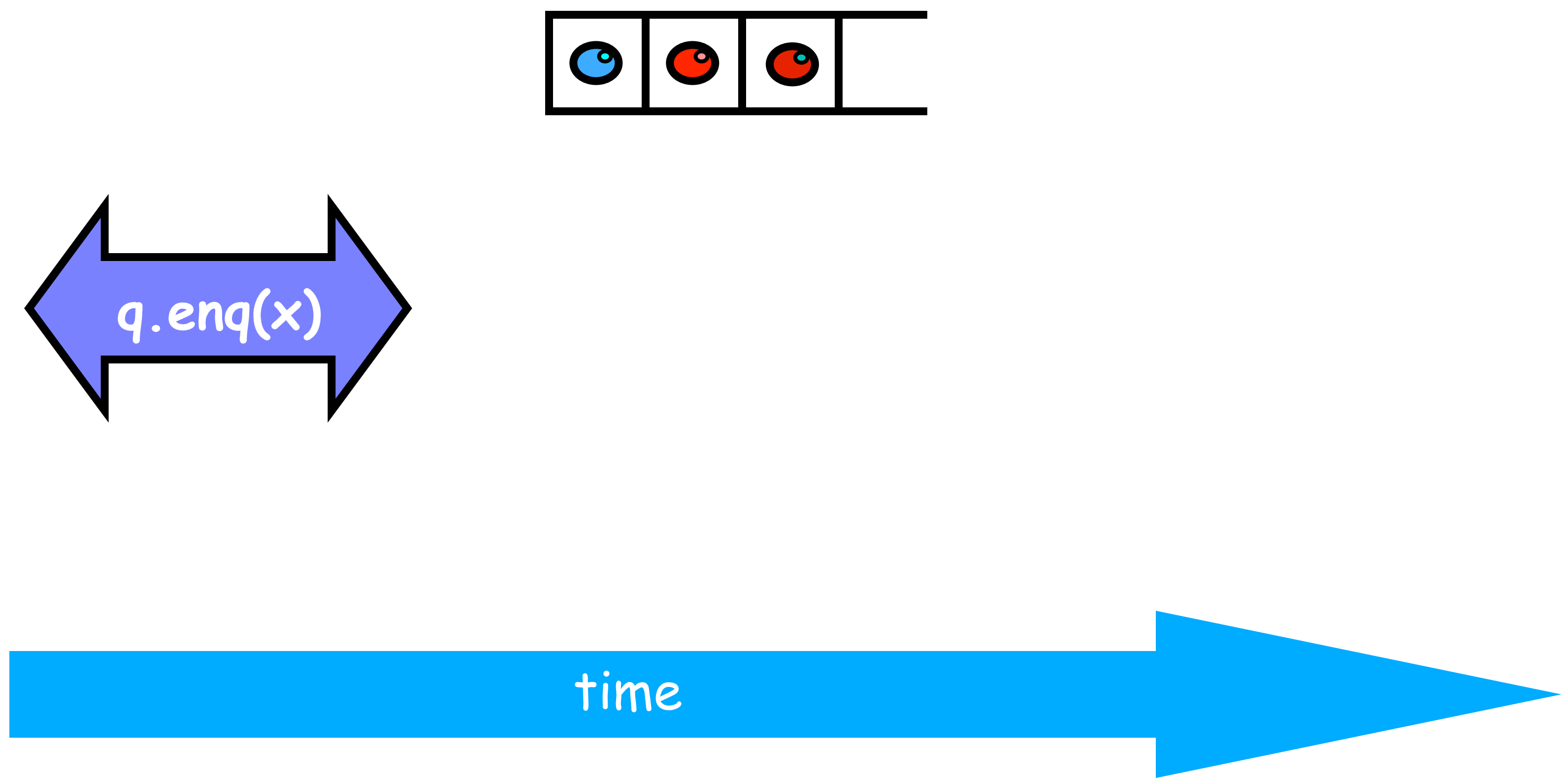
Example



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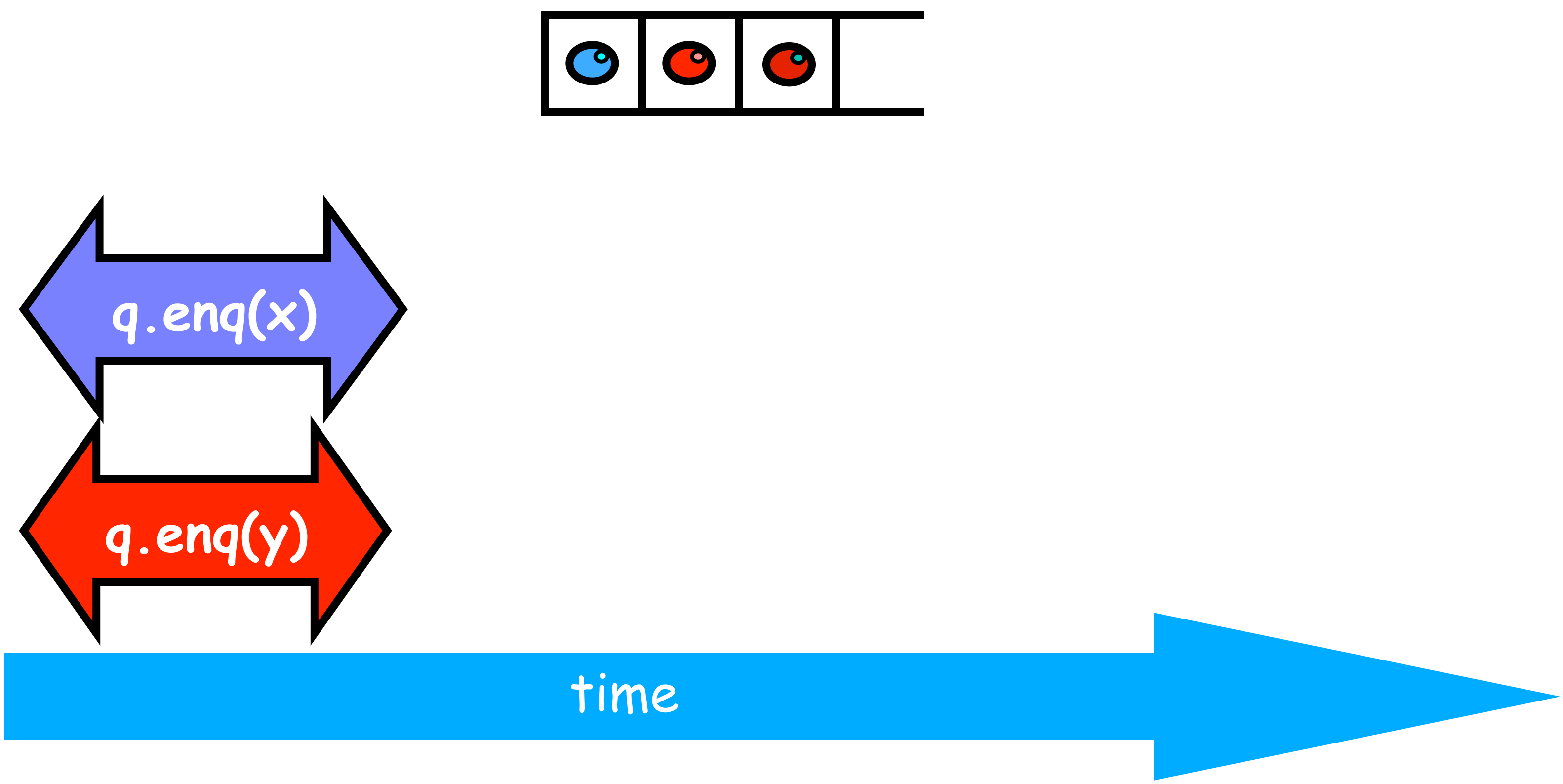
Example



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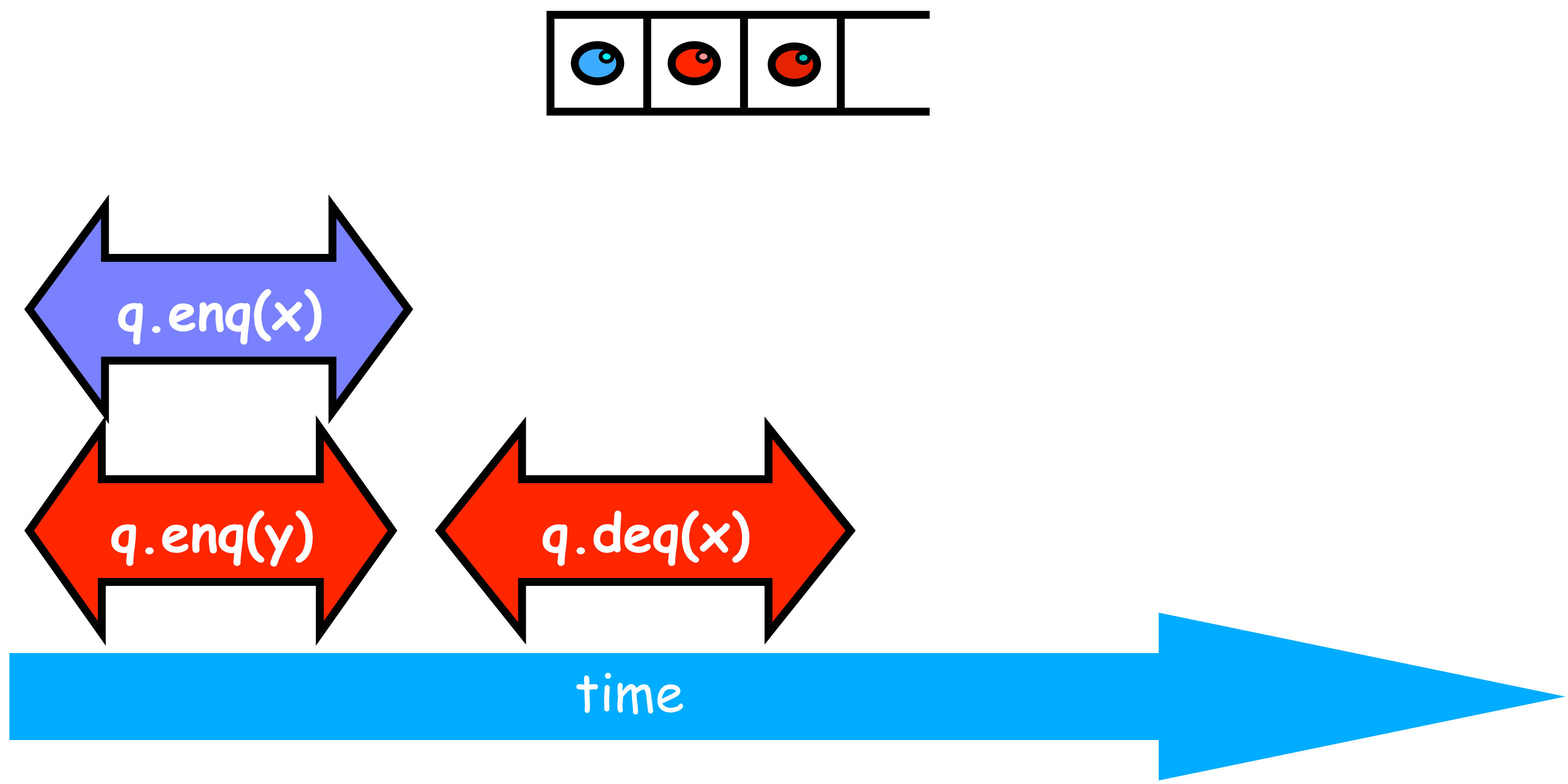
Example



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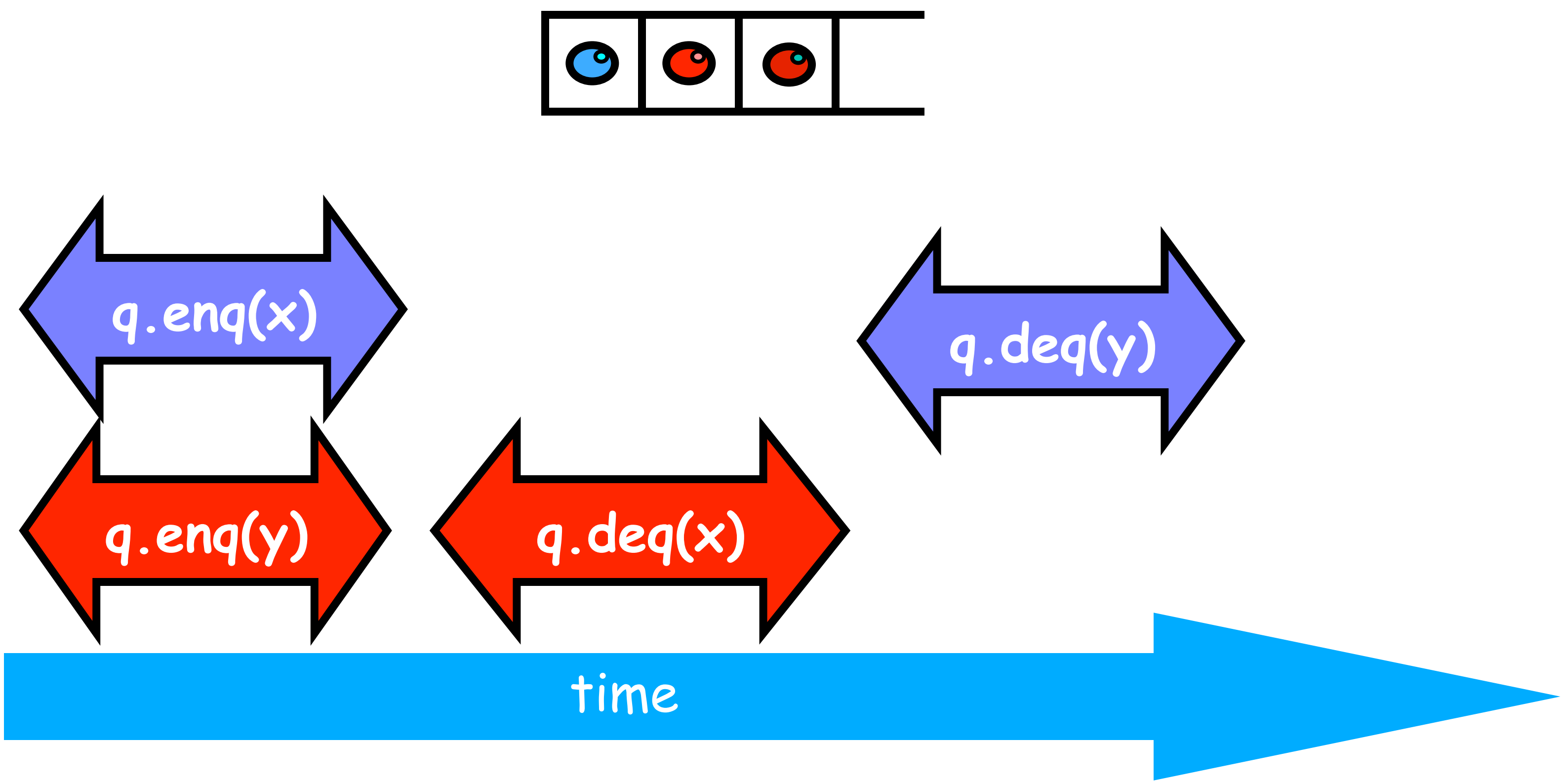
Example



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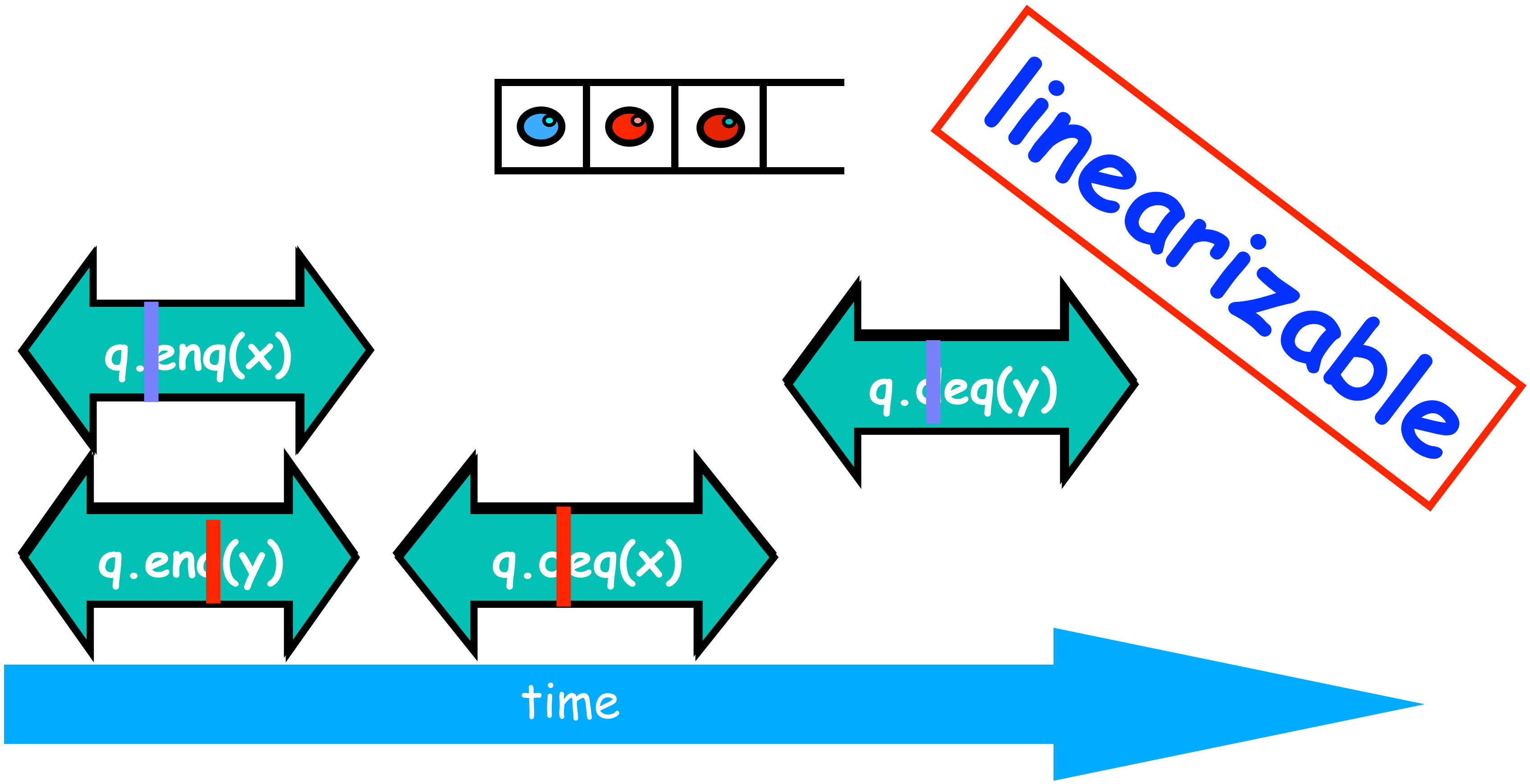
Example



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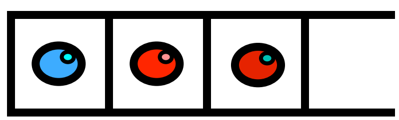
Another Example (like Table 3)



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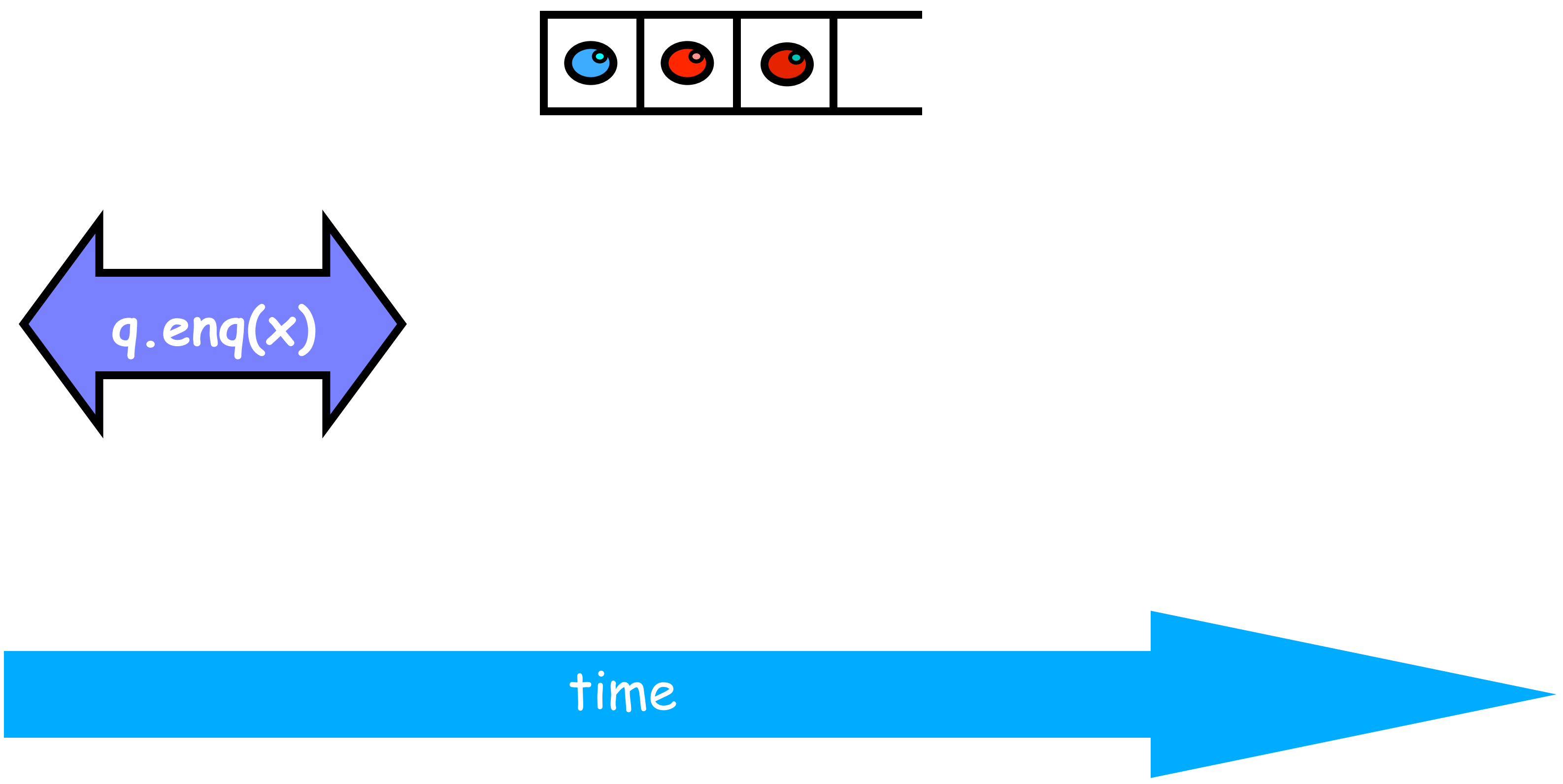
Another Example



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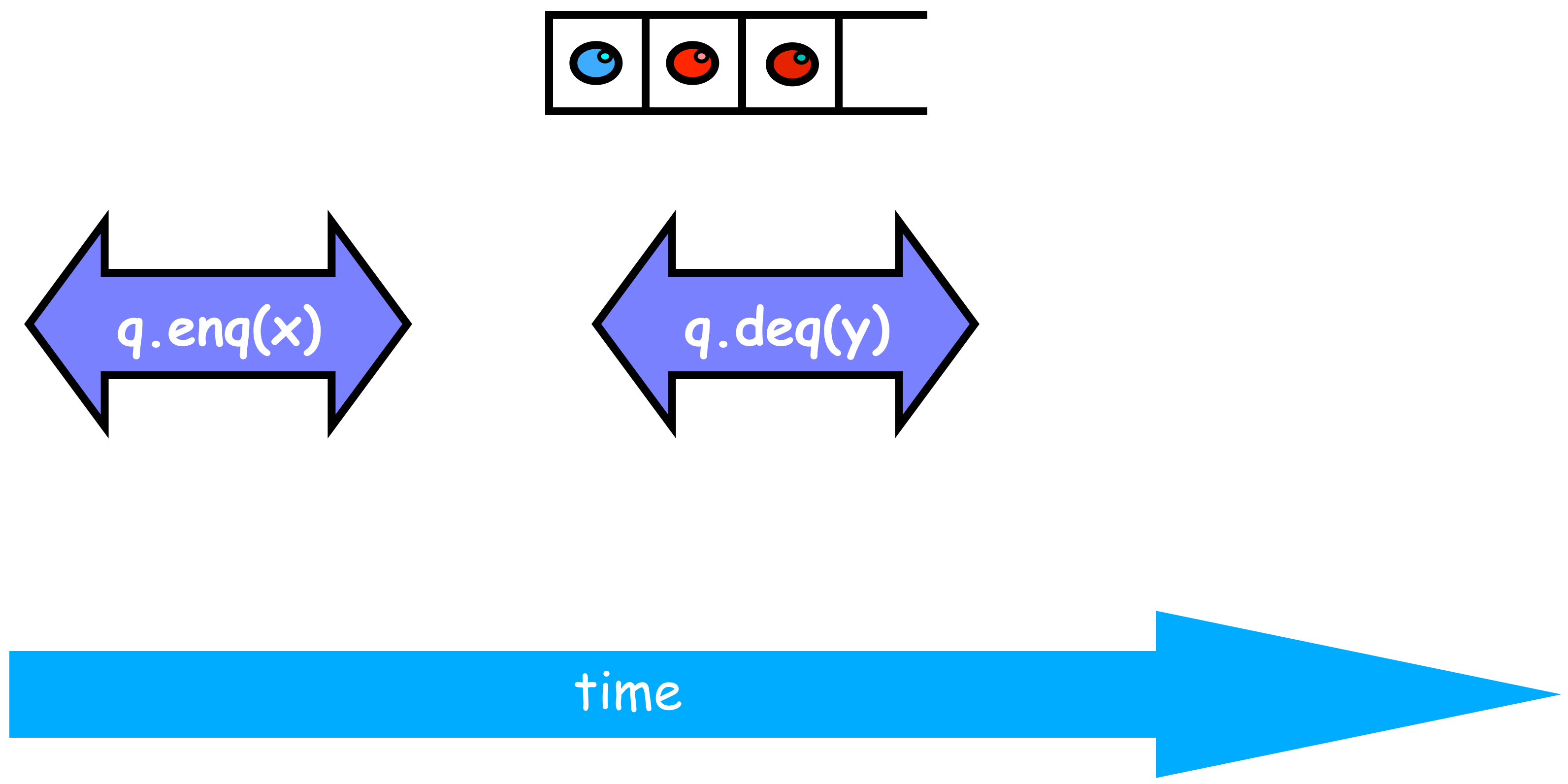

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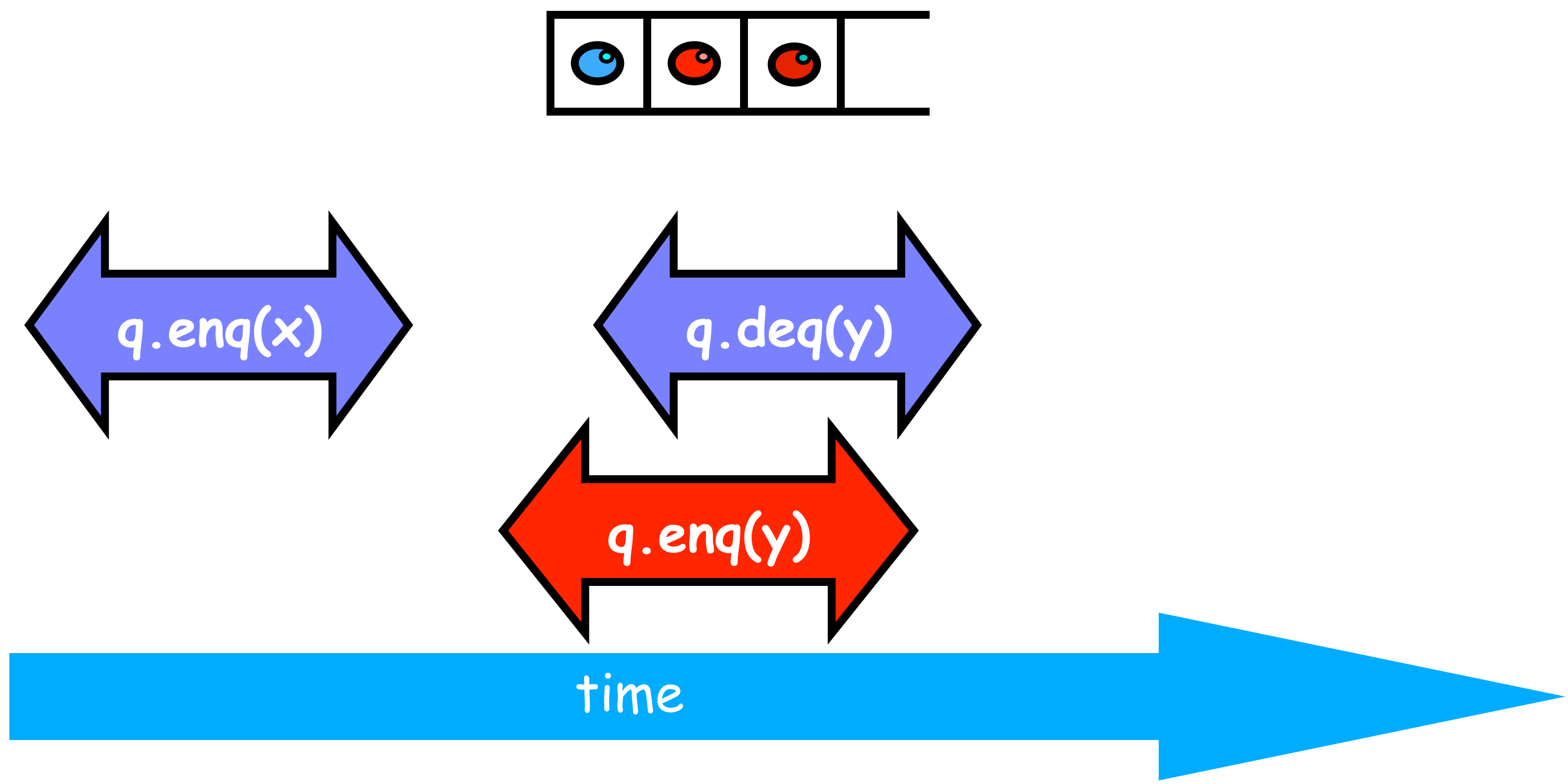

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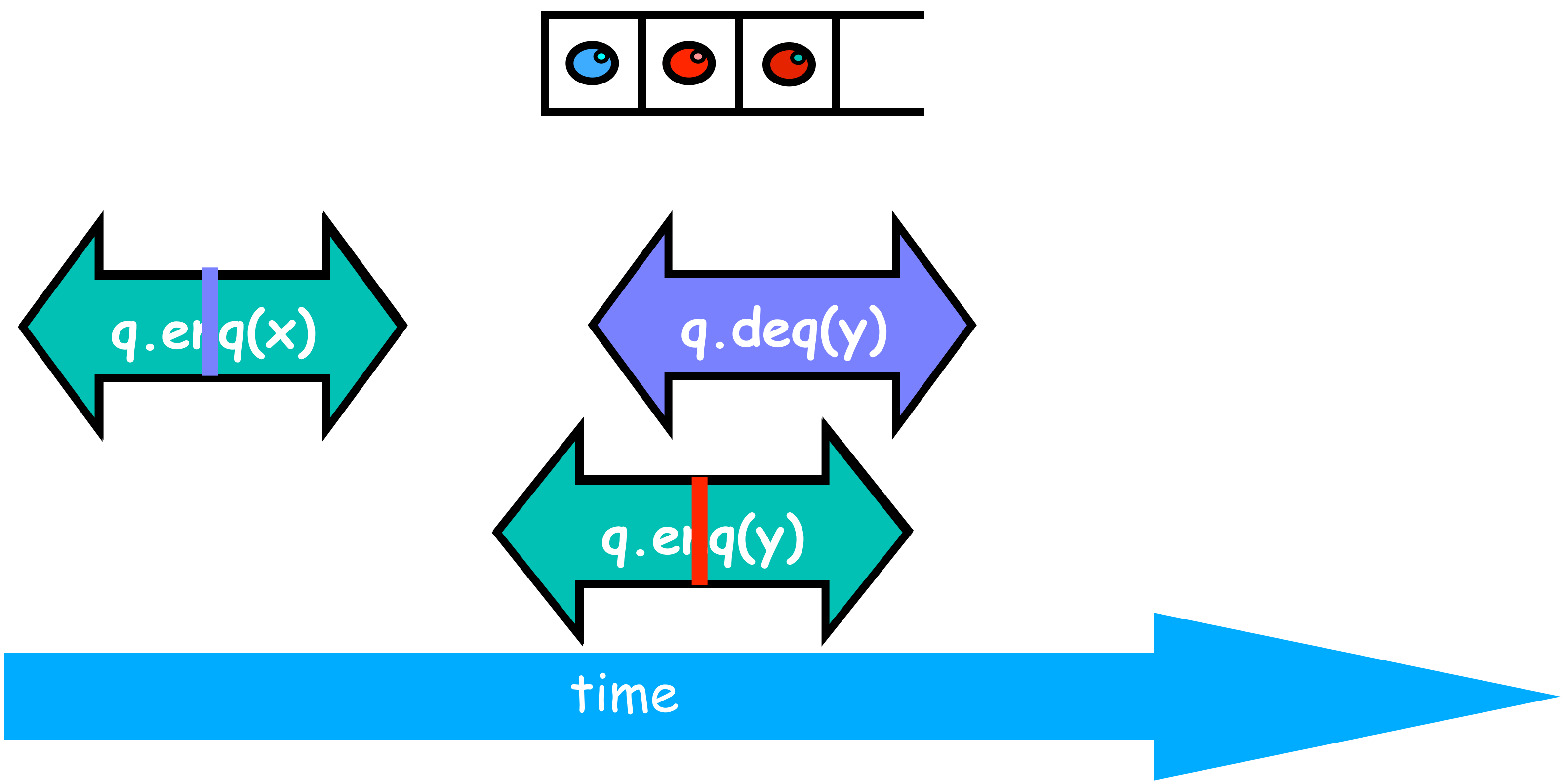

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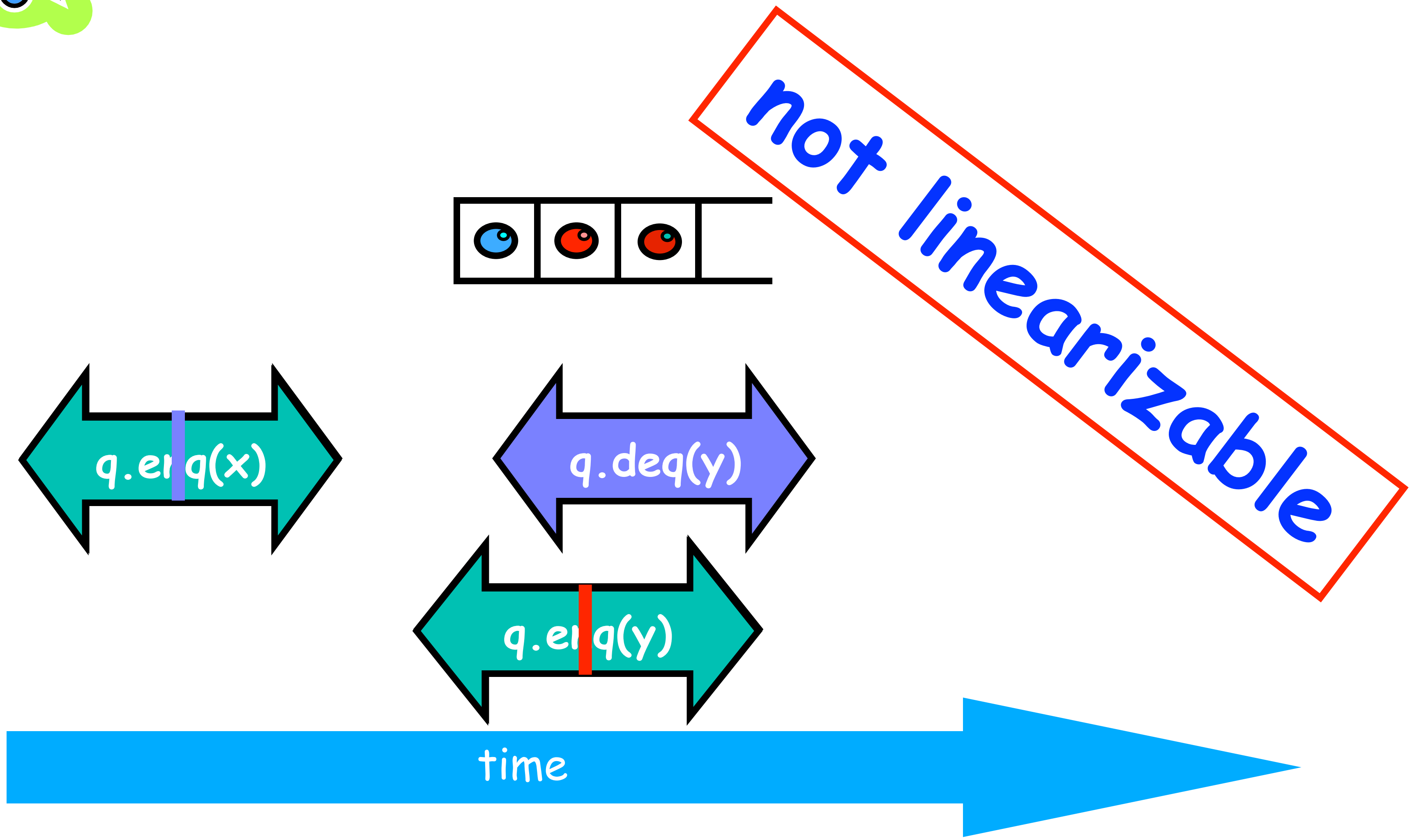

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Figure 1: Computation Graph for monitor-based implementation of FIFO queue (Table 1)

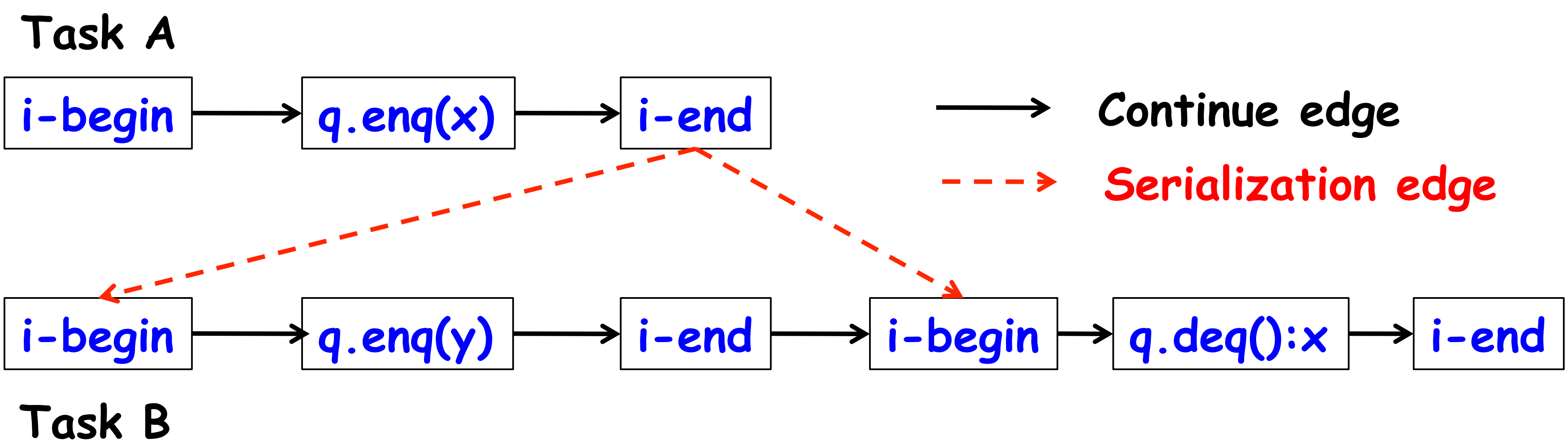
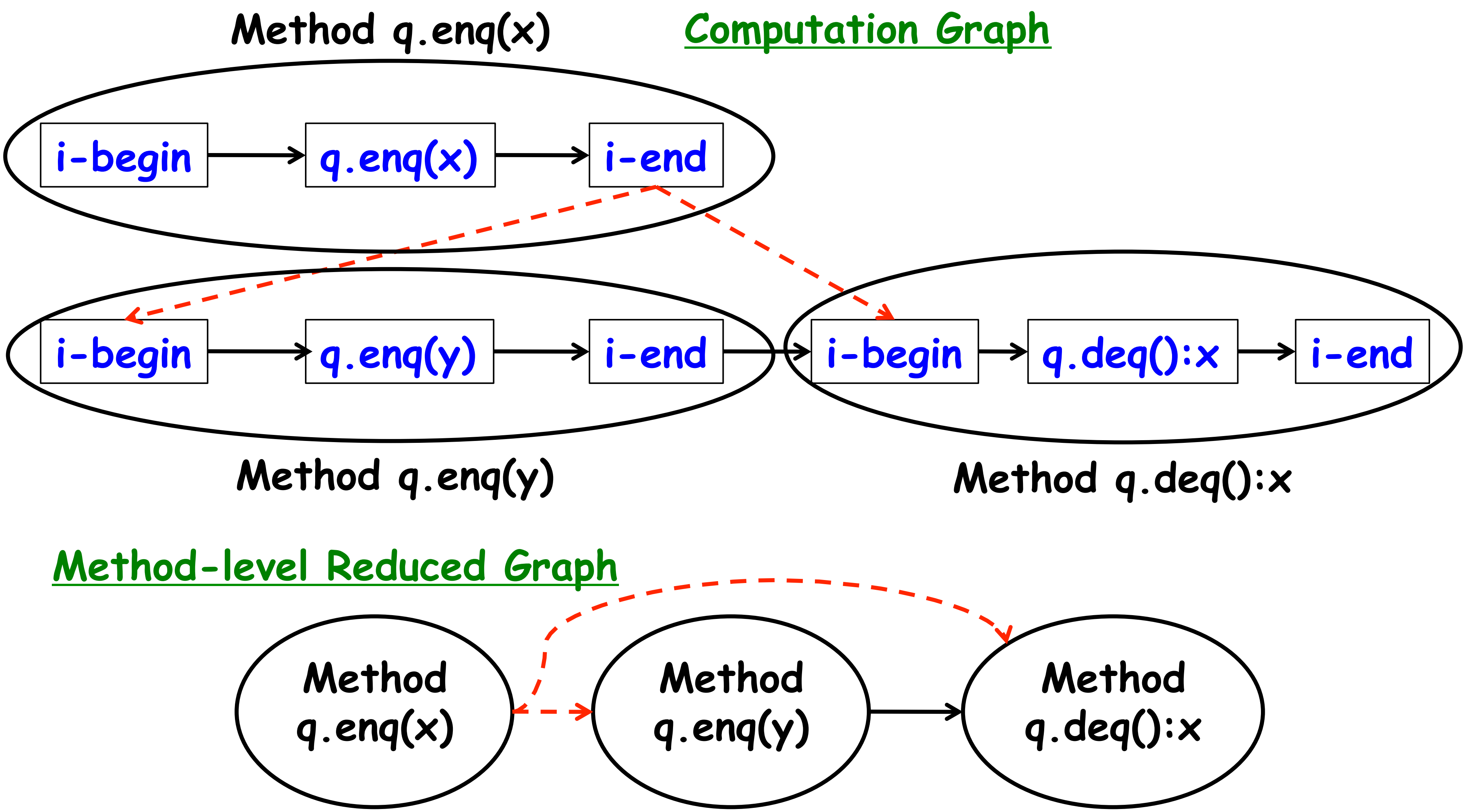


Figure 2: Creating a Reduced Graph to model Instantaneous Execution of Methods



Relating Linearizability to the Computation Graph model

- Given a reduced CG, a sufficient condition for linearizability is that the reduced CG is acyclic as in Figure 2.
- This means that if the reduced CG is acyclic, then the underlying execution must be linearizable.
- However, the converse is not necessarily true, as we will see.

Figure 3: example Computation Graph for concurrent implementation of FIFO queue (Table 2)

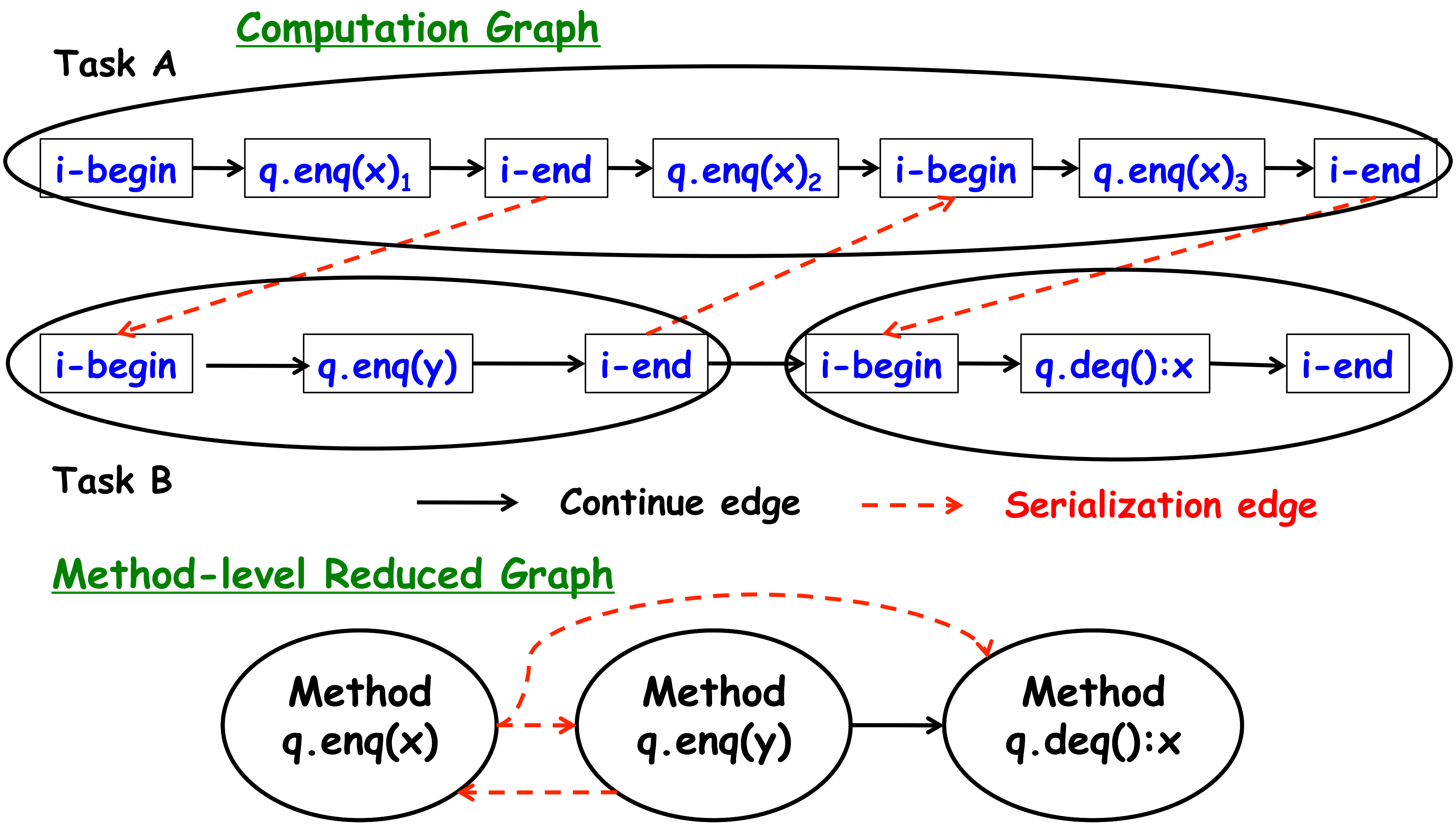
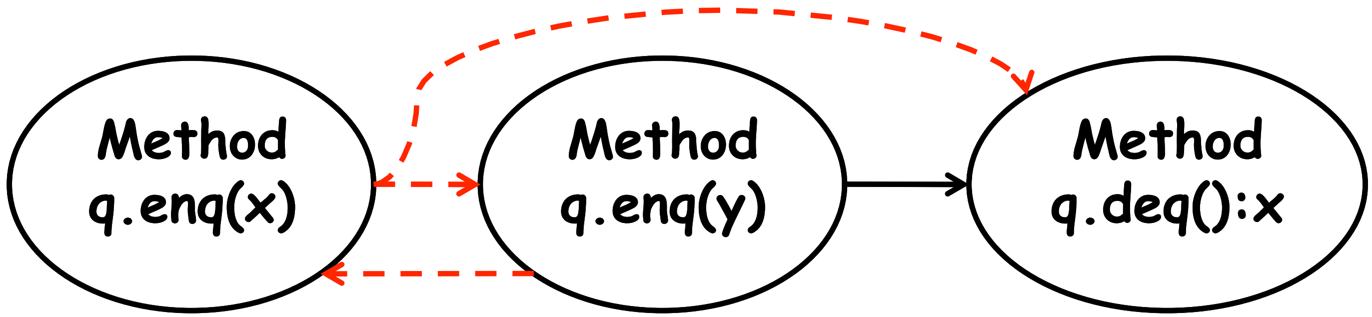


Figure 4: Reduced method-level graph for Computation Graph in Figure 3

- Example of linearizable execution graph for which reduced method-level graph is cyclic



- Approach to make cycle test more precise for linearizability
  - Decompose concurrent object method into a sequence of "try" steps followed by a sequence of "commit" steps
  - Assume that each "commit" step's execution does not use any input from any prior "try" step

  ➔ Reduced graph can just reduce the "commit" steps to a single node instead of reducing the entire method to a single node