COMP 322: Fundamentals of Parallel Programming

Lecture 10: Forasync chunking, Parallel Prefix Sum algorithm

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https://wiki.rice.edu/confluence/display/PARPROG/COMP322

Acknowledgments for Today's Lecture

- Prof. Kathy Yelick, UC Berkeley, CS 194 Lecture, Fall 2007
  - http://www.cs.berkeley.edu/~yelick/cs194f07/lectures/lect09-dataparallel.pdf

Goals for Today's Lecture

- <u>Chunking of forasync loops (contd)</u>
- Parallel prefix algorithm

Summary of forasync statement

```
forasync (point [i1] : [lo1:hi1]) <body>

forasync (point [i1,i2] : [lo1:hi1,lo2:hi2]) <body>

forasync (point [i1,i2,i3] : [lo1:hi1,lo2:hi2,lo3:hi3]) <body>

. . .
```

- forasync statement creates multiple async child tasks, one per iteration of the forasync
  - all child tasks can execute <body> in parallel
  - child tasks are distinguished by index "points" ([i1], [i1,i2], …)
- <body> can read local variables from parent (copy-in semantics like async)
- forasync needs a finish for termination, just like regular async tasks
  - Later, we will learn about replacing "finish forasync" by "forall"

Chunking of 1-D forasync loops

```
// BEFORE CHUNKING:
// Original forasync loop iterates over region R = [0:n-1]
forasync (point [i] : [0:n-1]) <body>
```

forasync								
	i=0	i=1	i=2	i=3	i=4	i=5	i=6	i=7

```
// AFTER PARTITIONING INTO Ci CHUNKS:
// Outer forasync-ii loop iterates over Ci chunks with
// point [ii] in region chunks([0:n-1],[Ci]).
// Inner for-i loop iterates over getChunk([0:n-1],[Ci],[ii])
forasync (point [ii] : [0:Ci-1])
  for (point [i] : getChunk([0:n-1],[Ci],[ii]))
```

forasync	ii = 0		ii = 1		ii = 2		ii = 3	
for	i=0	i=1	i=2	i=3	i=4	i=5	i=6	i=7

getChunk() method for 1-D regions

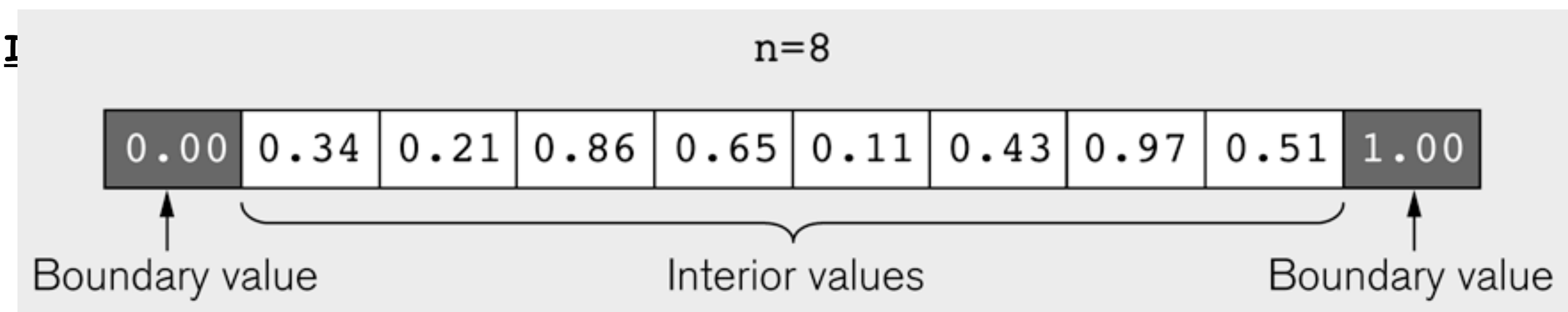
```
/*
 * @param r = region for original loop
 * @param c = tuple containing # chunks per dimension
 * @param p = current iteration for outer loop
 * @return region of iterations for chunk p
 */
static region getChunk(region r, point c, point p) {
  region retVal;
  // Set retVal = region for chunk p if region r is partitioned
  // according to number of chunks specified in c
  . . .
  return retVal
}
```

Implementing getChunk() method for 1-D regions

```
static region getChunk(region r, point c, point p) {
    // Assume that r, c, p all are 1D (rank = 1)
    int rLo = r.rank(0).low(); int rHi = r.rank(0).high();
    if (rLo > rHi) return [0:-1]; // Empty region
    int c0 = c.get(0); assert(c0>0); // numChunks must be > 0
    int ii = p.get(0); assert(0<=ii && ii<c0);
    // ii must be in the range [0:c0-1]
    int chunkSize = ceilDiv(rHi-rLo+1, c0);
    int myLo = rLo + ii*chunkSize;
    int myHi = Math.min(rHi, rLo + (ii+1)*chunkSize - 1);
    return [myLo:myHi];
}
```

One-Dimensional Iterative Averaging Example

- Initialize a one-dimensional array of (n+2) double's with boundary conditions, myVal[0] = 0 and myVal[n+1] = 1.
- In each iteration, each interior element myVal[i] in 1..n is replaced by the average of its left and right neighbors.
  - Two separate arrays are used in each iteration, one for old values and the other for the new values
- After a sufficient number of iterations, we expect each element of the array to converge to myVal[i] = i/(n+1)
  - In this case, myVal[i] = (myVal[i-1]+myVal[i+1])/2, for all i in 1..n



HJ code for One-Dimensional Iterative Averaging using nested for-finish-forasync structure

```
for (point [iter] : [0:iterations-1]) {
  // Compute MyNew as function of input array MyVal
  finish forasync (point [j] : [1:n]) { // Create n tasks
     myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
  } // finish forasync
  temp=myVal; myVal=myNew; myNew=temp;// Swap myVal & myNew;
  // myNew becomes input array for next iteration
} // for
```

- How many tasks does this version create?
- This is an idealized version with no “chunking” of forasync iterations

Example: HJ code for One-Dimensional Iterative Averaging with chunked for-finish-forasync-for

```
for (point [iter] : [0:iterations-1]) {
  // Compute MyNew as function of input array MyVal
  int Cj = ...; // Set to desired number of chunks
  finish forasync (point [jj]:[0:Cj-1]) {
     for (point [j]:getChunk([1:n],[Cj],[jj]))
       myNew[j] = (myVal[j-1] + myVal[j+1])/2.0;
  } // finish forasync
  temp=myVal; myVal=myNew; myNew=temp;// Swap myVal & myNew;
  // myNew becomes input array for next iteration
} // for
```

- How many tasks does this chunked version create?

Goals for Today's Lecture

- Chunking of forasync loops (contd)
- <u>Parallel prefix algorithm</u>
- Parallel quicksort algorithm

Prefix Sum (Scan) Problem Statement

Given input array A, compute output array X as follows

$$X[i] = \sum_{0 \le j \le i} A[j]$$

Observations:

- Mathematical specification may suggest that $O(n^2)$ additions are required since each X[i] is the sum of i terms
- However, it is easy to see that prefix sums can be computed sequentially in O(n) time

```
// Copy input array A into output array X
X = new int[A.length]; System.arraycopy(A,0,X,0,A.length);
// Update array X with prefix sums
for (int i=1 ; i < X.length ; i++ ) X[i] += X[i-1];
```

An Inefficient Parallel Prefix Sum program

```
finish {
  for (int i=0 ; i < X.length ; i++ )
    // invoke computeSum() function from Lecture 7
    // (see ArraySum2.hj)
    async X[i] = computeSum(A, 0, i);
}
```

Observations:

- Critical path length, CPL = O(log n)
- Total number of operations, WORK = $O(n^2)$

➔ With P = O(n) processors, the best execution time that you can achieve is T_P = max(CPL, WORK/P) = O(n), which is no better than sequential!

How can we do better?

Observation: each prefix sum can be decomposed into reusable terms of power-of-2-size e.g.

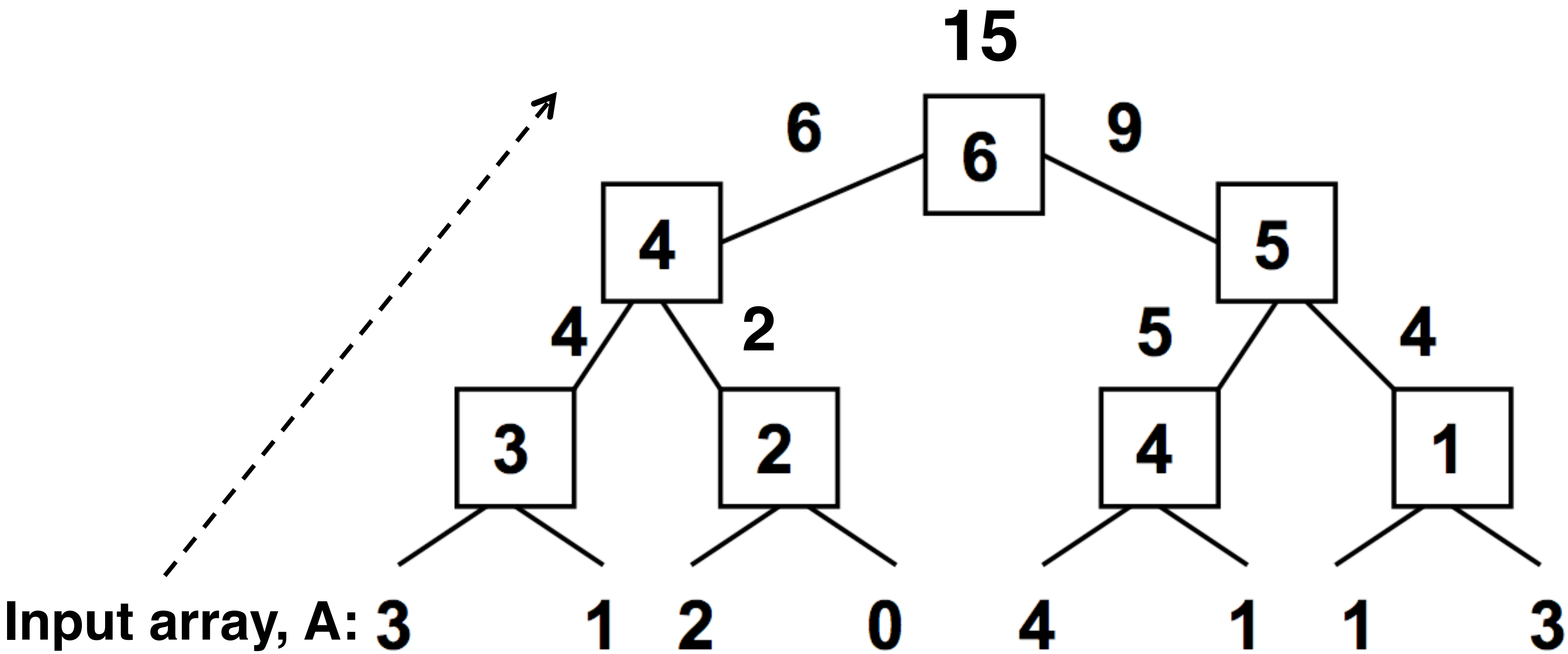
$$
\begin{aligned}
X[6] &= A[0]+A[1]+A[2]+A[3]+A[4]+A[5]+A[6] \\
&= (A[0]+A[1]+A[2]+A[3])+(A[4]+A[5])+A[6]
\end{aligned}
$$

Approach:

- Combine reduction tree idea from Parallel Array Sum with partial sum idea from Sequential Prefix Sum
- Use an "upward sweep" to perform parallel reduction, while storing partial sum terms in tree nodes
- Use a "downward sweep" to compute prefix sums while reusing partial sum terms stored in upward sweep

Parallel Prefix Sum: Upward Sweep

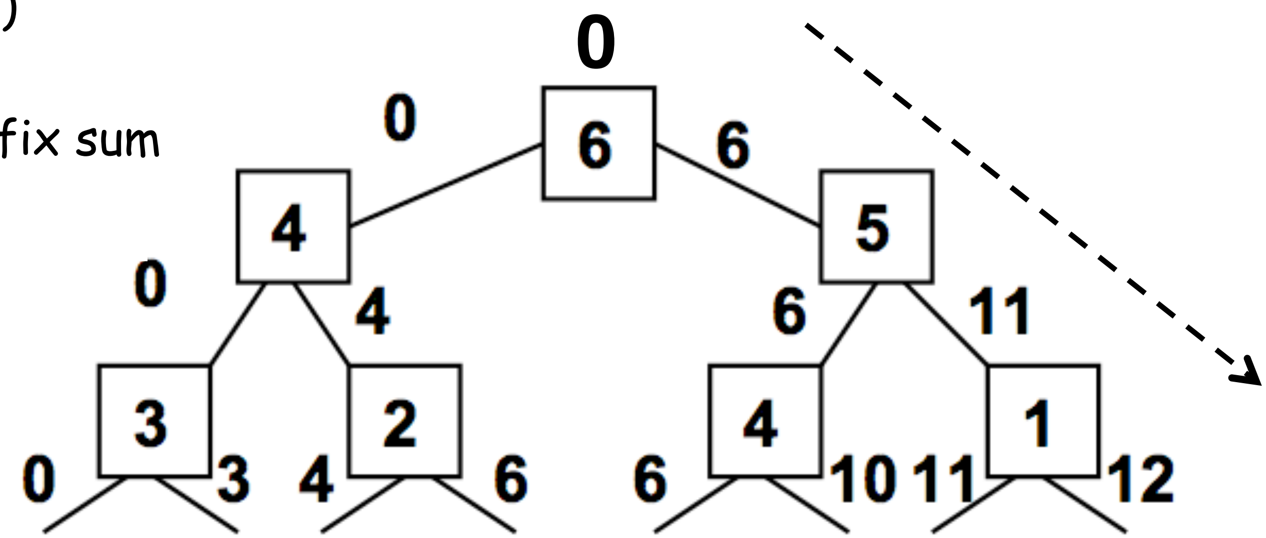
1. Receive values from children
2. Store left value in box (will contribute to prefix sum for right subtree in downward sweep)
3. Send left+right value to parent



Parallel Prefix Sum: Downward Sweep

1. Receive value from parent (root receives 0)
2. Send parent's value to left child (prefix sum for elements to left of left child's subtree)
3. Send parent+box value to right child (prefix sum for elements to left of right child's subtree)

Add A[i] to get final prefix sum



Prefix sum excluding A[i]

+ A[i]: 3 1 2 0 4 1 1 3

= Prefix sum including A[i]: 3 4 6 6 10 11 12 15

Summary of Parallel Prefix Sum Algorithm

- Critical path length, CPL = O(log n)
- Total number of add operations, WORK = O(n)
- Optimal algorithm for P = O(n/log n) processors
  - Adding more processors does not help
- Like Array Sum Reduction, Parallel Prefix Sum has several applications that go beyond computing the sum of array elements

Example Applications of Parallel Prefix Algorithm

- <u>Prefix Max with Index of First Occurrence</u>: given an input array A, output an array X of objects such that X[i].max is the maximum of elements A[0…i] and X[i].index contains the index of the first occurrence of X[i].max in A[0…i]
- <u>Filter and Packing of Strings</u>: given an input array A identify elements that satisfy some desired property (e.g., uppercase), and pack them in a new output array. (First create a 0/1 array for elements that satisfy the property, and then compute prefix sums to identify locations of elements to be packed.)