COMP 322: Fundamentals of Parallel Programming

Lecture 5: Parallel Array Sum and Array Reductions

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Announcements

- Homework 2 is due by 5pm today
- Homework 3 will be assigned on Monday, Jan 24th and will be due two weeks later on Monday, Feb 7th
  - This is a programming assignment with abstract performance metrics
  - To prepare for HW3, please make sure that you can compile and run the programs from Lab 2 on your own, using the –perf option. In case of problems, please send email to comp322-staff @ mailman.rice.edu
- Graded Homework 1 assignments will be emailed to you by Monday, Jan 24th

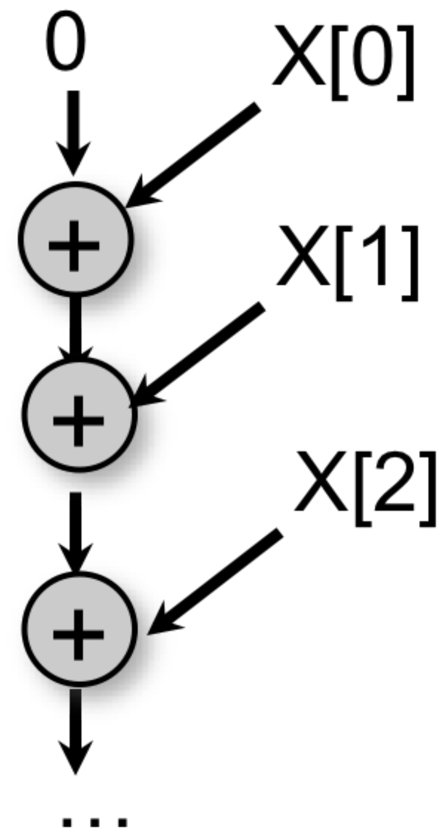
Acknowledgments for Today's Lecture

- **COMP 322 Lecture 5 handout**

Sequential Array Sum Program (Lecture 1)

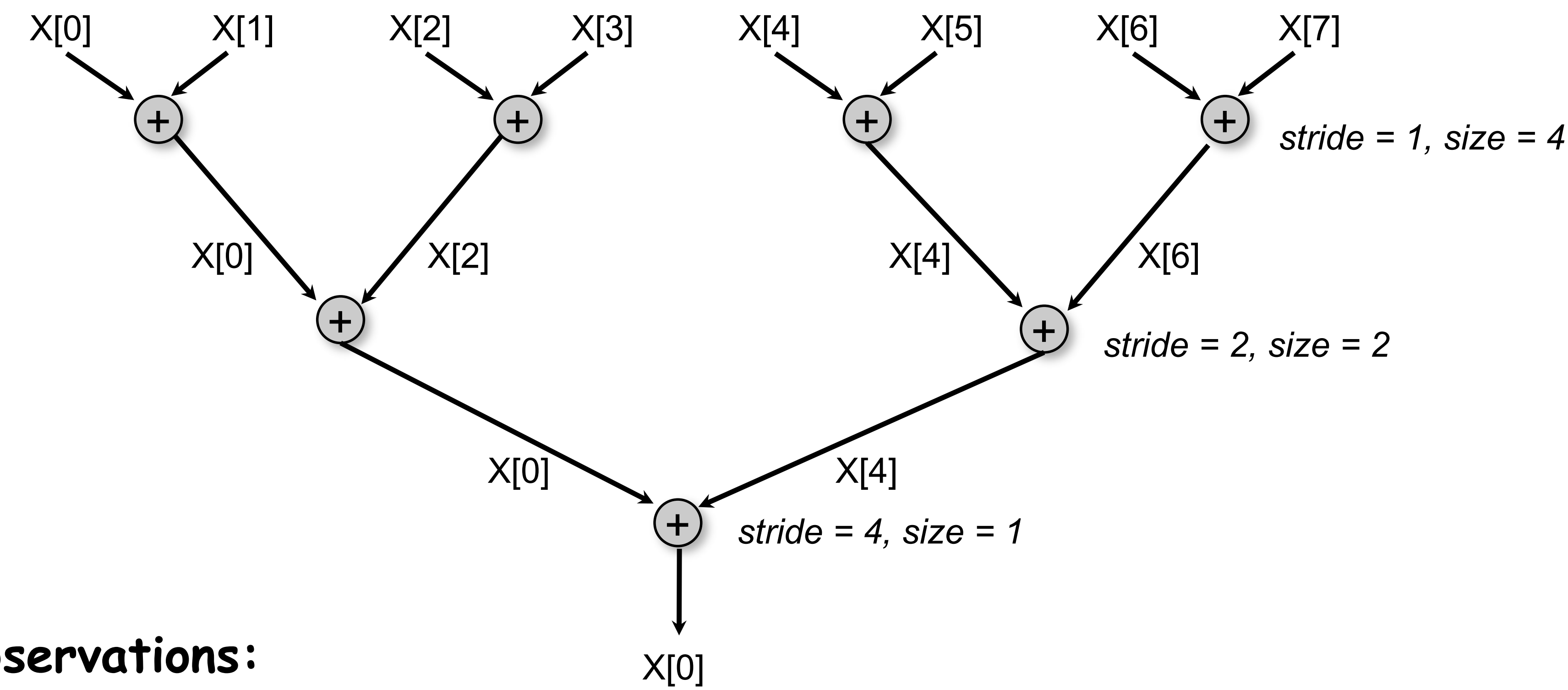
```
int sum = 0;
for (int i=0 ; i < X.length ; i++ )
    sum += X[i];
```

Computation Graph



- The original *computation graph* is sequential
- We studied a 2-task parallel program for this problem
- How can we expose more parallelism?

Reduction Tree Schema for computing Array Sum in parallel



Observations:

- This algorithm overwrites X (make a copy if X is needed later)
- stride = distance between array subscript inputs for each addition
- size = number of additions that can be executed in parallel in each level (stage)

Parallel Program that satisfies dependences in Reduction Tree schema (for X.length = 8)

```
finish { // STAGE 1: stride = 1, size = 4 parallel additions
  async X[0]+=X[1]; async X[2]+=X[3];
  async X[4]+=X[5]; async X[6]+=X[7];
}
finish { // STAGE 2: stride = 2, size = 2 parallel additions
  async X[0]+=X[2]; async X[4]+=X[6];
}
finish { // STAGE 3: stride = 4, size = 1 parallel additions
  async X[0]+=X[4];
}
```

Generalization to arbitrary sized arrays (ArraySum1)

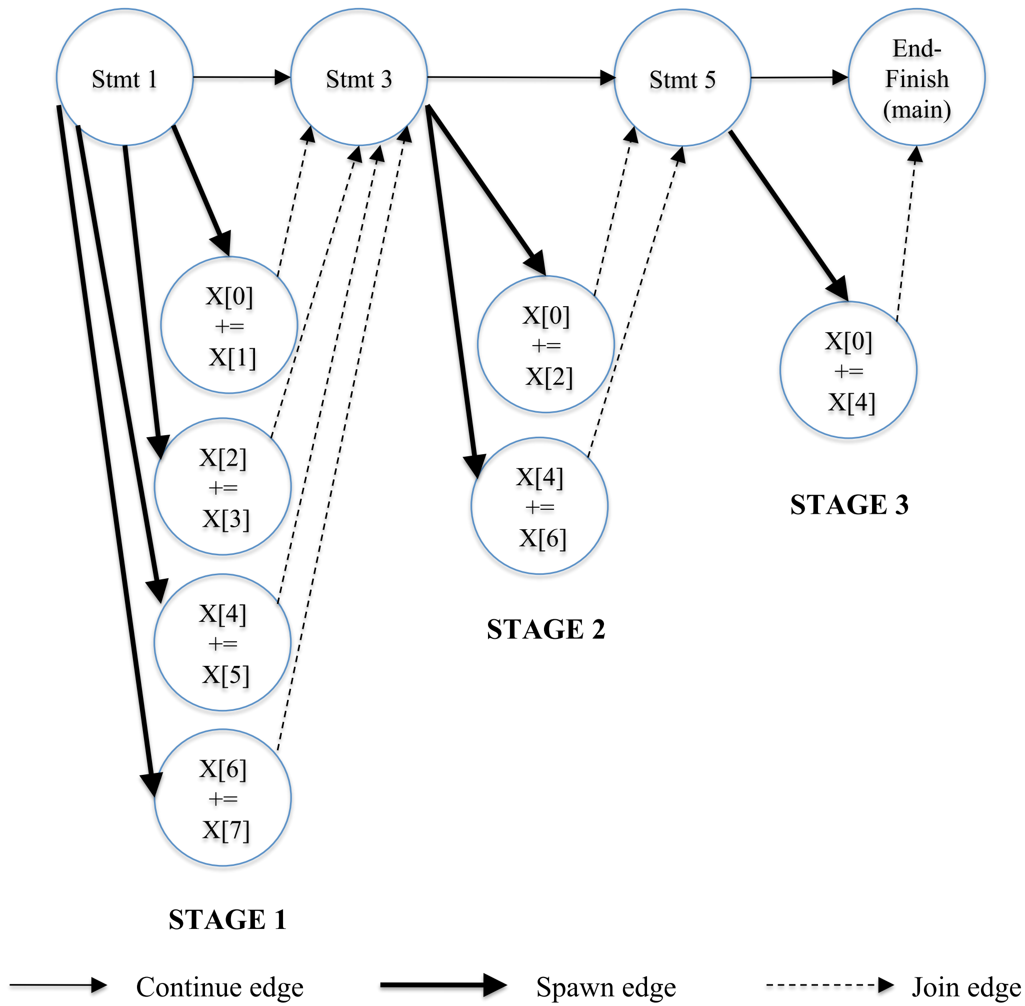
```
for ( int stride = 1; stride < X.length ; stride *= 2 ) {
  // Compute size = number of additions to be performed in stride
  int size=ceilDiv(X.length,2*stride);
  finish for(int i = 0; i < size; i++)
    async {
      if ( (2*i+1)*stride < X.length )
        X[2*i*stride]+=X[(2*i+1)*stride];
    } // finish-for-async
} // for

// Divide x by y, round up to next largest int, and return result
static int ceilDiv(int x, int y) { return (x+y-1) / y; }
```

Complexity Analysis of ArraySum1

- Define n = X.length
- Assume that each addition takes 1 unit of time
  - Ignore all other computations since they are related to the addition by some constant
- Total number of additions, WORK = n-1 = O(n)
- Critical path length (number of stages), CPL = ceiling($\log_2$(n)) = O(log(n))
- Ideal parallelism = WORK/CPL = O(n) / O(log(n))
- Consider an execution on *p* processors
  - Compute partial sum for n/p elements on each processor
  - Use ArraySum1 program to reduce p partial sums to one total sum
  - CPL for this version is O(n/p + log(p))
  - Parallelism for this version is O(n) / O(n/p + log(p))
  - Algorithm is optimal for p = n / log(n), or fewer, processors – why?

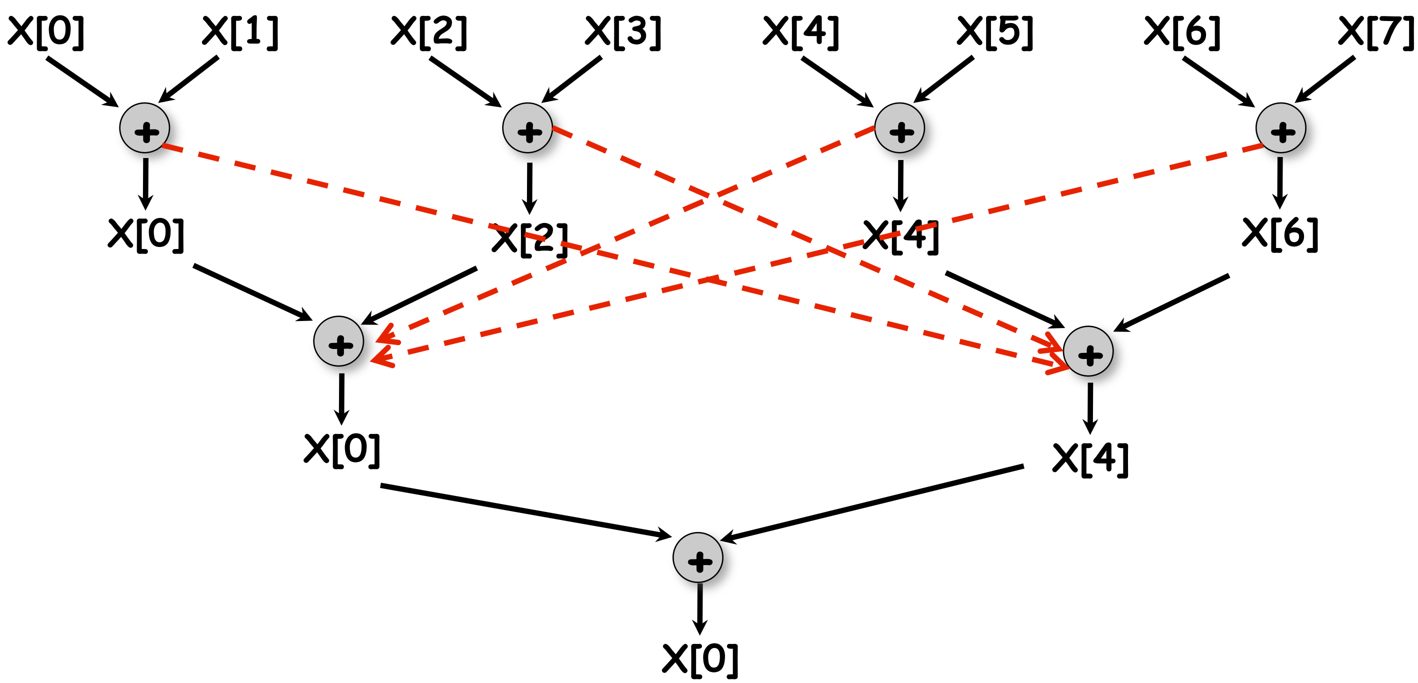
Computation Graph for ArraySum1



Observations:

- Computation graph has extra dependences relative to schema e.g., X[0]+=X[2] must follow X [4]+=X[5]
- Extra dependences can make a difference if computations in same stage take different times e.g., if X[4]+=X[5] and X[0]+=X[2] take 100 time units each
- How can we write a program that avoids these extra dependences?

Extra dependences in ArraySum1 program



Extra dependence edges due to finish-async stages

Summing an arbitrary sized array using a Recursive method and Future Tasks (ArraySum2)

```
static int computeSum(int[] X, int lo, int hi) {
  if ( lo > hi ) return 0;
  else if ( lo == hi ) return X[lo];
  else {
    int mid = (lo+hi)/2;
    final future<int> sum1 =
      async<int> {return computeSum(X, lo, mid);};
    final future<int> sum2 =
      async<int> {return computeSum(X, mid+1, hi);};
    return sum1.get() + sum2.get();
  }
} // computeSum
int sum = computeSum(X, 0, X.length-1); // main program code
```

Can be replaced by finish-async, but future tasks are more natural

Parallel Array Reductions

- Why all this focus on array sum?
- ArraySum1 and ArraySum2 programs can easily be adapted to reduce any associative function f
  - f(x,y) is said to be *associative* if f(a,f(b,c)) = f(f(a,b),c) for any inputs a, b, and c
- Sequential version of array reduction:

```
int result=X[0];
for(int i=1 ; i < X.length ; i++ ) result=f(result,X[i]);
```

- General reductions have many interesting applications in practice, as you will see when we learn about Google's Map Reduce framework
- Motivates complexity analysis where evaluation of a single call to f() is assumed to take 1 unit of time (could be much larger than an integer add, and justify the use of an async)

Extension of ArraySum1 to reduce an arbitrary associative function, f

```
for ( int stride = 1; stride < X.length ; stride *= 2 ) {
  // Compute size = number of additions to be performed in stride
  int size=ceilDiv(X.length,2*stride);
  finish for(int i = 0; i < size; i++)
    async {
      if ( (2*i+1)*stride < X.length )
        X[2*i*stride] = f(X[2*i*stride], X[(2*i+1)*stride]);
    } // finish-for-async
} // for

// Divide x by y, round up to next largest int, and return result
static int ceilDiv(int x, int y) { return (x+y-1) / x; }
```

Extension of ArraySum2 to reduce an arbitrary associative function, f

```
static int computeSum(int[] X, int lo, int hi) {
  if ( lo > hi ) return identity();
  else if ( lo == hi ) return X[lo];
  else {
    int mid = (lo+hi)/2;
    final future<int> sum1 =
      async<int> {return computeSum(X, lo, mid);};
    final future<int> sum2 =
      async<int> {return computeSum(X, mid+1, hi);};
    return f(sum1.get(), sum2.get());
  }
} // computeSum
int sum = computeSum(X, 0, X.length-1); // main program code
```