COMP 322: Fundamentals of Parallel Programming

Lecture 4: Parallel Speedup, Efficiency, Amdahl's Law

Vivek Sarkar
Department of Computer Science, Rice University
vsarkar@rice.edu

https://wiki.rice.edu/confluence/display/PARPROG/COMP322

Goals for Today's Lecture

- Recap of parallel complexity for ArraySum1
- Speedup, Efficiency, Amdahl's Law
- Use of Abstract Performance Metrics

Lower and Upper Bounds for Greedy Schedulers (Recap)

$$\max(WORK(G)/P,\ CPL(G)) \le T_P \le WORK(G)/P + CPL(G)$$

where

- G = computation graph
- WORK(G) = sum of time(N), for all nodes N in G
- CPL(G) = length of a longest directed path in CG G, when adding up the execution times of all nodes in the path
- The above bounds are for greedy schedulers and an idealized model of P parallel processors
- There may be cases when the lower and upper bounds are not achievable

Cases when Lower and Upper Bounds approach each other

$$\max(WORK(G)/P,\ CPL(G)) \le T_P \le WORK(G)/P + CPL(G)$$

Lower Bound — Upper Bound

Case 1: There's lots of parallelism, $WORK(G)/CPL(G) \gg P$

$\Rightarrow WORK(G)/P \gg CPL(G)$

$\Rightarrow WORK(G)/P \le T_P \le WORK(G)/P + CPL(G)$

$\Rightarrow T_P \approx WORK(G)/P$

Case 2: There's little parallelism, $WORK(G)/CPL(G) \ll P$

$\Rightarrow WORK(G)/P \ll CPL(G)$

$\Rightarrow CPL(G) \le T_P \le CPL(G) + WORK(G)/P$

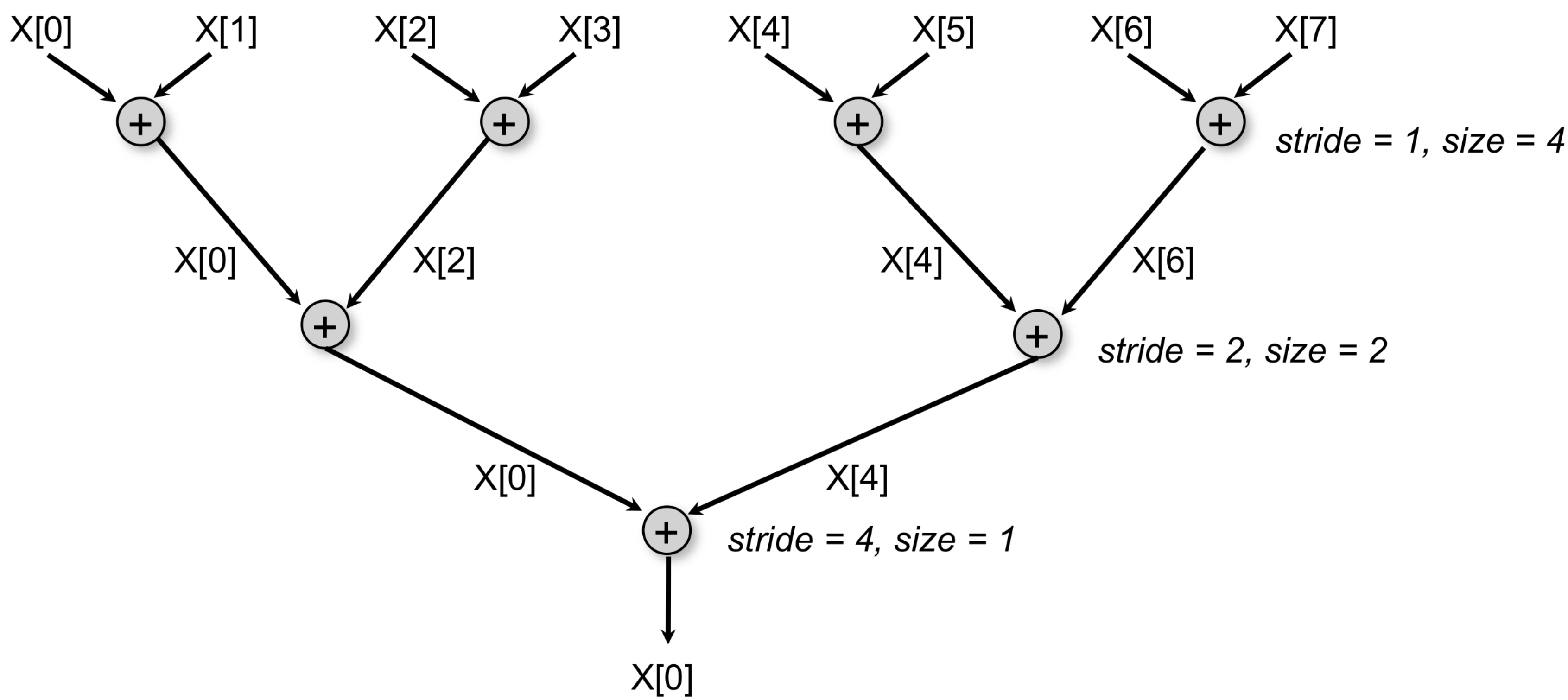
$\Rightarrow T_P \approx CPL(G)$

ArraySum1: Computing the sum of an array in parallel (Recap)

```
for ( int stride = 1; stride < X.length ; stride *= 2 ) {
  // size = number of additions to be performed in stride
  int size=ceilDiv(X.length,2*stride);
  finish for(int i = 0; i < size; i++)
    async {
      if ( (2*i+1)*stride < X.length )
        X[2*i*stride]+=X[(2*i+1)*stride];
    } // finish-for-async
} // for

// Divide x by y, and round up to next largest int
static int ceilDiv(int x, int y) { return (x+y-1) / y; }
```

Reduction Tree Schema for computing Array Sum in parallel



- Define N = X.length
- WORK = N-1 = O(N)
- Critical path length (number of stages), CPL = O(log(N))

ArraySum1 pre-pass when P < array length

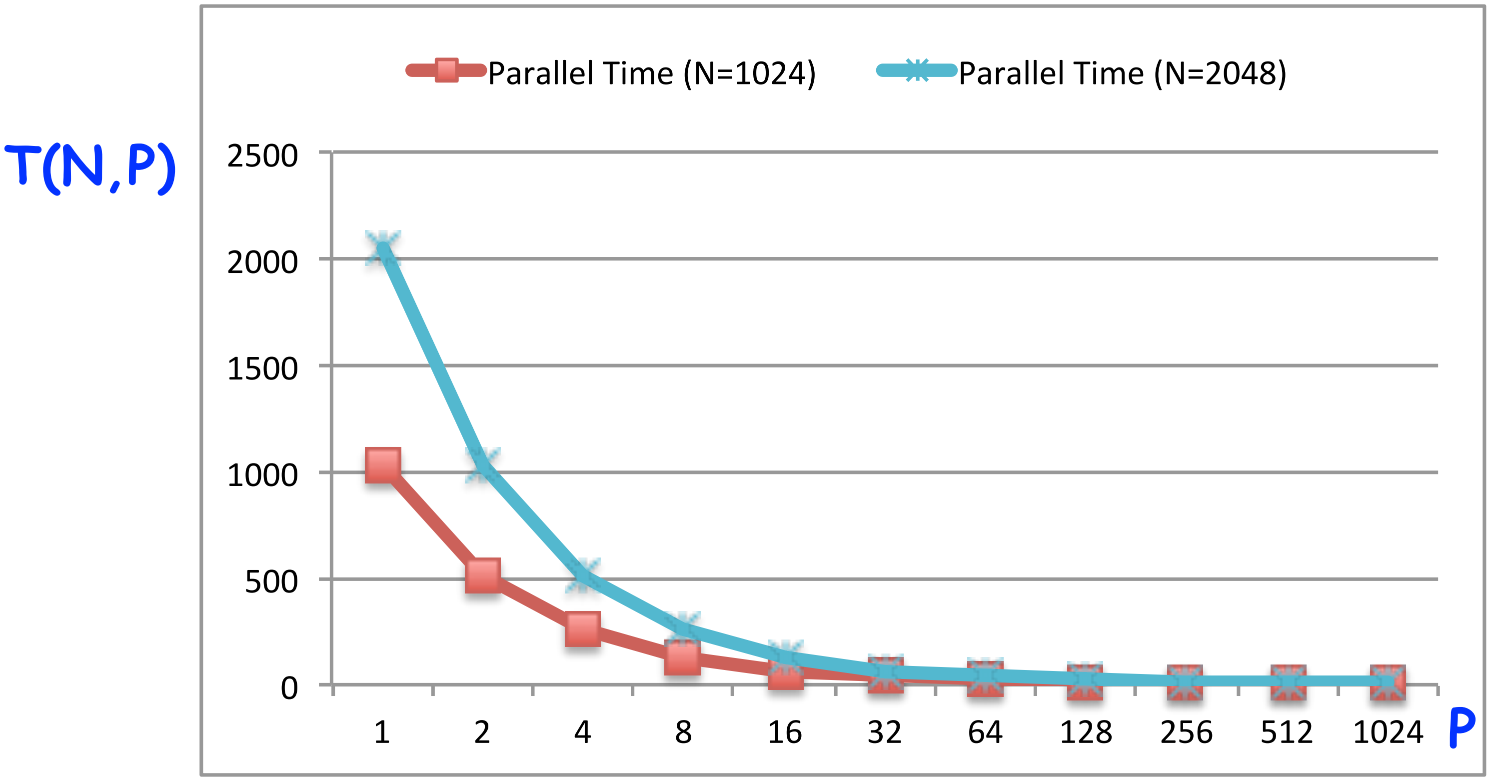
```
1. // Start of pre-pass: compute P partial sums in parallel
2. finish for(int j = 0; j < P; j++) // Create P tasks
3.    async {
4.      // Compute sum of A[j],A[j+P],... in task (processor) j
5.      // Any other decomposition into P partial sums is fine too
6.      for(int i = j; i < A.length; i += P) X[j] += A[i];
7.    } // finish-for-async
8. // End of pre-pass: now X[0..P-1] has P partial sums of array A
9. // Use ArraySum1 algorithm (slide 5) to obtain total sum
```

Complexity analysis

- Parallel time for pre-pass in lines 1-7 = O(N/P), where N = A.length
- Parallel time for ArraySum1 algorithm = O(log P)
- Total parallel time, T(N,P) = O(N/P + log P)

ArraySum: Ideal Parallel Time as function of P

- Total parallel time, $T(N,P) = N/P + \log_2(\min(P,N))$, depends on
  - Input size, N
  - Number of processors, P



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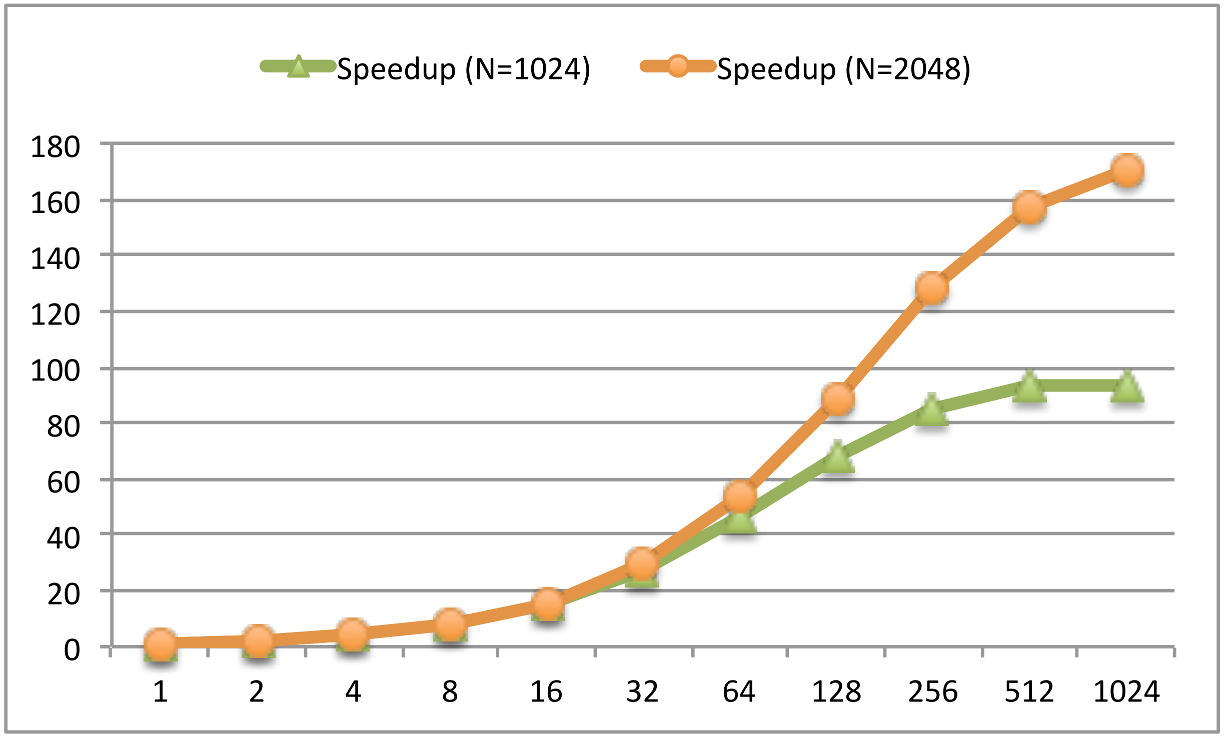
Speedup Definitions

- Speedup(N,P) = T(N,1)/T(N,P)
  - Factor by which the use of P processors speeds up execution time relative to 1 processor, for input size N
  - For ideal executions without overhead, 1 <= Speedup(P) <= P
- Strong scaling
  - Goal is linear speedup for a given input size
    - When Speedup(N,P) = k*P, for some constant k, 0 < k < 1
  - In practice, we may also see
    - Speedup(P) < 1 (slowdown)
    - Speedup(P) > P (super-linear speedup)
- Weak scaling
  - Increase problem size to use processors more efficiently
  - Define Weak-Speedup(N(P),P) = T(N(P),1)/T(N(P),P), where input size N(P) increases with P

ArraySum: Speedup as function of P

- Speedup(N,P) = T(N,1)/T(N,P) = N/(N/P + $\log_2$(min(P,N)))
- Asymptotically, Speedup(N,P) --> N/$\log_2$N, as P --> infinity

Speedup(N,P)



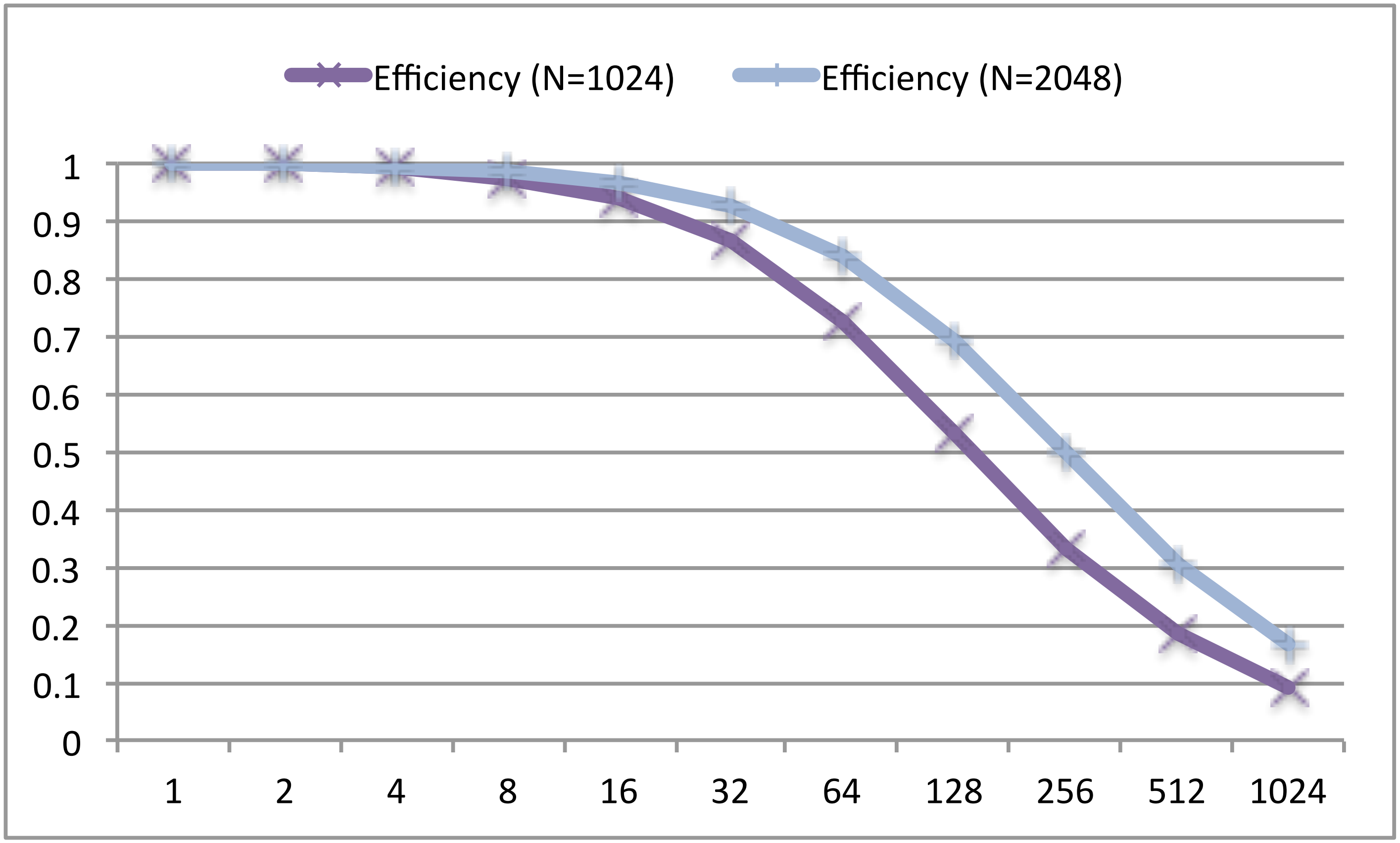
P

Efficiency Metrics

- Efficiency(P) = Speedup(P)/ P = $T_1/(P * T_P)$
  - Processor efficiency --- figure of merit that indicates how well a parallel program uses available processors
  - For ideal executions without overhead, 1/P <= Efficiency(P) <= 1
- Half-performance metric
  - $N_{1/2}$ = input size that achieves Efficiency(P) = 0.5 for a given P
  - Figure of merit that indicates how large an input size is needed to obtain efficient parallelism
  - A larger value of $N_{1/2}$ indicates that the problem is harder to parallelize efficiently

ArraySum: Efficiency as function of P

- **Common approach: choose largest number of processors that delivers efficiency above a given limit e.g., 50%**



Amdahl's Law [1967]

- If $q \leq 1$ is the fraction of WORK in a parallel program that must be executed sequentially for a given input size N, then the best speedup that can be obtained for that program is Speedup(N,P) $\leq 1/q$.
- Observation follows directly from critical path length lower bound on parallel execution time
  - CPL >= q * T(N,1)
  - T(N,P) >= q * T(N,1)
  - Speedup(N,P) = T(N,1)/T(N,P) <= 1/q
- This upper bound on speedup simplistically assumes that work in program can be divided into sequential and parallel portions
  - Sequential portion of WORK = q
    - also denoted as f_S (fraction of sequential work)
  - Parallel portion of WORK = 1-q
    - also denoted as f_p (fraction of parallel work)
- Computation graph is more general and takes dependences into account

Illustration of Amdahl's Law: Best Case Speedup as function of Parallel Portion

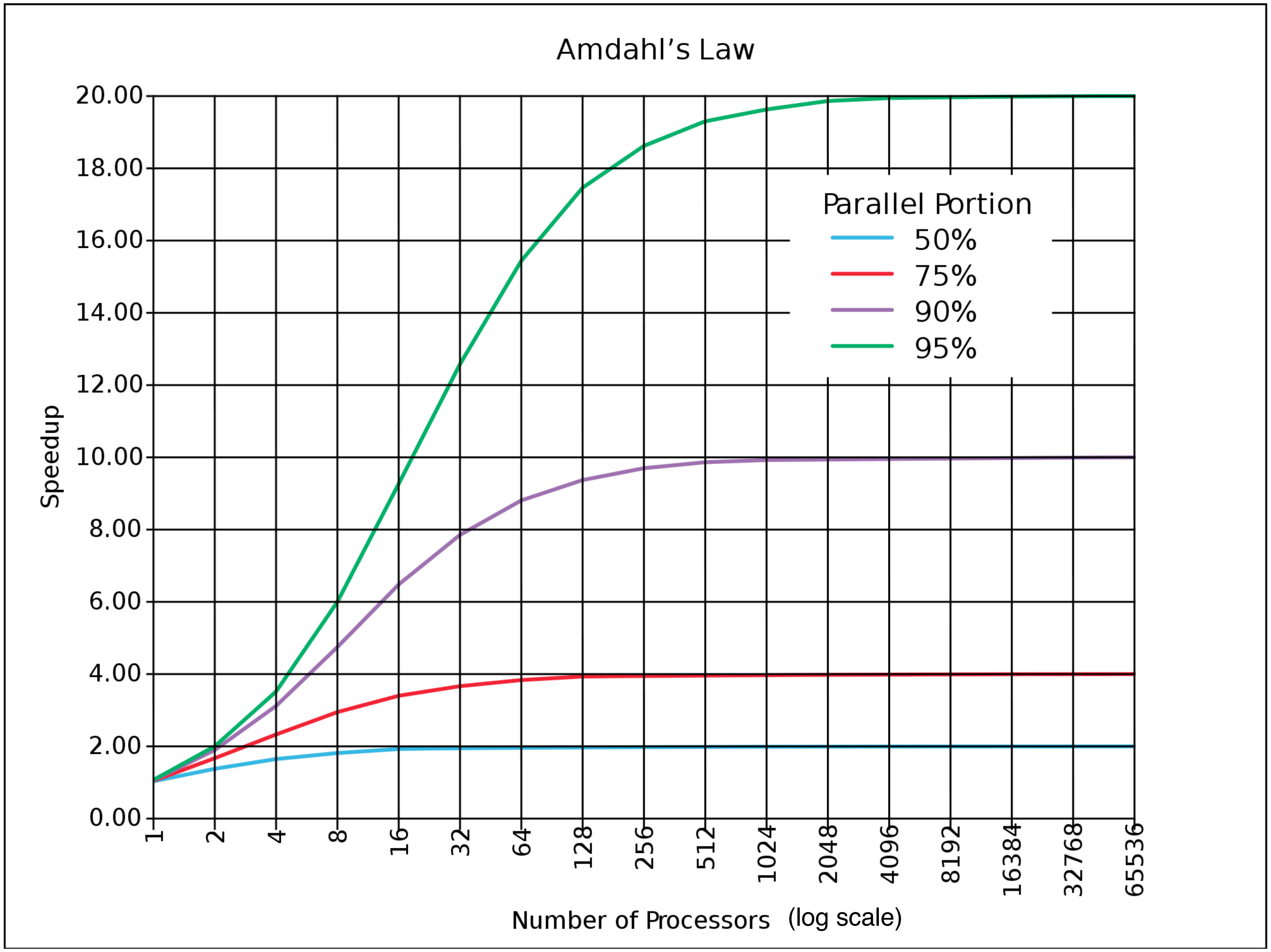


Figure source: http://en.wikipedia.org/wiki/Amdahl's law

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HJ Abstract Performance Metrics (Recap)

- **Basic Idea**
  - Count operations of interest, as in big-O analysis
  - Abstraction ignores overheads that occur on real systems

- **Calls to perf.addLocalOps()**
  - Programmer inserts calls of the form, perf.addLocalOps(N), within a step to indicate abstraction execution of N application-specific abstract operations
    - e.g., floating-point ops, stencil ops, data structure ops
  - Multiple calls add to the execution time of the step

- **Enabled by selecting "Show Abstract Execution Metrics" in DrHJ compiler options (or -perf=true runtime option)**
  - If an HJ program is executed with this option, abstract metrics are printed at end of program execution with WORK(G), CPL(G), Ideal Speedup = WORK(G)/ CPL(G)

Where should perf.addLocalOps() calls be placed?

- Answer: It depends. In HW2, we asked you to count each call to combine() as 1 unit, but here's the general idea …
- We'll say that a cost function Cost(n) is "order $f(n)$", or simply "$O(f(n))$" (read "Big-O of $f(n)$") if
  - Cost-X(n) < factor * $f(n)$, for sufficiently large n, for some constant factor
- Examples:
  - Cost-A(n) = $2 * n^3 + n^2 + 1$ Cost-A is $O(n^3)$
  - Cost-B(n) = $3 * n^2 + 10$ Cost-B is $O(n^2)$
  - Cost-C(n) = 2^n Cost-C is $O(2^n)$

Famous "Complexity Classes"

- $O(1)$ constant-time (head, tail)
- $O(\log n)$ logarithmic (binary search)
- $O(n)$ linear (vector multiplication)
- $O(n * \log n)$ "n logn" (sorting)
- $O(n^2)$ quadratic (matrix addition)
- $O(n^3)$ cubic (matrix multiplication)
- $n^{O(1)}$ polynomial (...many! ...)
- $2^{O(n)}$ exponential (guess password)

Where should perf.addLocalOps() calls be placed?

- Focus on key metric of interest in your algorithm
- Don't count operations that are incidental to your algorithm
  - They can be important implementation considerations, but may not contribute to understanding your algorithm
- Since big-O analysis does not care about differences within a constant factor, you can just use a unit cost as a stand-in for a constant number of operations