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# COMP 322: Fundamentals of Parallel Programming

## Lecture 2: Computation Graphs, Ideal Parallelism

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<https://wiki.rice.edu/confluence/display/PARPROG/COMP322>



# Async and Finish Statements for Task Creation and Termination (Recap)

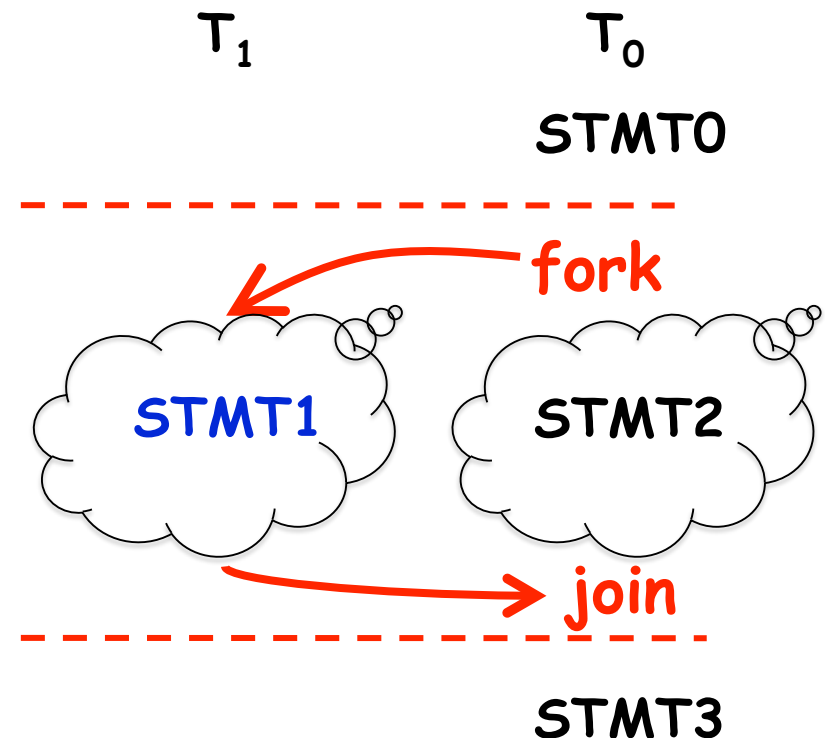
## async S

- Creates a new child task that executes statement S

```
// T0 (Parent task)
STMT0;
finish { //Begin finish
  async {
    STMT1; //T1 (Child task)
  }
  STMT2; //Continue in T0
           //Wait for T1
} //End finish
STMT3; //Continue in T0
```

## finish S

- Execute S, but wait until *all* asyncs in S's scope have terminated.



# Solution to Worksheet 1 (Parallel Matrix Multiplication)

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```
1. finish {
2.   for (int i = 0 ; i < N ; i++)
3.     for (int j = 0 ; j < N ; j++)
4.       async {
5.         for (int k = 0 ; k < N ; k++)
6.           C[i][j] += A[i][k] * B[k][j];
7.       } // async
8.} // finish
```

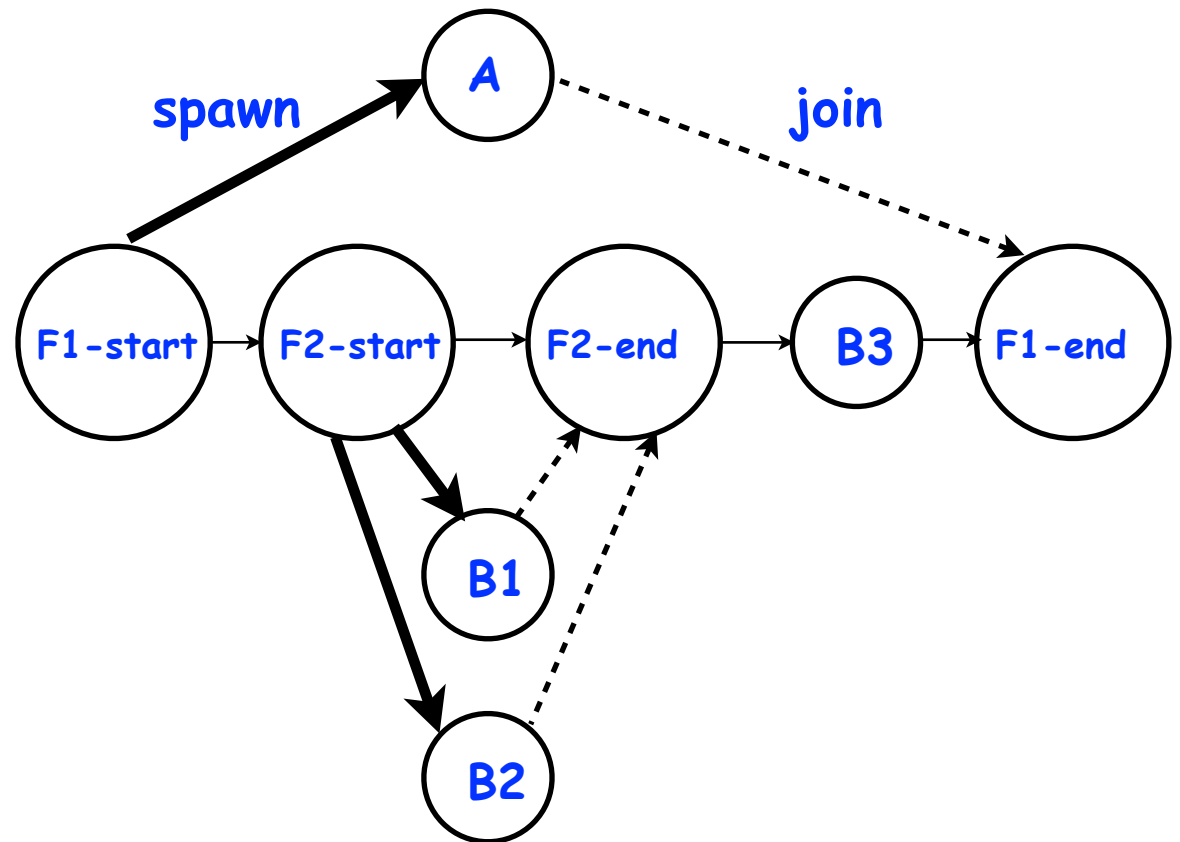
*This program generates  $N^2$  parallel async tasks, one to compute each  $C[i][j]$  element of the output array*



# Which statements can potentially be executed in parallel with each other?

```
1. finish { // F1
2.   async A;
3.   finish { // F2
4.     async B1;
5.     async B2;
6.   } // F2
7.   B3;
8. } // F1
```

## Computation Graph



# Computation Graphs

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- A Computation Graph (CG) captures the dynamic execution of a parallel program, for a specific input
- CG nodes are “steps” in the program’s execution
  - A step is a sequential subcomputation without any async, begin-finish and end-finish operations
- CG edges represent ordering constraints
  - “Continue” edges define sequencing of steps within a task
  - “Spawn” edges connect parent tasks to child async tasks
  - “Join” edges connect the end of each async task to its IEF’s end-finish operations
- All computation graphs must be acyclic
  - It is not possible for a node to depend on itself
- Computation graphs are examples of “directed acyclic graphs” (dags)



# Complexity Measures for Computation Graphs

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## Define

- $\text{TIME}(N)$  = execution time of node  $N$
- $\text{WORK}(G)$  = sum of  $\text{TIME}(N)$ , for all nodes  $N$  in CG  $G$ 
  - $\text{WORK}(G)$  is the total work to be performed in  $G$
- $\text{CPL}(G)$  = length of a longest path in CG  $G$ , when adding up execution times of all nodes in the path
  - Such paths are called *critical paths*
  - $\text{CPL}(G)$  is the length of these paths (critical path length)
  - $\text{CPL}(G)$  is also the smallest possible execution time for the computation graph



# What is the critical path length of this parallel computation?

```
1. finish { // F1
2.   async A; // Boil pasta
3.   finish { // F2
4.     async B1; // Chop veggies
5.     async B2; // Brown meat
6.   } // F2
7.   B3; // Make pasta sauce
8. } // F1
```

Step B1



Step B2



Step A



Step B3



# Ideal Parallelism

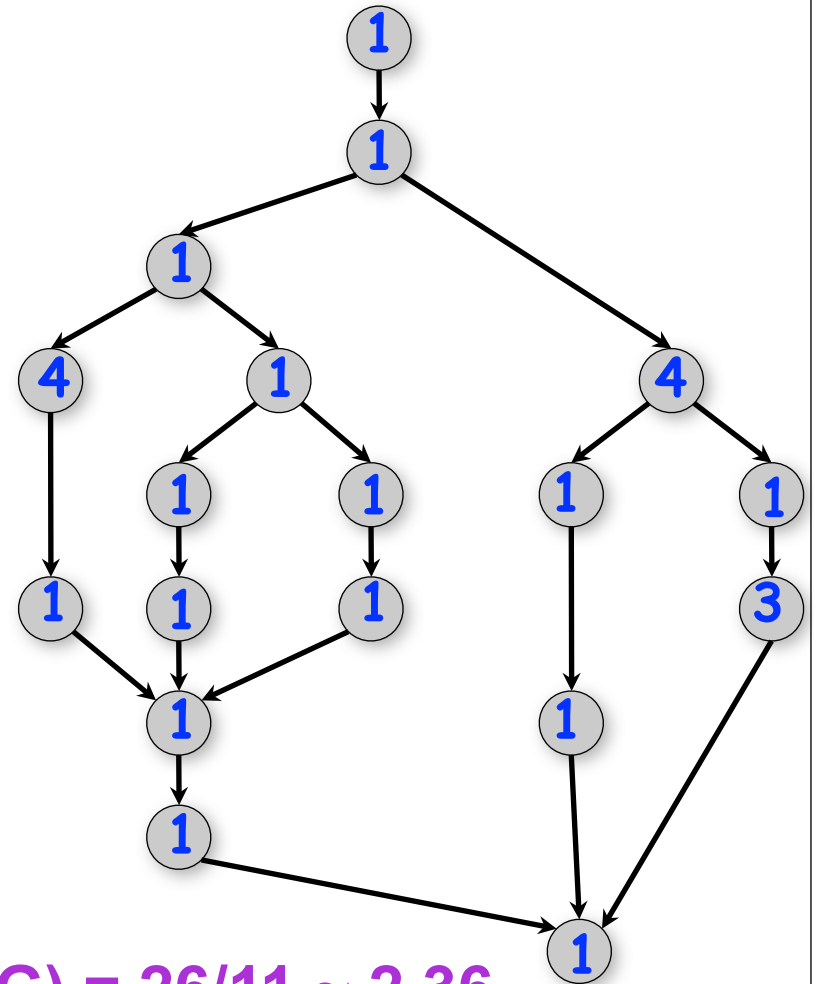
- Define **ideal parallelism** of Computation G Graph as the ratio,  $WORK(G)/CPL(G)$
- Ideal Parallelism is independent of the number of processors that the program executes on, and only depends on the computation graph

## Example:

$$WORK(G) = 26$$

$$CPL(G) = 11$$

$$\text{Ideal Parallelism} = WORK(G)/CPL(G) = 26/11 \sim 2.36$$





# String Search Problem

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- **Inputs**
  - text: a long string with N characters to search in
  - pattern: a short string of M characters to search for
- **Output**
  - Existence of an occurrence (boolean value)
- **Example**
  - text: “**ab**ac**ad**abr**abr**ac**abr**ac**ad**abab**ac**ad**abr**ac**abr**ac**ad**abr**abr**”
  - pattern: **aca**
  - output: true (pattern found)
- **Applications**
  - Word processing, virus scans, information retrieval, computational biology, web search engines, ...
- **Variations**
  - Count of occurrences, index of any occurrence, indices of all occurrences



# Brute Force Sequential Algorithm for String Search

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```
1. public static boolean search(char[] pattern, char[] text) {
2.     int M = pattern.length; int N = text.length;
3.     boolean found = false;
4.     for (int i = 0; i <= N - M; i++) {
5.         int j; // search for pattern starting at text[i]
6.         for (j = 0; j < M; j++) {
7.             // Count each char comparison as 1 unit of work
8.             if (text[i+j] != pattern[j]) break;
9.         } // for (j = ... )
10.        if (j == M) found = true; // found at offset i
11.    }
12.    return found;
13. }
```

What is the complexity (work) of this algorithm?



# Parallel Algorithm for String Search

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- Consider a parallel algorithm in which each  $i$  iteration is spawned as a separate async task
- For this above algorithm (assuming  $N \gg M$ )
  - WORK  $\sim M \cdot N$ ,
  - CPL  $\sim M$
  - Ideal Parallelism  $\sim N$
- Big-O notation: We say that a cost function  $\text{Cost}(n)$  is “order  $f(n)$ ”, or simply “ $O(f(n))$ ” (read “Big-O of  $f(n)$ ”) if
  - $\text{Cost}(n) < \text{factor} * f(n)$ , for sufficiently large  $n$ , for some constant **factor**
- If we consider  $M$  to be a constant in the String Search example then  $\text{WORK} = O(N)$ ,  $\text{CPL} = O(1)$ , and  $\text{Ideal Parallelism} = O(N)$



# Course Announcements

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- All Unit 1 lecture and demonstration quizzes are due by Jan 24th
  - Quizzes are still being uploaded into edX (see schedule on wiki)
- Homework 1 will be assigned on Jan 17th, and will be due on Jan 31st
- We will begin including programming exercises as in-class activities starting Jan 17th
  - Please bring laptops to class with HJlib set up for the exercises. Laptops can be shared within groups.
- Next week's schedule (Jan 20-24)
  - No lecture on Monday (MLK Jr Day)
  - No lab next week on Monday or Wednesday
  - We will have lectures on Wednesday & Friday as usual

