COMP 322: Fundamentals of Parallel Programming

Lecture 10: Loop-Level Parallelism, Parallel Matrix Multiplication, Iteration Grouping (Chunking)

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Worksheet #9 solution: Parallelizing Pascal's Triangle with Futures and Memoization

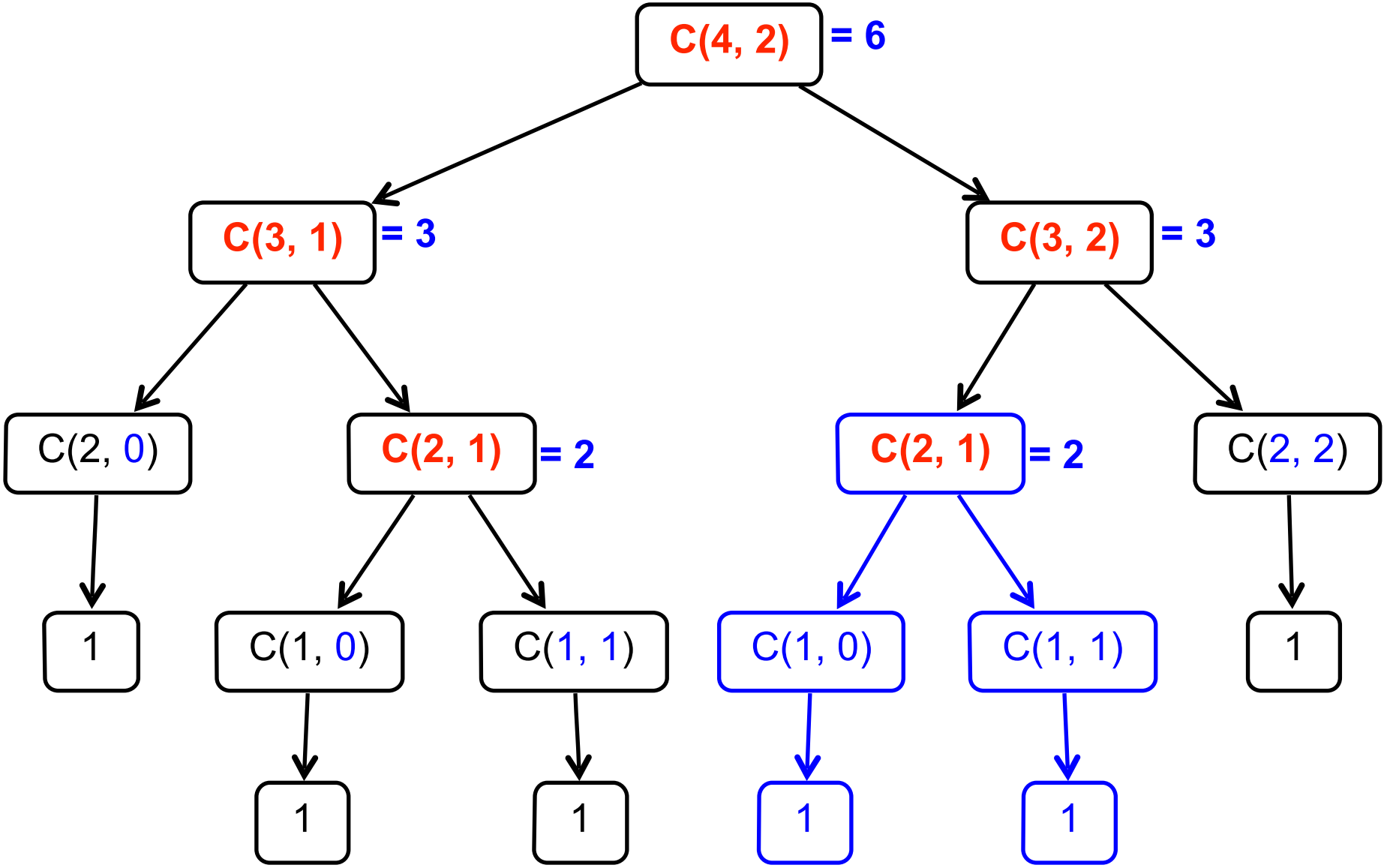
There are four variants of the Binomial Cooefficients program provided in four different HJlib methods in the next page:

a. Sequential Recursive without Memoization (chooseRecursiveSeq())
b. Parallel Recursive without Memoization (chooseRecursivePar())
c. Sequential Recursive with Memoization (chooseMemoizedSeq())
d. Parallel Recursive with Memoization (chooseMemoizedPar())

Your task is to analyze the WORK, CPL, and Ideal Parallelism for these four versions, for the input N = 4, and K = 2. Assume that each call to ComputeSum() has COST = 1, and all other operations are free. Complete all entries in the table:

Variant	Work	CPL	Ideal Parallelism
chooseRecursiveSeq	5	5	1
chooseRecursivePar	5	3	5/3 = 1.67
chooseMemoizedSeq	4	4	1
chooseMemoizedPar	4	3	4/3 = 1.33

REMINDER: computation structure of C(4,2) Nodes with calls to ComputeSum() are in red



Outline of Today's Lecture

- Multidimensional parallel loops
- Grouping/chunking of parallel loop iterations

Sequential Algorithm for Matrix Multiplication

$$c[i,j] = \sum_{0 \le k < n} a[i,k] * b[k,j]$$

```
1.  // Sequential version
2.  for (int i = 0 ; i < n ; i++)
3.    for (int j = 0 ; j < n ; j++)
4.      c[i][j] = 0;
5.  for (int i = 0 ; i < n ; i++)
6.    for (int j = 0 ; j < n ; j++)
7.      for (int k = 0 ; k < n ; k++)
8.        c[i][j] += a[i][k] * b[k][j];
9.  // Print first element of output matrix
10. System.out.println(c[0][0]);
```


Parallelizing the loops in Matrix Multiplication example using finish &

$$c[i,j] = \sum_{0 \le k < n} a[i,k] * b[k,j]$$

```
1.  // Parallel version using finish & async
2.  finish(() -> {
3.    for (int i = 0 ; i < n ; i++)
4.      for (int j = 0 ; j < n ; j++)
5.        async(() -> {c[i][j] = 0; });
6.  });
7.  finish(() -> {
8.    for (int i = 0 ; i < n ; i++)
9.      for (int j = 0 ; j < n ; j++)
10.       async(() -> {
11.         for (int k = 0 ; k < n ; k++)
12.           c[i][j] += a[i][k] * b[k][j];
13.       });
14.  });
15.  // Print first element of output matrix
16.  System.out.println(c[0][0])
```


Observations on finish-for-async version

- finish and async are general constructs, and are not specific to loops
  - —Not easy to discern from a quick glance which loops are sequential vs. parallel
- Loops in sequential version of matrix multiplication are "perfectly nested"
  - —e.g., no intervening statement between "for(i = ...)" and "for(j = ...)"
- The ordering of loops nested between finish and async is arbitrary
  - —They are parallel loops and their iterations can be executed in any order

Parallelizing the loops in Matrix Multiplication example using forall

$$c[i,j] = \sum_{0 \le k < n} a[i,k] * b[k,j]$$

```
1.  // Parallel version using finish & forall
2.   forall(0, n-1, 0, n-1, (i, j) -> {
3.       c[i][j] = 0;
4.     });
5.   forall(0, n-1, 0, n-1, (i, j) -> {
6.      forseq(0, n-1, (k) -> {
7.         c[i][j] += a[i][k] * b[k][j];
8.       });
9.    });
10. // Print first element of output matrix
11. System.out.println(c[0][0]);
```


forall API's in HJlib
(http://www.cs.rice.edu/~vs3/hjlib/doc/edu/rice/hj/Module1.html)

- `static void forall(edu.rice.hj.api.HjRegion.HjRegion1D hjRegion, edu.rice.hj.api.HjProcedureInt1D body)`
- `static void forall(edu.rice.hj.api.HjRegion.HjRegion2D hjRegion, edu.rice.hj.api.HjProcedureInt2D body)`
- `static void forall(edu.rice.hj.api.HjRegion.HjRegion3D hjRegion, edu.rice.hj.api.HjProcedureInt3D body)`
- `static void forall(int s0, int e0, edu.rice.hj.api.HjProcedure<java.lang.Integer> body)`
- `static void forall(int s0, int e0, int s1, int e1, edu.rice.hj.api.HjProcedureInt2D body)`
- `static <T> void forall(java.lang.Iterable<T> iterable, edu.rice.hj.api.HjProcedure<T> body)`
- **NOTE: all forall API's include an implicit finish. forasync is like forall, but without the finish.**

Observations on forall version

- The combination of perfectly nested finish-for–for–async constructs is replaced by a single API, forall
  - —forall includes an implicit finish
- Multiple loops can be collapsed into a single forall with a multi-dimensional iteration space (can be 1D, 2D, 3D, ...)
- The iteration variable for a forall is a HjPoint (integer tuple), e.g., (i,j)
- The loop bounds can be specified as a rectangular HjRegion (product of dimension ranges), e.g., (0:n−1) x (0:n−1)
- HJlib also provides a sequential forseq API that can also be used to iterate sequentially over a rectangular region
  - —Simplifies conversion between for and forall

forall examples: updates to a two-dimensional Java array

```
// Case 1: loops i,j can run in parallel
forall(0, m-1, 0, n-1, (i, j) -> { A[i][j] = F(A[i][j]);});

// Case 2: only loop i can run in parallel
forall(0, m-1, (i) -> {
  forseq(0, n-1, (j) -> { // Equivalent to "for (j=0;j<n;j++)"
    A[i][j] = F(A[i][j]) ;
}); });

// Case 3: only loop j can run in parallel
forseq(0, m-1, (i) -> { // Equivalent to "for (i=0;i<m;j++)"
  forall(0, n-1, (j) -> {
    A[i][j] = F(A[i][j]) ;
}); });
```


What about overheads?

- As you will see in today's lab, it is inefficient to create forall iterations in which each iteration (async task) does very little work
- An alternate approach is "iteration grouping" or "loop chunking"
  - e.g., replace

```
forall(0, 99, (i) -> BODY(i)); // 100 tasks
```

  - by

```
forall(0, 3, (ii) -> { // 4 tasks
// Each task executes a "chunk" of 25 iterations
   forseq(25*ii, 25*(ii+1)-1, (i) -> BODY(i));

}); // forall
```


forallChunked APIs

- `forallChunked(int s0, int e0, int chunkSize, edu.rice.hj.api.HjProcedure<Integer> body)`
- **Like** `forall(int s0, int e0, edu.rice.hj.api.HjProcedure<Integer> body)`
- **but forallChunked includes chunkSize as the third parameter!**
  - **e.g., replace**

```
forall(0, 99, (i) -> BODY(i)); // 100 tasks
```

  - **by**

```
forallChunked(0, 99, 100/4, (i)->BODY(i));
```


One-Dimensional Iterative Averaging Example

- Initialize a one-dimensional array of (n+2) double's with boundary conditions, myVal[0] = 0 and myVal[n+1] = 1.
- In each iteration, each interior element myVal[i] in 1..n is replaced by the average of its left and right neighbors.
  - —Two separate arrays are used in each iteration, one for old values and the other for the new values
- After a sufficient number of iterations, we expect each element of the array to converge to myVal[i] = (myVal[i-1]+myVal[i+1])/2, for all i in 1..n

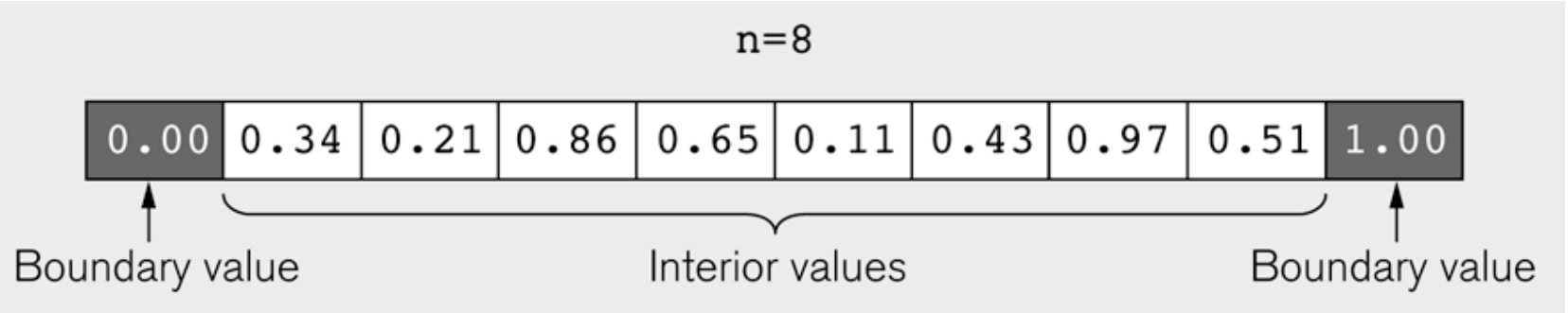


Illustration of an intermediate step for n = 8 (source: Figure 6.19 in Lin-Snyder book)

HJ code for One-Dimensional Iterative Averaging using nested forseq-forall structure

```
1. float[] myVal = new float[n+2];
2. float[] myNew = new float[n+2];
3.  … // Intialize myVal, m, n
4. forseq(0, m-1, (iter) -> {
5.   // Compute MyNew as function of input array MyVal
6.     forall(1, n, (j) -> { // Create n tasks
7.         myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
8.     }); // forall
9.   // What is the purpose of line 10 below?
10.  float[] temp=myVal; myVal=myNew; myNew=temp;
11.  // myNew becomes input array for next iteration
12.}); // for
```


Example: HJ code for One-Dimensional Iterative Averaging with forseq-forall structure w/ chunking

```
1.  int nc = numWorkerThreads();
2.   … // Initializations
3.  forseq(0, m-1, (iter) -> {
4.    // Compute MyNew as function of input array MyVal
5.      forallChunked(1, n, n/nc, (j) -> { // Create n/nc tasks
6.        myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
7.      }); // forall
8.    // Swap myVal & myNew;
9.    float[] temp=myVal; myVal=myNew; myNew=temp;
10.   // myNew becomes input array for next iteration
11. }); // for
```