COMP 322: Fundamentals of Parallel Programming

Lecture 3: Computation Graphs, Abstract Performance Metrics, Array Reductions

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https://wiki.rice.edu/confluence/display/PARPROG/COMP322

Acknowledgments for Today's Lecture

- Cilk lectures, http://supertech.csail.mit.edu/cilk/

Goals for Today's Lecture

- <u>Lower and upper bounds for abstract parallel execution time</u>
- Parallel Array sum and Complexity Analysis
- Abstract execution metrics in HJ

Computation Graphs for HJ Programs (Recap)

- A Computation Graph (CG) captures the dynamic execution of an HJ program, for a specific input
- CG nodes are "steps" in the program's execution
  - A step is a sequential subcomputation without any async, begin-finish and end-finish operations
- CG edges represent ordering constraints
  - "Continue" edges define sequencing of steps within a task
  - "Spawn" edges connect parent tasks to child async tasks
  - "Join" edges connect the end of each async task to its IEF's end-finish operations
- All computation graphs must be acyclic
  - It is not possible for a node to depend on itself
- Computation graphs are examples of "directed acyclic graphs" (dags)

Lower Bounds on Execution Time

- Let T_P = execution time of computation graph on P processors
  - Assume an idealized machine where node N takes TIME(N) regardless of which processor it executes on, and that there is no overhead for creating parallel tasks
- Observations
  - T_1 = WORK(G)
  - T_∞ = CPL(G)
- Lower bounds
  - Capacity bound: $T_P \geq$ WORK(G)/P
  - Critical path bound: $T_P \geq$ CPL(G)
- Putting them together
  - $T_P \geq$ max(WORK(G)/P, CPL(G))

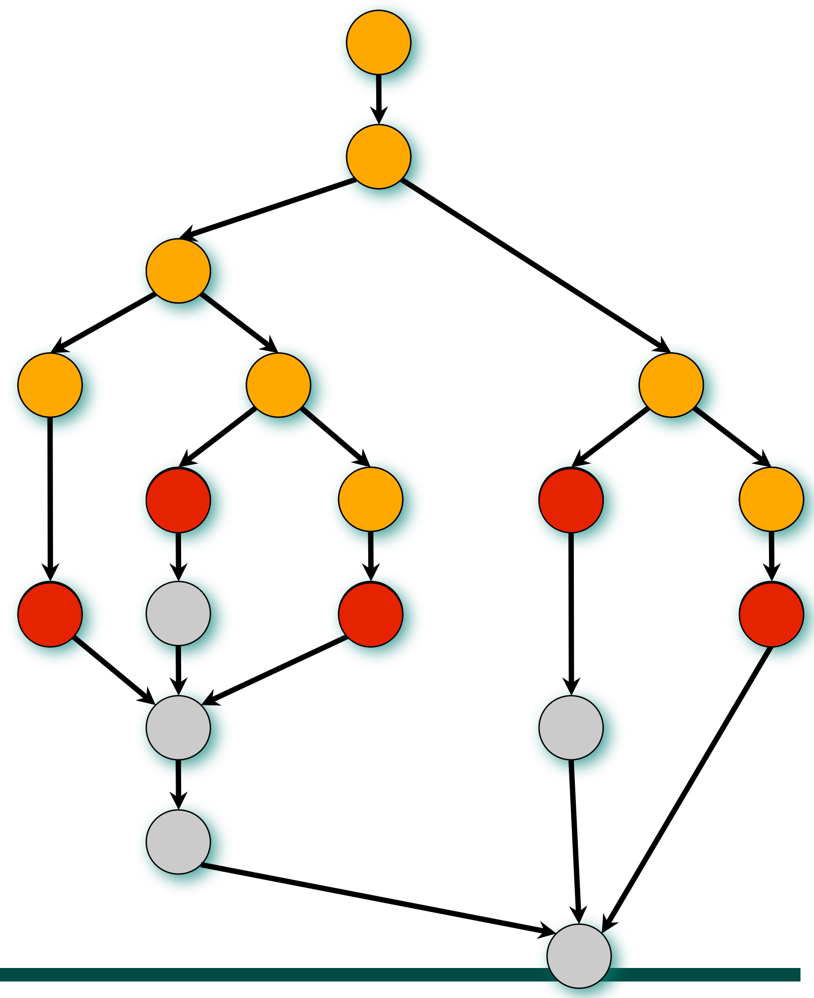
Upper Bound for Greedy Scheduling

Theorem [Graham '66]. Any "greedy scheduler" achieves

$$T_P \le WORK(G)/P + CPL(G)$$

- A greedy scheduler is one that never forces a processor to be idle when one or more nodes are ready for execution

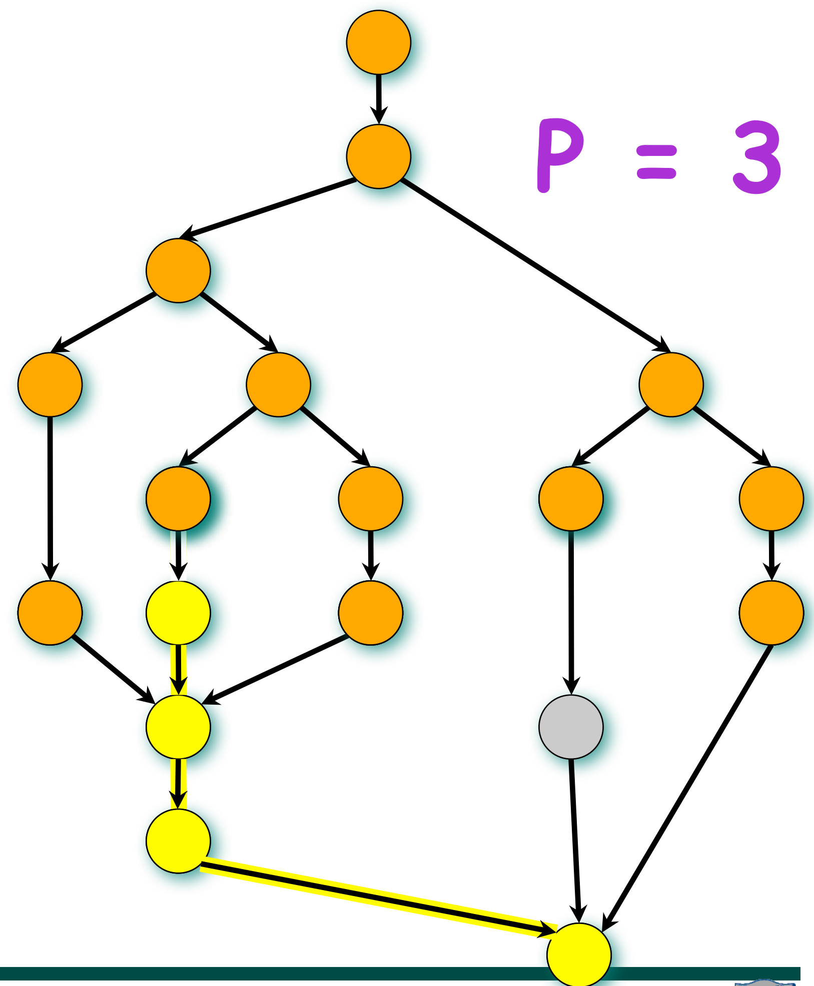
- A node is ready for execution if all its predecessors have been executed



Upper Bound on Execution Time: Greedy-Scheduling Theorem

Theorem [Graham '66]. Any greedy scheduler achieves

$$T_P \leq WORK(G)/P + CPL(G)$$



Proof sketch:

- Define a time step to be complete if $\geq$ P nodes are ready at that time, or incomplete otherwise

complete time steps $\leq$ WORK(G)/P, since each complete step performs P work.

incomplete time steps $\leq$ CPL(G), since each incomplete step reduces the span of the unexecuted dag by 1.

Optimality of Greedy Schedulers

Combine lower and upper bounds to get

$\max(WORK(G)/P,\ CPL(G)) \le T_P \le WORK(G)/P + CPL(G)$

Corollary 1: Any greedy scheduler achieves execution time T_P that is within a factor of 2 of the optimal time (since max(a,b) and (a+b) are within a factor of 2 of each other, for any $a \ge 0, b \ge 0$).

Corollary 2: Lower and upper bounds approach the same value whenever

- There's lots of parallelism, WORK(G)/CPL(G) >> P
- Or there's little parallelism, WORK(G)/CPL(G) << P

Goals for Today's Lecture

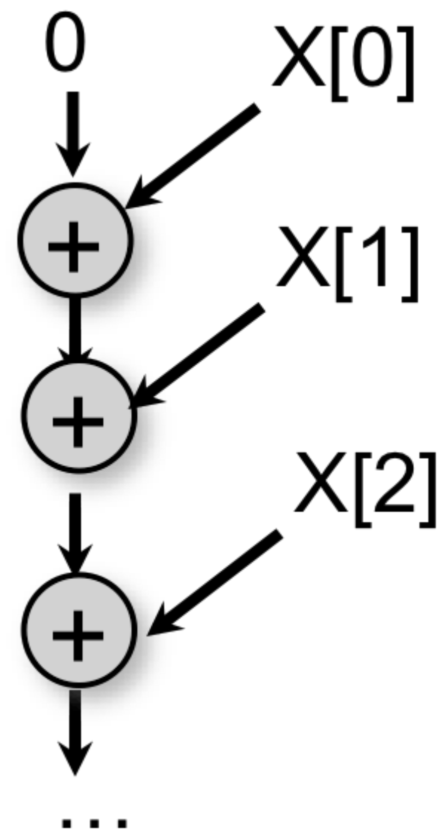
- Lower and upper bounds for abstract parallel execution time
- <u>Parallel Array sum and Complexity Analysis</u>
- Abstract execution metrics in HJ

Sequential Array Sum Program (Lecture 1)

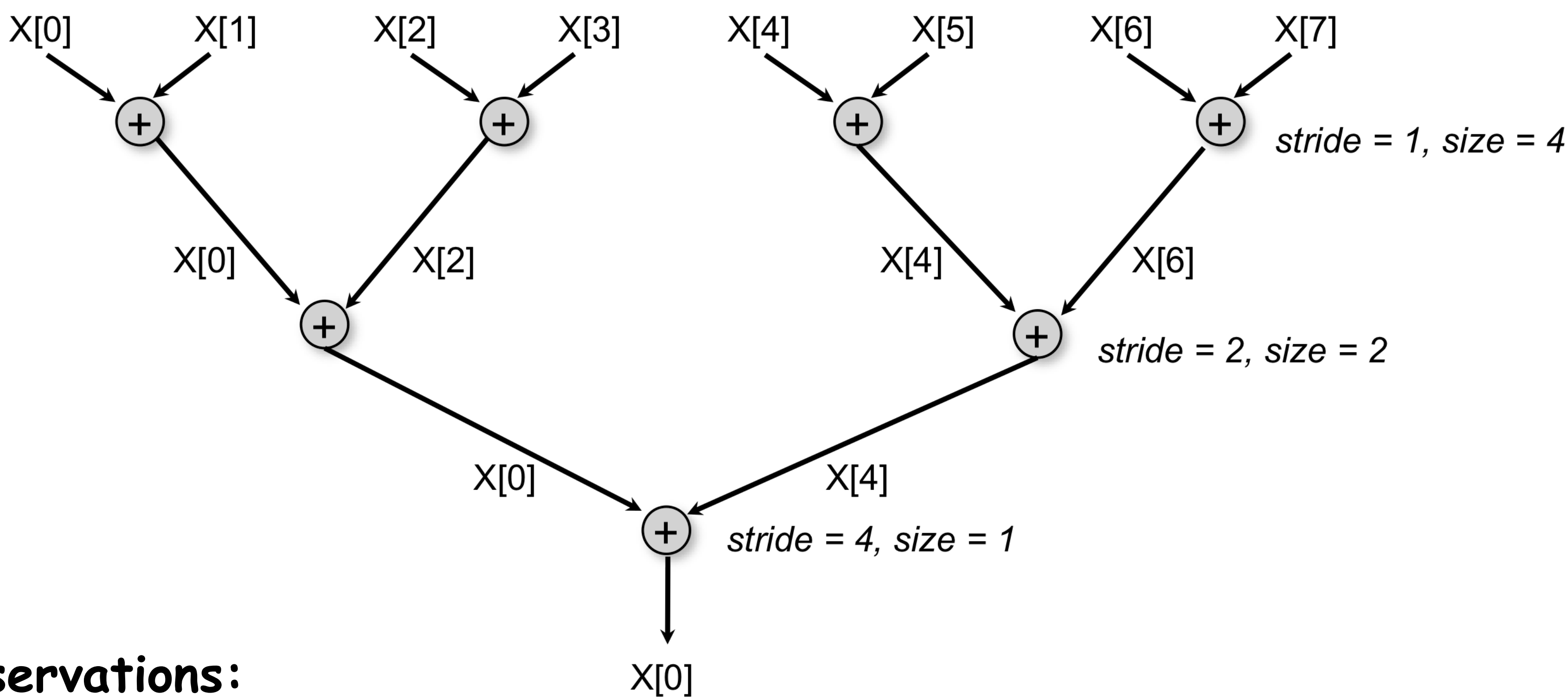
```
int sum = 0;
for (int i=0 ; i < X.length ; i++ )
    sum += X[i];
```

- The original computation graph is sequential
- We studied a 2-task parallel program for this problem
- How can we expose more parallelism?

Computation Graph



Reduction Tree Schema for computing Array Sum in parallel



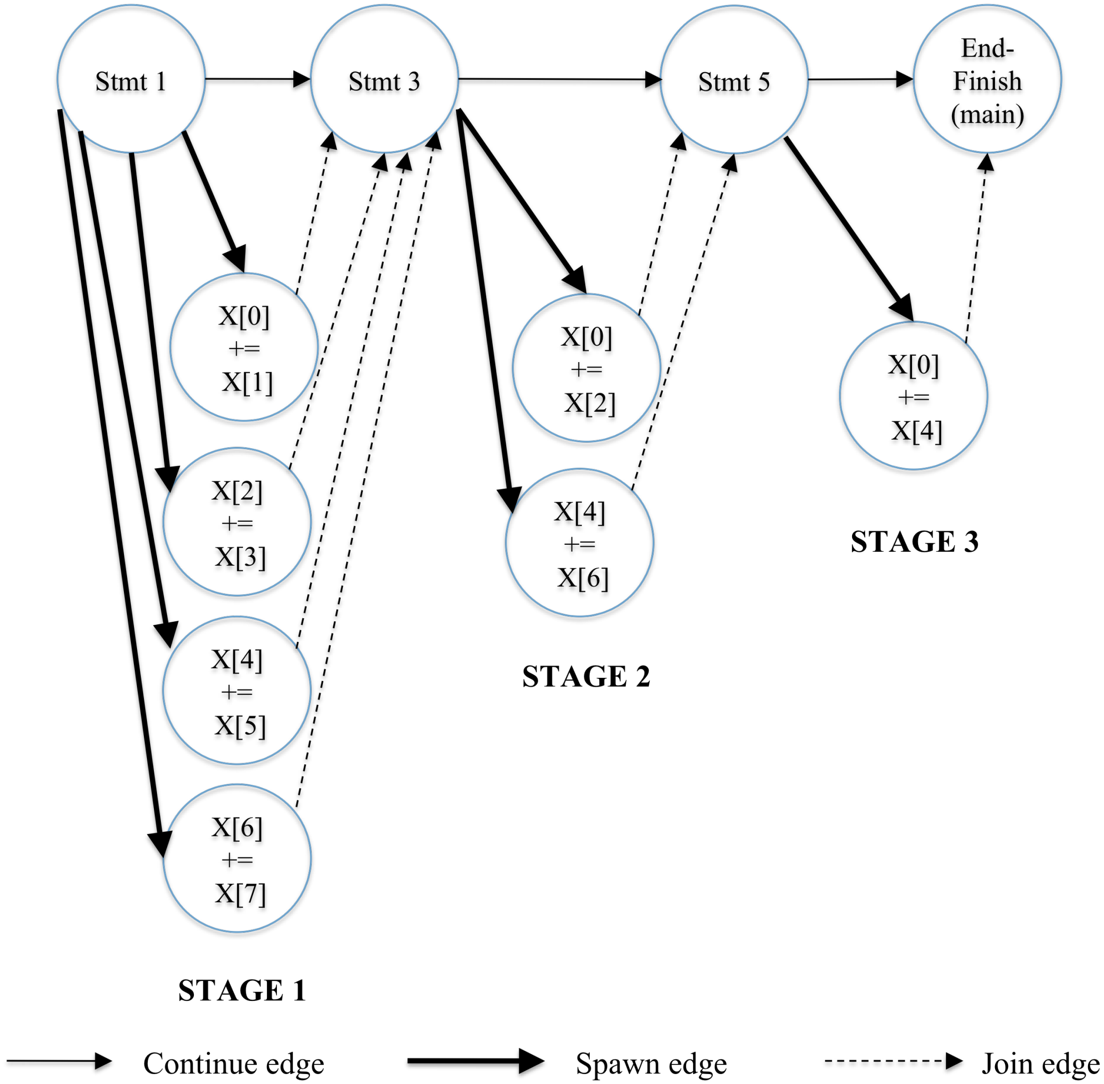
Observations:

- This algorithm overwrites X (make a copy if X is needed later)
- stride = distance between array subscript inputs for each addition
- size = number of additions that can be executed in parallel in each level (stage)

Parallel Program that satisfies dependences in Reduction Tree schema (for X.length = 8)

```
finish { // STAGE 1: stride = 1, size = 4 parallel additions
  async X[0]+=X[1]; async X[2]+=X[3];
  async X[4]+=X[5]; async X[6]+=X[7];
}
finish { // STAGE 2: stride = 2, size = 2 parallel additions
  async X[0]+=X[2]; async X[4]+=X[6];
}
finish { // STAGE 3: stride = 4, size = 1 parallel addition
  async X[0]+=X[4];
}
```

Computation Graph for ArraySum1



Generalization to arbitrary sized arrays (ArraySum1)

```
for ( int stride = 1; stride < X.length ; stride *= 2 ) {
  // Compute size = number of additions to be performed in stride
  int size=ceilDiv(X.length,2*stride);
  finish for(int i = 0; i < size; i++)
    async {
      if ( (2*i+1)*stride < X.length )
        X[2*i*stride]+=X[(2*i+1)*stride];
    } // finish-for-async
} // for


// Divide x by y, round up to next largest int, and return result
static int ceilDiv(int x, int y) { return (x+y-1) / y; }
```

Complexity Analysis of ArraySum1

- Define n = X.length
- Assume that each addition takes 1 unit of time
  - Ignore all other computations since they are related to the addition by some constant
- Total number of additions, WORK = n-1 = O(n)
- Critical path length (number of stages), CPL = ceiling($\log_2$(n)) = O(log(n))
- Ideal parallelism = WORK/CPL = O(n) / O(log(n))
- Consider an execution on p processors
  - Compute partial sums for batches of n/p elements on each processor
  - Use ArraySum1 program to reduce p partial sums to one total sum
  - CPL for this version is O(n/p + log(p))
  - Parallelism for this version is O(n) / O(n/p + log(p))
  - Algorithm is optimal for p = n / log(n), or fewer, processors – why?

Generalized Array Reductions

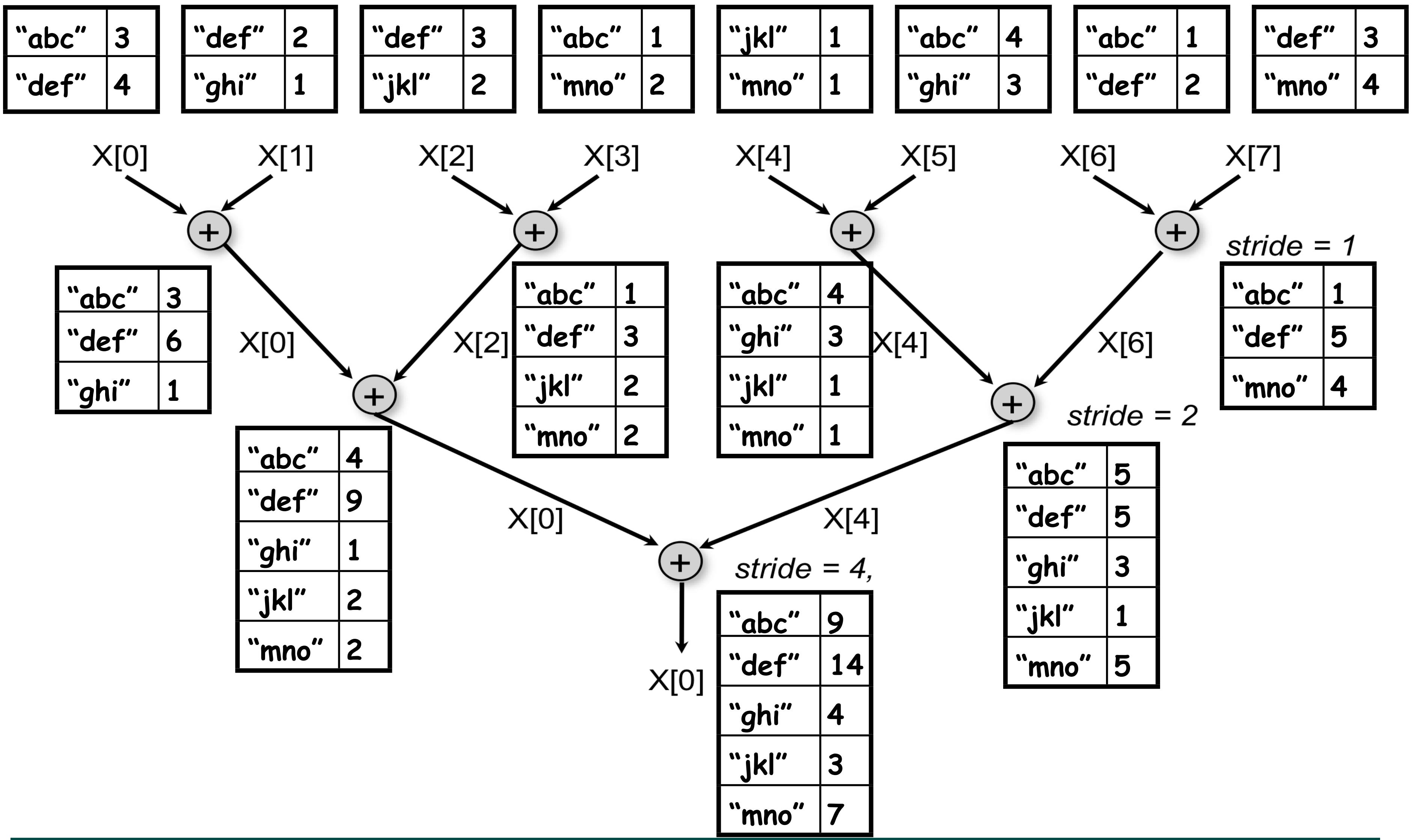
- ArraySum1 can easily be adapted to reduce any associative function f
  - f(x,y) is said to be associative if f(a,f(b,c)) = f(f(a,b),c) for any inputs a, b, and c
- Sequential reduction of X, an array of objects of type T:

```
T result=X[0];

for(int i=1 ; i < X.length ; i++ ) result=f(result,X[i]);
```

- Generalized reductions have many interesting applications in practice, as you will see when we learn about Google's Map Reduce framework
- Execution time of f() could be much larger than an integer add, and justify the use of an async

Generalized Reduction of WordCount



Extension of ArraySum1 to reduce an arbitrary associative function, f

```
for ( int stride = 1; stride < X.length ; stride *= 2 ) {
  // Compute size = number of additions to be performed in stride
  int size=ceilDiv(X.length,2*stride);
  finish for(int i = 0; i < size; i++)
    async {
      if ( (2*i+1)*stride < X.length )
        X[2*i*stride] = f(X[2*i*stride], X[(2*i+1)*stride]);
    } // finish-for-async
} // for

// Divide x by y, round up to next largest int, and return result
static int ceilDiv(int x, int y) { return (x+y-1) / x; }
```

HJ Abstract Performance Metrics

- **Basic Idea**
  - Count operations of interest, as in big-O analysis
  - Abstraction ignores overheads that occur on real systems
- **Calls to perf.addLocalOps()**
  - Programmer inserts calls of the form, perf.addLocalOps(N), within a step to indicate abstraction execution of N application-specific abstract operations
    - e.g., floating-point ops, stencil ops, data structure ops
  - Multiple calls add to the execution time of the step
- **Enabled by selecting "Show Abstract Execution Metrics" in DrHJ compiler options (or -perf=true runtime option)**
  - If an HJ program is executed with this option, abstract metrics are printed at end of program execution with WORK(G), CPL(G), Ideal Speedup = WORK(G)/ CPL(G)

Homework 1 Reminder

- Written assignment, due today
- Submit a softcopy of your solution in Word, PDF, or plain text format
  - Try and use turn-in script for submission, if possible
  - Otherwise, email your homework to comp322-staff at mailman.rice.edu
- See course web site for penalties for late submissions
  - Send me email if you have an extenuating circumstance for delay