COMP 322: Fundamentals of Parallel Programming

Lecture 11: Loop-Level Parallelism, Parallel Matrix Multiplication, Iteration Grouping (Chunking)

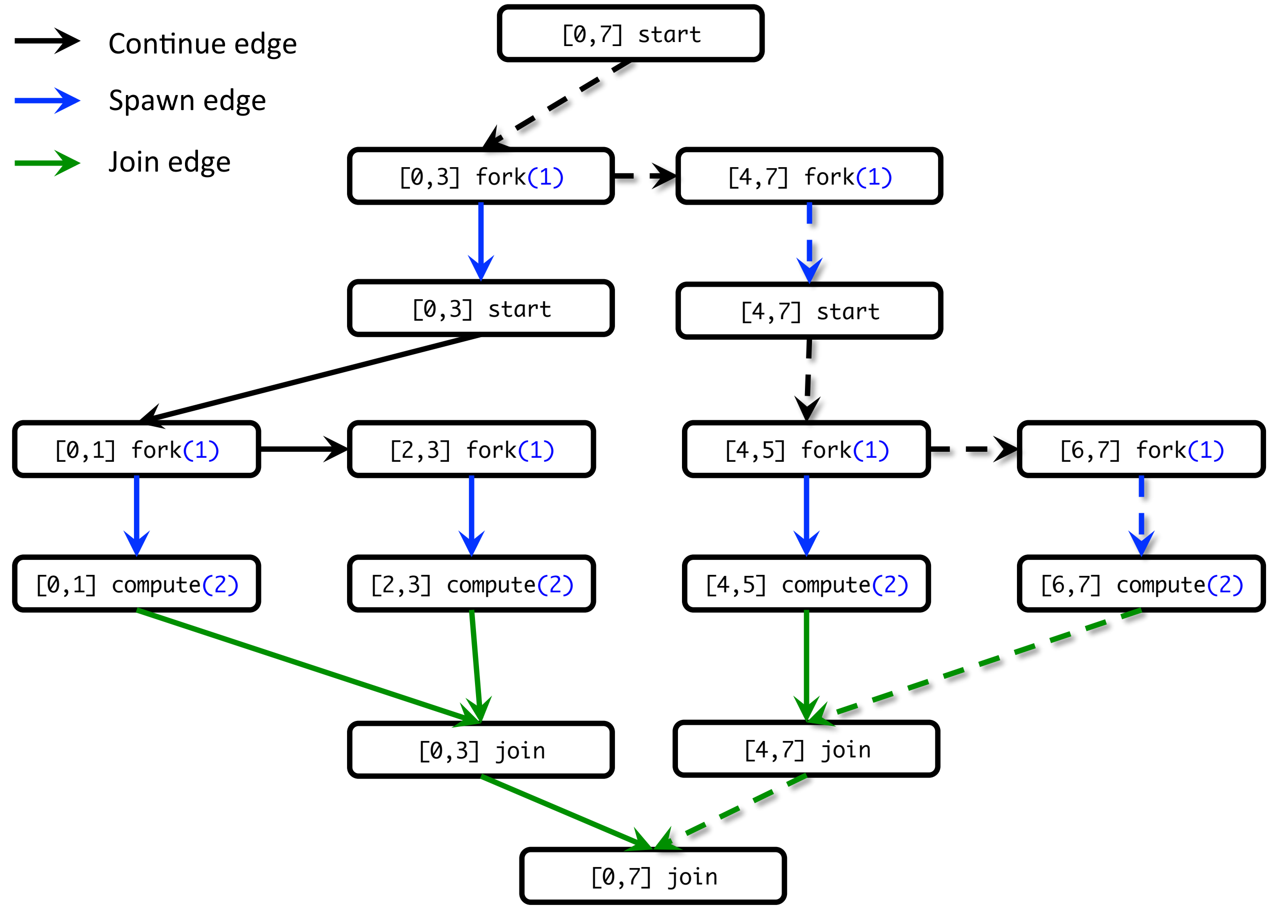
Mack Joyner and Zoran Budimlić
{mjoyner, zoran}@rice.edu

http://comp322.rice.edu

Worksheet #10 solution: RecursiveAction Computation Graph

1) Consider the compute method on slide 9. Let us suppose we supply it with an 8 element array with values [0,1,2,3,4,5,6,7] and THRESHOLD value of 2. Draw a computation graph corresponding to a call to `compute` with the appropriate fork and join edges.

2) Define each direct (sequential) computation as 2 units of work and each recursive subdivision as one unit of work. What is the total work? What is the critical path length?



TOTAL WORK = 14, CPL = 6 (critical path is highlighted as dashed edges)

NOTE: each call to compute() takes 2 units because THRESHOLD = 2

Outline of Today's Lecture

- Loop-Level Parallelism, Parallel Matrix Multiplication
  - [Topics 3.1, 3.2]

- Grouping/chunking of parallel loop iterations
  - [Topic 3.3]

Sequential Algorithm for Matrix Multiplication

```
// Sequential version
for (int i = 0 ; i < n ; i++)
  for (int j = 0 ; j < n ; j++)
    c[i][j] = 0;
for (int i = 0 ; i < n ; i++)
  for (int j = 0 ; j < n ; j++)
    for (int k = 0 ; k < n ; k++)
      c[i][j] += a[i][k] * b[k][j];
// Print first element of output matrix
println(c[0][0]);
```

$$c[i,j] = \sum_{0 \leq k < n} a[i,k] * b[k,j]$$

Parallelizing the loops in Matrix Multiplication example using finish & async

```
// Parallel version using finish & async
finish(() -> {
  for (int ii = 0 ; ii < n ; ii++)
    for (int jj = 0 ; jj < n ; jj++) {
      int i = ii; int j = jj;
      async(() -> {c[i][j] = 0; });
    }
});
finish(() -> {
  for (int ii = 0 ; ii < n ; ii++)
    for (int jj = 0 ; jj < n ; jj++){
      int i = ii; int j = jj;
      async(() -> {
        for (int k = 0 ; k < n ; k++)
          c[i][j] += a[i][k] * b[k][j];
      });
    }
});
// Print first element of output matrix
println(c[0][0])
```

$$c[i,j] = \sum_{0 \le k < n} a[i,k] * b[k,j]$$

Observations on finish-for-async version

- `finish` and `async` are general constructs, and are not specific to loops
  - Not easy to discern from a quick glance which loops are sequential vs. parallel
- Loops in sequential version of matrix multiplication are "perfectly nested"
  - e.g., no intervening statement between "`for`(i = ...)" and "`for`(j = ...)"
- The ordering of loops nested between `finish` and `async` is arbitrary
  - They are parallel loops and their iterations can be executed in any order

Parallelizing the loops in Matrix Multiplication example using forall

$$c[i,j] = \sum_{0 \le k < n} a[i,k] * b[k,j]$$

```
// Parallel version using forall
forall(0, n-1, 0, n-1, (i, j) -> {
    c[i][j] = 0;
 });
forall(0, n-1, 0, n-1, (i, j) -> {
   forseq(0, n-1, (k) -> {
      c[i][j] += a[i][k] * b[k][j];
   });
});
// Print first element of output matrix
println(c[0][0]);
```

forall API's in HJlib
(http://www.cs.rice.edu/~vs3/hjlib/doc/edu/rice/hj/Module1.html)

- static void forall(edu.rice.hj.api.HjRegion.HjRegion1D hjRegion, edu.rice.hj.api.HjProcedureInt1D body)
- static void forall(edu.rice.hj.api.HjRegion.HjRegion2D hjRegion, edu.rice.hj.api.HjProcedureInt2D body)
- static void forall(edu.rice.hj.api.HjRegion.HjRegion3D hjRegion, edu.rice.hj.api.HjProcedureInt3D body)
- static void forall(int s0, int e0, edu.rice.hj.api.HjProcedure<java.lang.Integer> body)
- static void forall(int s0, int e0, int s1, int e1, edu.rice.hj.api.HjProcedureInt2D body)
- static <T> void forall(java.lang.Iterable<T> iterable, edu.rice.hj.api.HjProcedure<T> body)
- NOTE: all `forall` API's include an implicit `finish`. `forasync` is like `forall`, but without the `finish`. Also e0 is the "end" value, not 1 + end value.

Observations on `forall` version

- The combination of perfectly nested finish-for–for–async constructs is replaced by a single API, `forall`
  - `forall` includes an implicit `finish`
- Multiple loops can be collapsed into a single `forall` with a multi-dimensional iteration space (can be 1D, 2D, 3D, ...)
- The iteration variable for a `forall` is a `HjPoint` (integer tuple), e.g., (i,j) is a 2-dimensional point
- The loop bounds can be specified as a rectangular `HjRegion` (product of dimension ranges), e.g., (0:n−1) x (0:n−1)
- HJlib also provides a sequential `forseq` API that can also be used to iterate sequentially over a rectangular region
  - Simplifies conversion between `forseq` and `forall`

forall examples: updates to a two-dimensional Java array

```
// Case 1: loops i,j can run in parallel
forall(0, m-1, 0, n-1, (i, j) -> { A[i][j] = F(A[i][j]);});

// Case 2: only loop i can run in parallel
forall(0, m-1, (i) -> {
  forseq(0, n-1, (j) -> { // Equivalent to “for (j=0;j<n;j++)”
     A[i][j] = F(A[i][j-1]) ;
}); });

// Case 3: only loop j can run in parallel
forseq(0, m-1, (i) -> { // Equivalent to “for (i=0;i<m;j++)”
  forall(0, n-1, (j) -> {
     A[i][j] = F(A[i-1][j]) ;
}); });
```

What about overheads?

- It is inefficient to create `forall` iterations in which each iteration (`async` task) does very little work
- An alternate approach is "iteration grouping" or "loop chunking"
  - e.g., replace

```
forall(0, 99, (i) -> BODY(i)); // 100 tasks
```

  - by

```
forall(0, 3, (ii) -> { // 4 tasks
  // Each task executes a "chunk" of 25 iterations
    forseq(25*ii, 25*(ii+1)-1, (i) -> BODY(i));

}); // forall
```

  - This is better, but it's still inconvenient for the programmer to do the "iteration grouping" or "loop chunking" explicitly

forallChunked APIs

- forallChunked(int s0, int e0, int chunkSize, edu.rice.hj.api.HjProcedure<Integer> body)
- Like forall(int s0, int e0, edu.rice.hj.api.HjProcedure<Integer> body)
- but `forallChunked` includes chunkSize as the third parameter!
  - e.g., replace

```
forall(0, 99, (i) -> BODY(i)); // 100 tasks
```

  - by

```
forallChunked(0, 99, 100/4, (i)->BODY(i));
```

One-Dimensional Iterative Averaging Example

- Initialize a one-dimensional array of (n+2) double's with boundary conditions, myVal[0] = 0 and myVal[n+1] = 1.
- In each iteration, each interior element myVal[i] in 1..n is replaced by the average of its left and right neighbors.
  - —Two separate arrays are used in each iteration, one for old values and the other for the new values
- After a sufficient number of iterations, we expect each element of the array to converge to myVal[i] = (myVal[i-1]+myVal[i+1])/2, for all i in 1..n

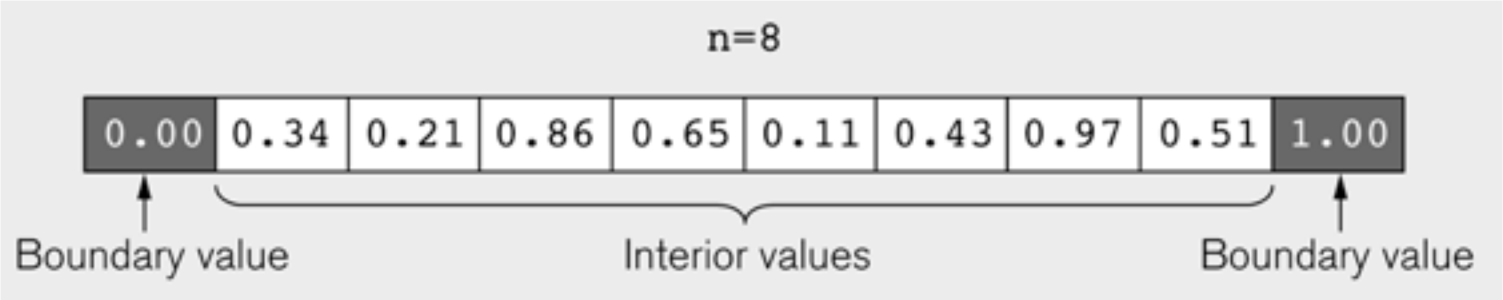


Illustration of an intermediate step for n = 8 (source: Figure 6.19 in Lin-Snyder book)

Sequential code for One-Dimensional Iterative Averaging that uses two copies of the array

```
1.  // Intialize m, n, myVal, newVal
2.  m = … ; n = … ;
3.  float[] myVal = new float[n+2];
4.  float[] myNew = new float[n+2];
5.   forseq(0, m-1, (iter) -> {
6.     // Compute MyNew as function of input array MyVal
7.       forseq(1, n, (j) -> { // Create n tasks
8.           myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
9.       }); // forseq
10.    // What is the purpose of line 11 below?
11.    float[] temp=myVal; myVal=myNew; myNew=temp;
12.    // myNew becomes input array for next iteration
13.  }); // forseq
```

QUESTION: can either forseq() loop execute in parallel?

Example: HJ code for One-Dimensional Iterative Averaging using nested forseq-forall structure

```
1.  // Intialize m, n, myVal, newVal
2.  m = … ; n = … ;
3.  float[] myVal = new float[n+2];
4.  float[] myNew = new float[n+2];
5.  forseq(0, m-1, (iter) -> {
6.    // Compute MyNew as function of input array MyVal
7.      forall(1, n, (j) -> { // Create n tasks
8.          myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
9.      }); // forseq
10.  // What is the purpose of line 11 below?
11.  float[] temp=myVal; myVal=myNew; myNew=temp;
12.  // myNew becomes input array for next iteration
13. }); // forseq
```

Example: HJ code for One-Dimensional Iterative Averaging with forseq-forall structure w/ chunking

```
int nc = numWorkerThreads();
 … // Initializations
forseq(0, m-1, (iter) -> {
  // Compute MyNew as function of input array MyVal
    forallChunked(1, n, n/nc, (j) -> { // Create n/nc tasks
      myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
    }); // forall
  // Swap myVal & myNew;
  float[] temp=myVal; myVal=myNew; myNew=temp;
  // myNew becomes input array for next iteration
}); // for
```

NOTS Maintenance: Feb 6th - 8th