COMP 322: Fundamentals of Parallel Programming

Lecture 12: Parallelism in Java Streams, Parallel Prefix Sums

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Worksheet #12: Forall Loops and Barriers

Draw a “barrier matching” figure similar to lecture 12 slide 11 for the code fragment below.

```
String[] a = { “ab”, “cde”, “f” };
. . . int m = a.length; . . .
forallPhased (0, m-1, (i) -> {
  for (int j = 0; j < a[i].length(); j++) {
    // forall iteration i is executing phase j
    System.out.println("(" + i + "," + j + ")");
    next();
  }
});
```

Solution

```
i=0          i=1          i=2
 |            |            |
(0,0)       (1,0)        (2,0)
 |            |            |
next ----- next ----- next
 |            |            |
(0,1)       (1,1)          |
 |            |            |
next ----- next ----- end
 |            |
 |          (1,2)
 |            |
end ----- next
              |
              |
              |
             end
```

How Java Streams addressed pre-Java-8 limitations of Java Collections

1. Iteration had to be performed explicitly using for/foreach loop, e.g.,

```
// Iterate through students (collection of Student objects)
for (Student s in students) System.out.println(s);
```

⇒ Simplified using Streams as follows

```
students.stream().foreach(s -> System.out.println(s));
```

2. Overhead of creating intermediate collections

```
List<Student> activeStudents = new ArrayList<Student>();
for (Student s in students)
     if (s.getStatus() == Student.ACTIVE) activeStudents.add(s);
for (Student a in activeStudents) totalCredits += a.getCredits();
```

⇒ Simplified using Streams as follows

```
totalCredits = students.stream().filter(s -> s.getStatus() == Student.ACTIVE)
                                .mapToInt(a -> a.getCredits()).sum();
```

3. Complexity of parallelism simplified (for example by replacing `stream()` by `parallelStream()`)

Parallelism in processing Java Streams

- Parallelism can be introduced at a stream source …
  - e.g., library.parallelStream()…
- … or as an intermediate operation
  - e.g., library.stream().sorted().parallel()…
- Stateful intermediate operations should be avoided on parallel streams …
  - e.g., distinct, sorted, user-written lambda with side effects
- … but stateless intermediate operations work just fine
  - e.g., filter, map

Beyond Sum/Reduce Operations — Prefix Sum (Scan) Problem Statement

Given input array A, compute output array X as follows

$$X[i] = \sum_{0 \leq j \leq i} A[j]$$

- The above is an inclusive prefix sum since X[i] includes A[i]
- For an exclusive prefix sum, perform the summation for 0 <=j <i
- It is easy to see that inclusive prefix sums can be computed sequentially in O(n) time …

```
// Copy input array A into output array X
X = new int[A.length]; System.arraycopy(A,0,X,0,A.length);
// Update array X with prefix sums
for (int i=1 ; i < X.length ; i++ ) X[i] += X[i-1];
```

- … and so can exclusive prefix sums

An Inefficient Parallel Algorithm for Exclusive Prefix Sums

```
forall(0, X.length-1, (i) -> {
    // computeSum() adds A[0..i-1]
    X[i] = computeSum(A, 0, i-1);
}
```

Observations:

- Critical path length, CPL = O(log n)
- Total number of operations, WORK = $O(n^2)$
- With P = O(n) processors, the best execution time that you can achieve is T_P = max(CPL, WORK/P) = O(n), which is no better than sequential!

How can we do better?

Assume that input array A = [3, 1, 2, 0, 4, 1, 1, 3]

Define scan(A) = exclusive prefix sums of A = [0, 3, 4, 6, 6, 10, 11, 12]

Hint:

- Compute B by adding pairwise elements in A to get B = [4, 2, 5, 4]
- Assume that we can recursively compute scan(B) = [0, 4, 6, 11]
- How can we use A and scan(B) to get scan(A)?

Another way of looking at the parallel algorithm

Observation: each prefix sum can be decomposed into reusable terms of power-of-2-size e.g.

$$
\begin{aligned}
X[6] &= A[0] + A[1] + A[2] + A[3] + A[4] + A[5] + A[6] \\
&= (A[0] + A[1] + A[2] + A[3]) + (A[4] + A[5]) + A[6]
\end{aligned}
$$

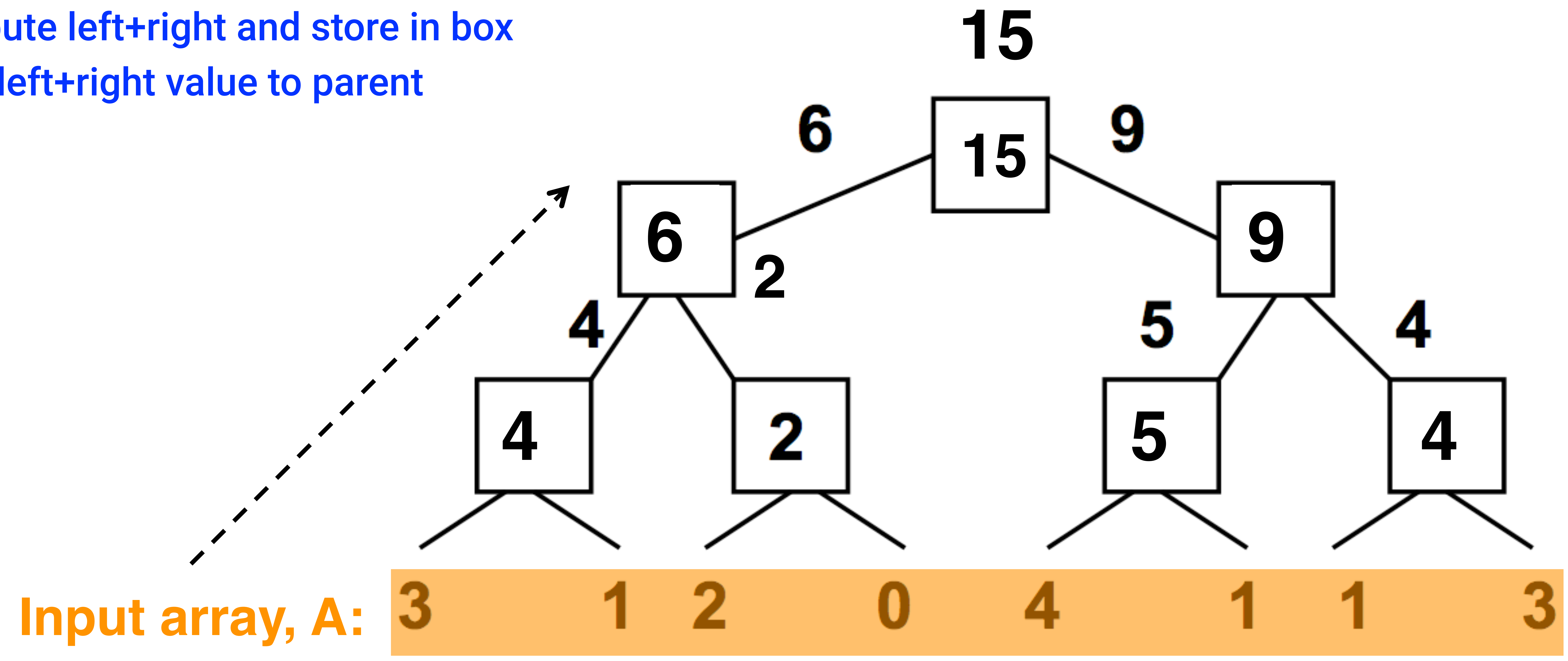
Approach:

- Combine reduction tree idea from Parallel Array Sum with partial sum idea from Sequential Prefix Sum
- Use an “upward sweep” to perform parallel reduction, while storing partial sum terms in tree nodes
- Use a “downward sweep” to compute prefix sums while reusing partial sum terms stored in upward sweep

Parallel Prefix Sum: Upward Sweep (while calling scan recursively)

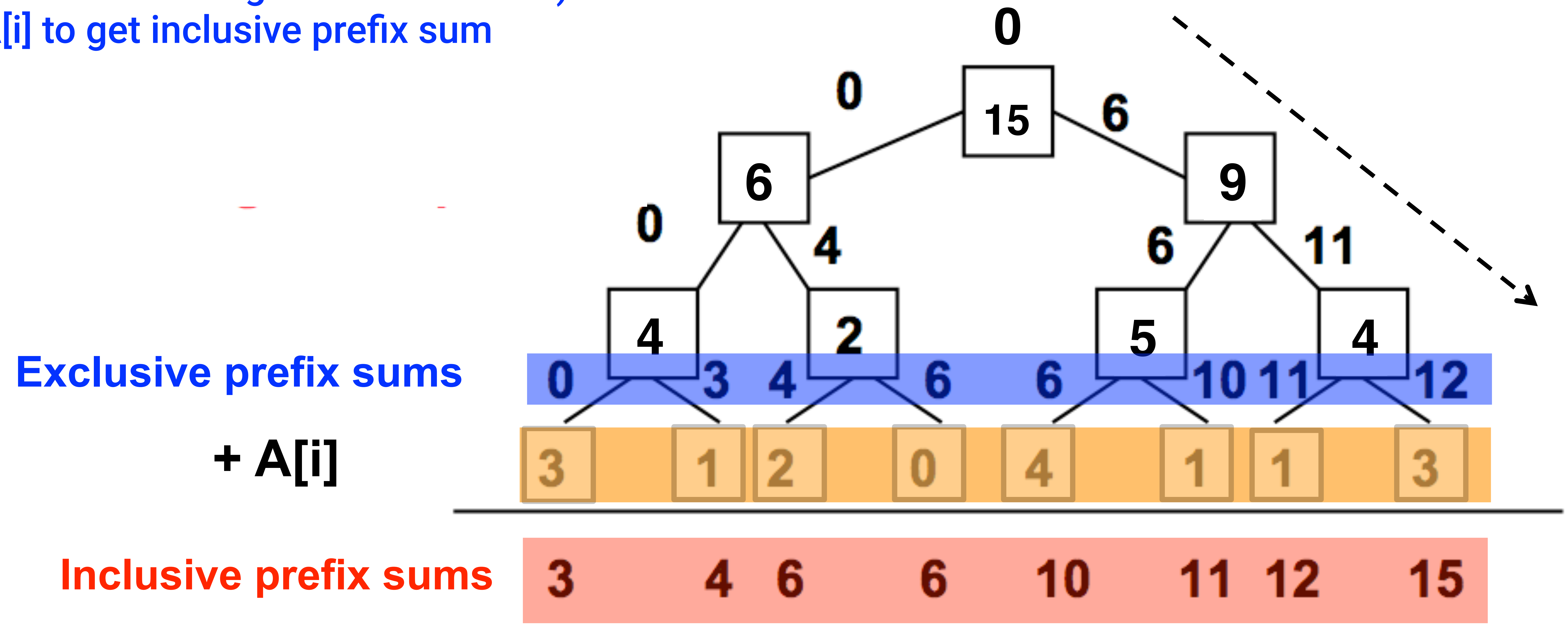
Upward sweep is just like Parallel Reduction, except that partial sums are also stored along the way

1. Receive values from left and right children
2. Compute left+right and store in box
3. Send left+right value to parent



Parallel Prefix Sum: Downward Sweep (while returning from recursive calls to scan)

1. Receive value from parent (root receives 0)
2. Send parent's value to LEFT child (prefix sum for elements to left of left child's subtree)
3. Send parent's value+ left child's box value to RIGHT child (prefix sum for elements to left of right child's subtree)
4. Add A[i] to get inclusive prefix sum



Summary of Parallel Prefix Sum Algorithm

- Critical path length, CPL = O(log n)
- Total number of add operations, WORK = O(n)
- Optimal algorithm for P = O(n/log n) processors
  - Adding more processors does not help
- Parallel Prefix Sum has several applications that go beyond computing the sum of array elements
  - Parallel Prefix Sum can be used for any operation that is associative (need not be commutative)
    - In contrast, finish accumulators required the operator to be both associative and commutative

Parallel Filter Operation

[Credits: David Walker and Andrew W. Appel (Princeton), Dan Grossman (U. Washington)]

Given an array **`input`**, produce an array **`output`** containing only elements such that **`f(elt)`** is true, i.e., `output = input.parallelStream().filter(f).toArray()`

Example: **`input [17, 4, 6, 8, 11, 5, 13, 19, 0, 24]`**

`f: is elt > 10`

`output [17, 11, 13, 19, 24]`

Parallelizable?

– Finding elements for the output is easy
– But getting them in the right place seems hard

Parallel prefix to the rescue

1. Parallel map to compute a bit-vector for true elements (can use Java streams)

```
input  [17, 4, 6, 8, 11, 5, 13, 19, 0, 24]
bits   [1,  0, 0, 0,  1, 0,  1,  1, 0,  1]
```

2. Parallel-prefix sum on the bit-vector (not available in Java streams)

```
bitsum [1,  1, 1, 1,  2, 2,  3,  4, 4,  5]
```

3. Parallel map to produce the output (can use Java streams)

```
output [17, 11, 13, 19, 24]
```

```
output = new array of size bitsum[n-1]
FORALL(i=0; i < input.length; i++){
  if(bits[i]==1)
    output[bitsum[i]-1] = input[i];
}
```

Announcements & Reminders

- Quiz for Unit 2 (topics 2.1 - 2.8) is due today by 11:59pm
- HW2 is due Wednesday by 11:59pm
- Watch the topic 3.5, 3.6 videos for the next lecture
- Midterm Exam on Thursday, Feb. 27th from 7-9pm in DH McMurtry Aud.