COMP 322: Fundamentals of Parallel Programming

Lecture 4: Parallel Speedup and Amdahl's Law

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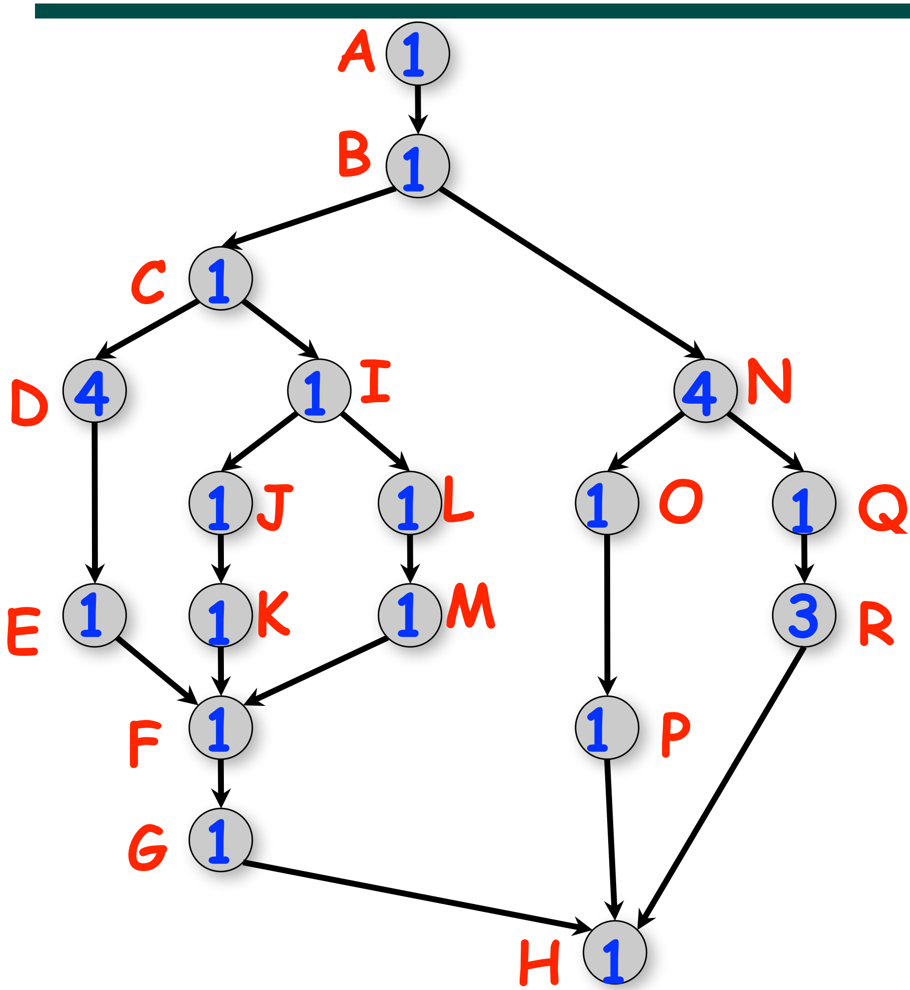
One Possible Solution to Worksheet 3 (Multiprocessor Scheduling)



There are 4 idle slots in this schedule — can we do better than T_2 = 15 ?

Start time	Proc 1	Proc 2
0	A	
1	B	
2	C	N
3	D	N
4	D	N
5	D	N
6	D	O
7	I	Q
8	J	R
9	L	R
10	K	R
11	M	E
12	F	P
13	G	
14	H	
15		

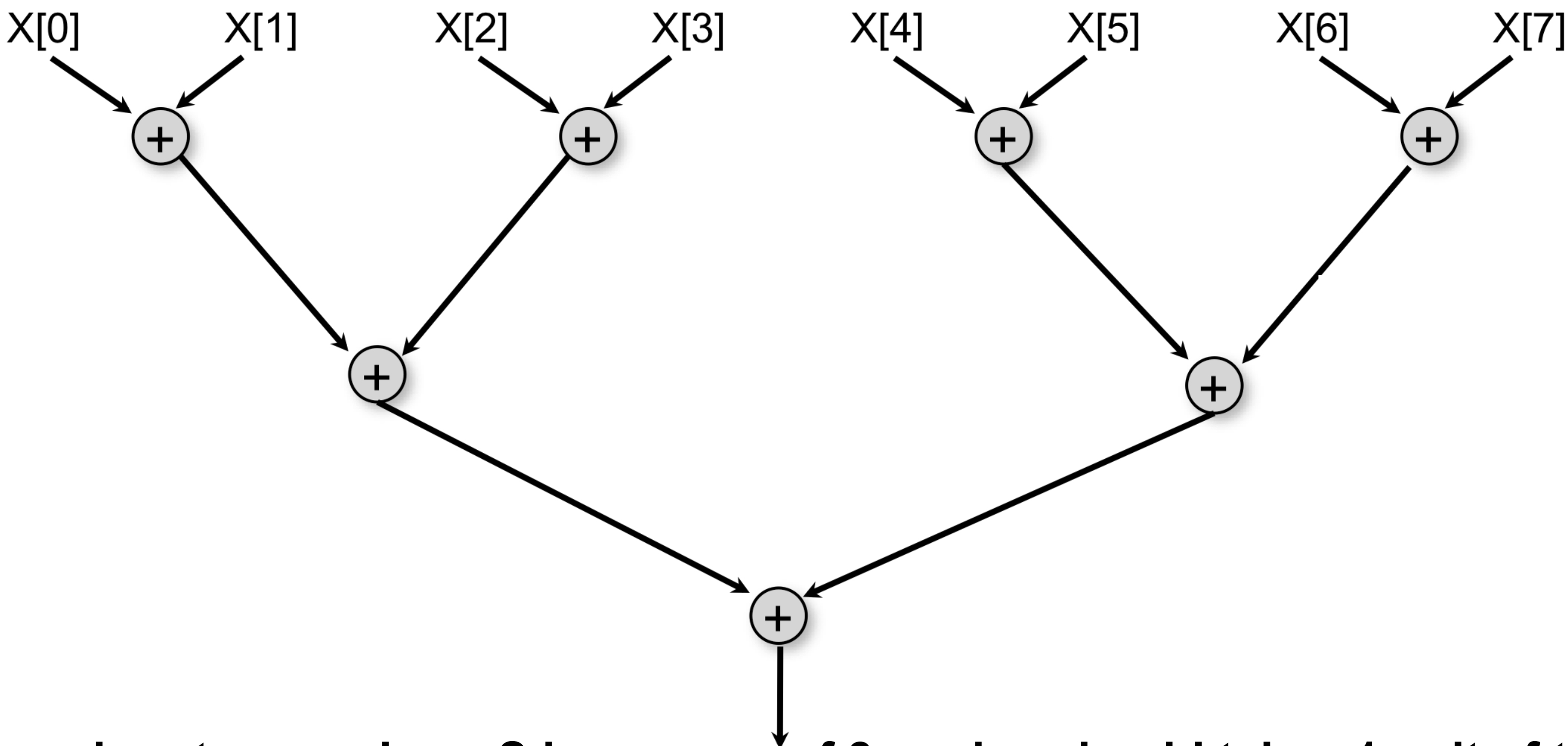
- As before, WORK = 26 and CPL = 11 for this graph
- T_2 = 15, for the 2-processor schedule on the right
- We can also see that
  $\max(CPL, WORK/2) <= T_2 < CPL + WORK/2$

Parallel Speedup

- Define Speedup(P) = T_1 / T_P
  - —Factor by which the use of P processors speeds up execution time relative to 1 processor, for a fixed input size
  - —For ideal executions without overhead, 1 <= Speedup(P) <= P
    - —This is what you will see with abstract metrics, but these bounds may not hold when we start measuring real execution times with real overheads
  - —Linear speedup
    - When Speedup(P) = k*P, for some constant k, 0 < k < 1
- Ideal Parallelism = WORK / CPL = T_1 / T_∞

  = Parallel Speedup on an unbounded (infinite) number of processors

Computation Graph for Recursive Tree approach to computing Array Sum in parallel



Assume input array size = S is a power of 2, and each add takes 1 unit of time:

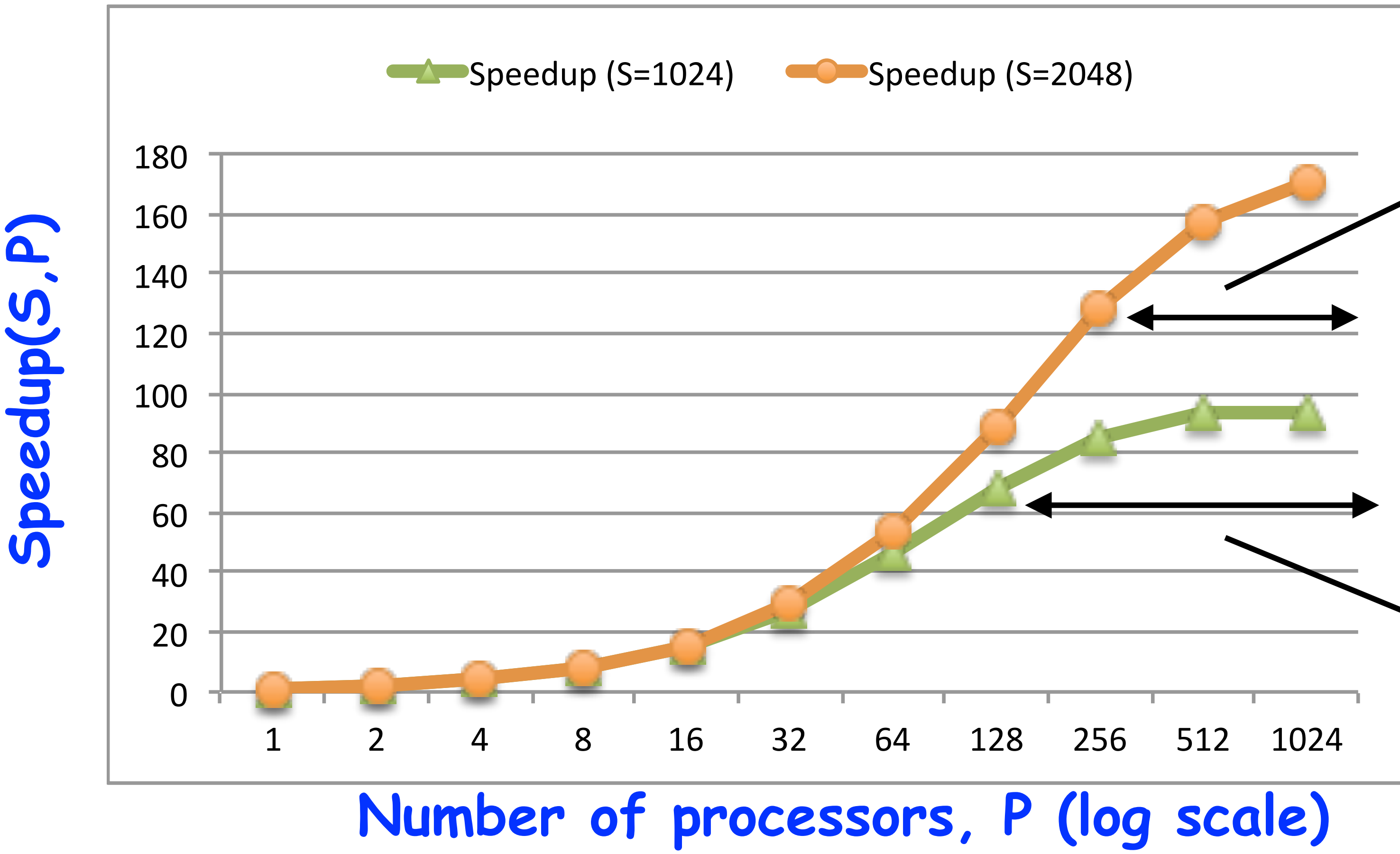
- WORK(G) = S-1, and CPL(G) = log2(S)
- Define T(S,P) = parallel execution time for Array Sum with size S on P processors
- Use upper bound T(S,P) <= WORK(G)/P + CPL(G) as a worst-case estimate
  - T(S,P) = WORK(G)/P + CPL(G) = (S-1)/P + log2(S)
- ⇒ Speedup(S,P) = T(S,1)/T(S,P) = S/(S/P + log2(S))

How many processors should we use?

- Define Efficiency(P) = Speedup(P)/ P = $T_1/(P * T_P)$
  - Processor efficiency --- figure of merit that indicates how well a parallel program uses available processors
  - For ideal executions without overhead, 1/P <= Efficiency(P) <= 1
  - Efficiency(P) = 1 (100%) is the best we can hope for.
- Half-performance metric
  - $S_{1/2}$ = input size that achieves Efficiency(P) = 0.5 for a given P
  - Figure of merit that indicates how large an input size is needed to obtain efficient parallelism
  - A larger value of $S_{1/2}$ indicates that the problem is harder to parallelize efficiently
- How many processors to use?
  - Common goal: choose number of processors, P for a given input size, S, so that efficiency is at least 0.5 (50%)

ArraySum: Speedup as function of array size, S, and number of processors, P

- $Speedup(S,P) = T(S,1)/T(S,P) = S/(S/P + log_2(S))$
- Asymptotically, $Speedup(S,P) \rightarrow S/log_2S$, as $P \rightarrow$ infinity



Efficiency(P) ≤ 0.5, for P ≥ 258 ==> wasteful to use more than 256 processors for S=2048

Efficiency(P) ≤ 0.5, for P ≥ 128 ==> wasteful to use more than 128 processors for S=1024

Amdahl's Law [1967]

- If q ≤ 1 is the fraction of WORK in a parallel program that must be executed sequentially for a given input size S, then the best speedup that can be obtained for that program is Speedup(S,P) ≤ 1/q.
- Observation follows directly from critical path length lower bound on parallel execution time
  - CPL >= q * T(S,1)
  - T(S,P) >= q * T(S,1)
  - Speedup(S,P) = T(S,1)/T(S,P) <= 1/q
- This upper bound on speedup simplistically assumes that work in program can be divided into sequential and parallel portions
  - Sequential portion of WORK = q
    - also denoted as f_S (fraction of sequential work)
  - Parallel portion of WORK = 1-q
    - also denoted as f_p (fraction of parallel work)
- Computation graph is more general and takes dependences into account

Illustration of Amdahl's Law: Best Case Speedup as function of Parallel Portion

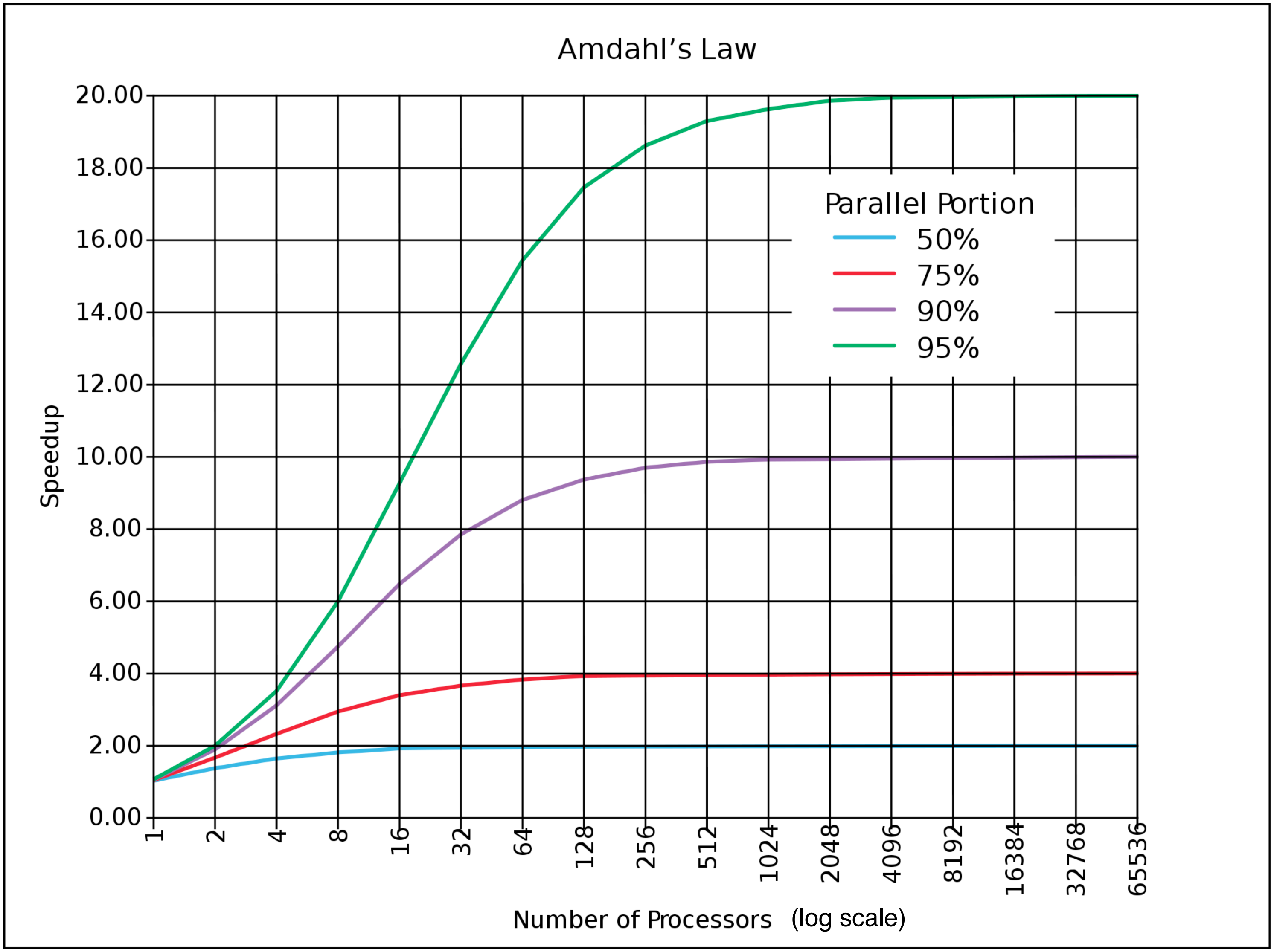


Figure source: http://en.wikipedia.org/wiki/Amdahl's law

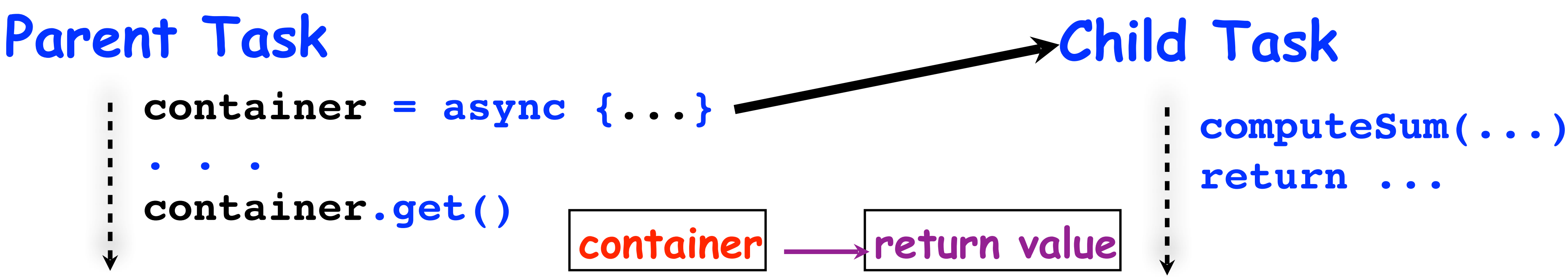
Functional Parallelism: Adding Return Values to Async Tasks

Example Scenario (PseudoCode)

```
// Parent task creates child async task
future<Integer> container = async { return computeSum(X,low,mid); };
. . .
// Later, parent examines the return value
int sum = container.get();
```

Two issues to be addressed:

1) Distinction between container and value in container (box)
2) Synchronization to avoid race condition in container accesses



Example: Two-way Parallel Array Sum using Future Tasks (PseudoCode)

```
// Parent Task T1 (main program)
// Compute sum1 (lower half) & sum2 (upper half) in parallel
future<Integer> sum1 = async { // Future Task T2
  int sum = 0;
  for(int i = 0; i < X.length / 2; i++) sum += X[i];
  return sum;
};
future<Integer> sum2 = async { // Future Task T3
  int sum = 0;
  for(int i = X.length / 2; i < X.length; i++) sum += X[i];
  return sum;
};
// Task T1 waits for Tasks T2 and T3 to complete
int total = sum1.get() + sum2.get();
```

Future Task Declarations and Uses

- Variable of type future is a reference to a future object
  - Container for return value from future task
  - The reference to the container is also known as a "handle"
- Two operations that can be performed on variable V of type future:
  - Assignment: V can be assigned value of type future
  - Blocking read: V.get() waits until the future task referred to by V has completed, and then propagates the return value
    - Supports waiting on selected tasks, in contrast to finish which waits on all tasks in scope

Announcements & Reminders

- IMPORTANT:
  - Watch video & read handout for topic 2.1 for next lecture on Friday, Jan 20th
- HW1 was posted on the course web site (http://comp322.rice.edu) on Jan 11th, and is due on Jan 25th
- Quiz for Unit 1 (topics 1.1 - 1.5) is due by Jan 27th on Canvas
- See course web site for all work assignments and due dates
- Use Piazza (public or private posts, as appropriate) for all communications re. COMP 322
- See Office Hours link on course web site for latest office hours schedule. Group office hours are now scheduled during 3pm - 4pm on MWF in DH 3092 (default room but alternate room may need to be used on some days — an announcement will be made in the lecture on those days)